Weak Signal Detection Research Based on Duffing Oscillator Used for Downhole Communication

Xuanchao Liu
School of Electronic Engineering, Xi’an Shiyou University, Xi’an, China
xcliu@xsyu.edu.cn

Xiaolong Liu
School of Electronic Engineering, Xi’an Shiyou University, Xi’an, China
Lxlong_006@126.com

Abstract—Weak signal detection is very important in the downhole acoustic telemetry system. This paper introduces the Duffing oscillator weak signal detection method for the downhole acoustic telemetry systems. First, by solving the Duffing equation, analyzed the dynamics characteristic of Duffing oscillator and weak signal detection principle; and then on this basis, built Duffing oscillator circuit based on the Duffing equation, by circuit simulation to study the Duffing circuit sensitive to different initial parameters, conducted a detailed analysis for how the different parameters impacted the system statuses and the low-pass filter with simplicity and availability was proposed for signal demodulation. The results show that the method could effectively detect the weak changes of input signal and suppress strong noise; it is feasible, advanced and practical used for downhole acoustic telemetry system.

Index Terms—weak signal detection, Duffing oscillator, Duffing chaos circuit simulation, downhole communication, downhole acoustic telemetry system

I. INTRODUCTION

In the oil field exploration and development process, which needs timely to obtain downhole information, downhole acoustic telemetry system (D-ATS) is an effective wireless transmission way to achieve downhole information [1] [2]. It based on the theory of acoustic can be transmitted along the steel medium (oil pipe or drill pipe) to complete information transmission. The main realization of ways including: downhole high-power and low-frequency sine acoustic wave signal transmitters, transmission medium and ground weak acoustic receiver. The ground weak signal detection is very important part of it. Because of poor working conditions, high temperature, little space and only the case of battery-powered, therefore, the signal intensity of downhole acoustic generator is subject to certain restrictions. When it is transmitting, not only subjected to natural absorption attenuation, but a variety of random noise disturber. Finally, we get a weak acoustic signal which drowned in the noise at the ground. Whether can accurately detect the useful signal is the key to success or failure of information transmission. Thus, the study of weak signal detection technology is of great significance.

Weak signal detection is an emerging technical discipline, mainly researches detection principle and method of the weak signal buried in the noise, which is a kind of comprehensive technology in the signal processing technology, mainly utilizes the modern electronics, the information theory and the physics method, to analyze reason and rule the noise producing, to study statistical nature and difference of the detecting signal and the noise; uses a series of signal processing method, achieves the goal that weak signal buried in the background noise is detected successfully.

Generally, weak signal detection mostly makes periodic signal as the main detecting signal. Fully makes use of the additivity of periodic signal and the randomness of noise to extract useful signal effectively, achieves the goal of signal detection. Because the sinusoidal signal has the simplicity and the easy determinism in the extraction of frequency characteristic and phase feature as well as in the signal processing aspect, specially its Single frequency characteristic makes it has unique advantage in the anti-interference processing aspect, makes sinusoidal signal become an important object of study in the weak signal detection domain.

The most common of several mature weak signal detection methods are mainly the following [3]:

A. Periodic signal relevant detection method

This method can pick-up the periodic signal drowned in random noise, it uses the feature of signal related in time. The way in achieving is divided into self-correlation detection method and the cross-correlation detection method. Self-correlation detection method uses the signal characteristics of periodic and noise characteristics of random, through the relevant operation to remove noise. Cross-correlation detection method is based on the frequency of known signal and needed a same frequency reference signal, which through reference signal and detecting signal make cross-correlation to remove noise. The detection effect of cross-correlation method is better.
than self-correlation method, but the detected signal frequency must be known.

B. Periodic signal sampling points integral method

This method uses the signal characteristics of periodic and noise characteristics of random, divides periodic signal into some time intervals, and then sample signal at these time intervals, and averaged the sample values which in the each cycle same position by analog circuits integral or through computer with digitally processing method. This method is aimed at enhancing useful signal and suppressing noise to achieve effective detection for signal.

C. Single frequency lock detection method

This method uses the signal’s single frequency or narrow-band cosine (or sine) characters to complete narrow band processing, and then making use of cross-correlation theory for cross-correlation processing of the signal under test and reference signal with the same frequency. At last, it can detect the useful signal buried in noise.

With continuous development of the integrated circuits and computer technology, the people still continue to explore various weak signal detection methods, for example, based on wavelet transform detection method, based on Chaos theory detection method, based on fuzzy mathematics detection method, based on artificial neural network detection method and so on.

In the above methods of weak signal detection, the traditional weak signal detection mainly uses the method of linear filtering and signal superimposition to extract the signal, but it is often unable to survey signal when the background noise is very strong and the detecting signal is very weak, can not meet the needs of weak signal detection requirements. In the new field of study. Although modern detection methods have obvious effect, strong adaptability, but most of them have complex structure, difficult to achieve and to promote. Therefore, the weak signal detection technology has a new breakthrough when the chaos theory is introduced in the signal detection domain successfully. The Chaos detection method is different from a variety of existing measuring methods, which is a new signal processing method.

Particularly, the method based on Duffing oscillator of chaos theory received considerable attention. This method has made a lot of research results. Study finds that this method is especially suited for downhole acoustic telemetry system with good detecting effect, simple structure, more easy achievement and other advantages. But, most studies are still at the stage of theory exploratory; those are involved with less physical implementation aspects [4][5]. In this paper about problems of Duffing oscillator simulation circuit and detection method were studied.

II. WEAK SIGNAL DETECTION PRINCIPLE BASED ON CHAOS THEORY

Signal detection based on chaos was developed from the 1990s; this method had high sensitivity and low detection threshold to weak signal, as well as characteristic of strong abatement ability to noise, and hopefully to reduce the cost of testing equipment, so became hot spot in the weak signal detection research areas as soon as it appeared, and once had presented many outstanding methods [6][7].

Chaos theory for the detection of weak signals derived from the discovery of chaos in nonlinear dynamic system. The research of chaos theory indicates that certain nonlinear chaos systems have the sensitivity to small signal and the immunity to noise under the controlled condition, utilizing this characteristic, takes detecting signal as the driving force of the chaos system, although the noise is intense, it has no influence on system mode's change; however once has specific signal, even if its amplitude is quite small, also will make the system change. The computer recognizes system modes by the method of image recognition or envelope abstraction and so on, and then judges whether signal is exist, thus achieve the goal of detecting weak signal from strong background noise. The extreme sensitivity of the chaotic dynamic system behavior for the initial parameters results in to study it on weak signal detection. Especially, because of clear physical meaning and convenient debugging of Duffing oscillator, it became the hotspot of weak signal detection research.

Duffing equation is a mathematical model which describes weak damping motion of Duffing oscillator (soft spring oscillator) [8][9]. It is a second order differential equation containing a cubic item. In the external excitation, the equation behavior would oscillate which can be expressed as the periodic motion and chaotic motion. The concrete expression is the following.

\[ \chi'' + k\chi' + \chi + \chi^3 = F(t) = \gamma \sin(t). \]  

\( \gamma \sin(t) \) for period driving force, k for damping ratio, \(-x(t) + x3(t)\) for nonlinear restoring force. When k is fixed (such as k=0.5), the system status would present the marvelous change with \( \gamma \) changing.

As \( \gamma \) is zero, let \(-x(t) + x3(t)=0\), we could achieved 3 stable points, \(x=0, x=-1, x=1\). According to original status difference, it would fall on the different position, showed as Fig 1~Fig 3.

![Image](image_url)

**Figure 1. Phase orbit in [ x, x’]=[ 0, 0 ]**
As $\gamma$ changes from small to big, the phase diagram of the system will go through different statuses from homoclinic trajectory, bifurcation trajectory, chaotic trajectory (critical period status), large-scale periodic status changes. showed as Fig 4 ~Fig 9, especially, before and after the status where $\gamma$ changes to critical period status ($\gamma = \gamma_c$ ), the system status changes would be dramatic, when $\gamma$ is less than $\gamma_c$, the system was in chaotic status, as show in Fig 8. When $\gamma$ is greater than $\gamma_c$, the system would be in periodic status, as show in Fig 9.
The process of these dramatic changes could be used to weak signal detection. Because if added a reference signal which with the same or similar frequency and phase as detecting signal and the amplitude equivalent or slightly smaller than the point $\gamma$. When the detecting weak signal is injected in the chaotic system, it may lead the dynamics of chaotic systems to change from chaotic status to periodic status. When the system was in periodic status, the phase trajectory would be landed the largest-scale track, and its behavior is the period signal in the time domain, and it was very different from the chaotic status. By distinguishing between two different statuses, we could achieve the purpose of weak signal detection. In other words, the thinking of Duffing chaotic oscillator for weak signal detection and its realization is utilized initial sensitivity characteristics of the system.

III. THE CIRCUIT AND SIMULATION FOR DUFFING OSCILLATOR

We had known the theory of Duffing oscillator by calculating equation in theoretical way, but it did not meet the needs of real-time detection signal, thus, we have to continue to research its implementation circuit and effective detection method.

A. simulation circuit implementation on the $\omega=1 \ k=0.5$ mode

In order to convert Duffing oscillator to circuit implementation (1) converted the following form:

$$\chi'' = -\gamma \sin(t + \pi) - 0.5 \chi - (\chi) - \chi^3 .$$  (2)

According to (2), we could build Duffing oscillator simulation circuit through sinusoidal voltage source, analog operational amplifier, analog multiplier, resistors and capacitors in Proteus simulation software [10]. As showing in Fig 10, all components are the ideal virtual electronic components.

According to the basic feature of an ideal operational amplifier, we could easily derive the circuit equation of Duffing chaotic system from Fig 10.

$$U_1 = (-R_1C_1)U'_2$$

$$U_2 = (-R_4C_2)U'_1$$

$$U_3 = (-\frac{R_8}{R_7})(-R_6C_2)U'_1$$

$$U_1 = -\frac{R_5}{R_4}V_1 + \frac{R_5}{R_4}U_6 + \frac{R_5}{R_7}(-\frac{R_{11}}{R_{10}}U_1) + \frac{R_5}{R_7}U'_3$$

$$(-R_6C_2)(-R_6C_2)U'_1 = -\frac{R_5}{R_4}V_1 + \frac{R_5}{R_4}U_6 + \frac{R_5}{R_2}(-\frac{R_{11}}{R_{10}}U_1) + \frac{R_5}{R_7}U'_3$$  (3)

After comparing (3) with (2), assuming: $R_1=R_2=R_3=R_5=R_7=R_8=R_{10}=R_{11}=10K\Omega$, $R_4=20K\Omega$, $R_6=R_9=1M\Omega$, $C_1=C_2=1\mu F$, the phase of sinusoidal voltage source is $\pi$, so (3) became:

$$U'_3 = V_1 - 0.5U'_2 + U_3 - U'_2.$$  (4)

By comparing, in this simulation circuit: $U'_3$ representative of the displacement, $U_3$ representative of the speed.

Initial parameter sensitivity test is showed below:
In order to verify the Duffing oscillator effectiveness of simulation circuits, mainly about initial parameter sensitivity, firstly, we should adjust the amplitude of input sinusoidal signal to find the point $\gamma_c$, and then, inputted a different range and frequency signals in the before and after $\gamma_c$, we could get the waveform as showed from Fig 11 to Fig 14. The simulation circuit also could utilize the feature of initial parameter sensitivity to detect weak signal, comparing with the numerical results, we could visually and in real-time see the process of dynamic changes.

In order to facilitate Duffing oscillator converting circuit, we changed (5) as the following form.

$$\frac{1}{\omega^2}\dot{x}'(\omega\tau) + \frac{1}{\omega}\dot{x}(\omega\tau) - x(\omega\tau) + x'(\omega\tau) = \gamma \sin(\omega\tau).$$

(5)

After comparing (6) and (5), we could suppose following form in the simulation circuit.

$$U_1 = \left(-\frac{R_6C_1}{\omega}\right)U'_3$$

$$U_2 = \left(-\frac{R_9C_2}{\omega}\right)U'_3$$

Under general circumstances, if $\tau = R6C1=R9C2=1$, we could get response equation in different frequency.

$$\frac{1}{\omega^2}U''_3 = -V_1 - \frac{0.5}{\omega}U'_3 + U_3 - U_3^3$$

When the parameters of practical circuit was selected, the original value of $\tau = 1$ must be reduced the $\omega$ times, in the case of low frequency, could reduce the value of R6 and R9 in $\omega$ times.

Now, inputted signal frequency of 600Hz as an example:

In the $\omega = 1200\pi$, k=0.5 status simulation circuit realization, made sure R6 and R9 for 0.2653KΩ in the Fig 3, and then changing input signal frequency for 600Hz for testing. The same way as before, we could get the results as shown in table I.
TABLE I.
THE OUTPUT STATUSES OF DIFFERENT SIGNAL FREQUENCY
AND AMPLITUDE

<table>
<thead>
<tr>
<th>Input Signal Frequency (Hz)</th>
<th>Input Signal Amplitude (v)</th>
<th>Circuit Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>0. 867</td>
<td>Chaotic</td>
</tr>
<tr>
<td>600</td>
<td>0. 868</td>
<td>Periodic</td>
</tr>
<tr>
<td>610</td>
<td>0. 868</td>
<td>Chaotic</td>
</tr>
<tr>
<td>580</td>
<td>0. 868</td>
<td>Periodic</td>
</tr>
<tr>
<td>580</td>
<td>0. 768</td>
<td>Chaotic</td>
</tr>
</tbody>
</table>

IV. EXPERIMENTAL RESULTS AND ANALYSIS

Simulation results verified the correctness of the circuit design. Utilizing simulation software Proteus to design diagram of Duffing oscillator system, studied the feature of Duffing oscillator through circuit simulation, not only avoids the complexity of the programming operation, but also achieves real-time dynamic detection of the signal. Through a simple parameter setting, we could study Duffing circuit adaptability for different frequencies response. It has advantages of simple circuit and easy to implement. From table 1 we could see that the circuit has the sensitive for input signal in the case of input signal frequency for 600Hz and the amplitude for 0.8674V. If pre-add a reference signal with the amplitude for 0.8670V or a little small, when the weak signal with the same or similar frequency and phase as this reference signal is injected, it could lead to the dynamic behavior of chaotic system from chaotic status to large-scale periodic status. Comparing with Fig 12 and Fig 14, we could find that: when the system was in large-scale periodic status, the time interval of output signal crossing zero point is invariable; however, when the system was in chaotic status, the time interval of output signal crossing zero point is variable. It also showed which included some lower frequency components. We could judge the system is in a status of large-scale periodic status or a chaotic status by comparing the output signal zero-crossing time intervals and low-pass filter. But, how to make sure reference signal phase the same or similar as the detecting signal and implement detection circuit, needed much more research on this.

V. THE INFLUENCE ON SIGNAL RECEPTION
WITH CHANGES OF THE FREQUENCY

We discussed the basic principle of weak signal detection which based on the duffing oscillator under the ideal situation above. The research of the Duffing oscillator indicated that the system would generate intermittent chaos phenomenon of the chaotic status and the periodic status if there was an extremely small frequency difference between detecting signal and the reference driving force frequency, shows as Fig 15.

The intermittent chaos phenomenon which appears alternately order and disorder in the time is a special dynamics phenomenon with growth and decline of the vector sum of the cycle driving force amplitude and the weak signal amplitude nearby the marginal value \( \gamma_c \).

According to the following Duffing equation, we would explain the principle of the intermittent chaos phenomenon theoretically.

Rewrote the equation (1) as followed:

\[
\chi'' + \kappa \chi' - \chi + \chi^3 = F(t) = F_r(t) + F_i(t) .
\]

\[
F(t) = \gamma_r \sin(t) + a \sin((1 + \Delta \omega)t + \varphi)
\]

Among which, \( \gamma_r \) is amplitude of the cycle reference driving force (be equal to or slightly be smaller than marginal value \( \gamma_c \)), \( a \sin((1 + \Delta \omega)t + \varphi) \) represents the external detecting signal, \( a \) is amplitude of the detecting signal, \( \varphi \) is its initial phase. From formula (8), we could see that \( \gamma \) is the vector sum of the cycle reference driving force amplitude and the weak signal amplitude, because there was a frequency difference \( \Delta \omega \), the total driving force amplitude \( \gamma(t) \) would change between \( \gamma_r + a \) and \( \gamma_r - a \) constantly.

\[
\gamma_r - a \leq \gamma(t) \leq \gamma_r + a
\]

We also could express the total driving force as the form of radius vector synthesis, shows as Fig 16. We could clearly see the rule of the growth and decline of the total driving force.

We could consider that the driving force was static, so the external tested signal’s radius vector would rotate around itself in \( \Delta \omega \) frequency (clockwise or counterclockwise) slowly. When they were in the same
direction, the result of the synthetic of radius vector would cause the total driving force amplitude greater than the threshold $\gamma_c$, the system would change from chaotic status to periodic status; When they were in the opposite direction, the result of the synthetic of radius vector would cause the total driving force amplitude less than the threshold $\gamma_c$, the system would change to the previous chaotic status. So the system status would show intermittent chaos phenomenon between chaotic and periodic sometimes.

We could transform the total driving force $Y(t)$ in mathematical way:

$$ F(t) = \gamma_c \sin(t) + a \sin((1 + \Delta \omega)t + \varphi) $$

$$ = \gamma(t) \sin(t + \lambda(t)) $$

Among which:

$$ \gamma(t) = \sqrt{\gamma_r^2 + 2\gamma_c a \cos(\Delta \omega \cdot t + \varphi) + a^2} . \quad (9) $$

$$ \lambda(t) = \text{artc} \frac{a \sin(\Delta \omega \cdot t + \varphi)}{\gamma_r + a \cos(\Delta \omega \cdot t + \varphi)} . \quad (10) $$

In the case of small frequency difference, according to the formula (9), we could see that the system’s status will come into periodic status when $\gamma(t)$ is greater than Chaotic threshold $\gamma_c$, and sometimes the system’s status would come into chaotic status when $\gamma(t)$ is less than chaotic threshold $\gamma_c$, therefore the system’s status would come into specific intermittent chaos phenomenon. In the intermittent chaotic status, when $\Delta \omega$ is very small, the $\gamma(t)$ would change very slowly and much slower than the status transition. Generally, the status transition needed several cycles. However, the system had a lot of cycles to keep steady periodic status. So the system had a good responds to the slow changes of driving force. Therefore, the periodic status and chaotic status would change in cyclical way; the cycle T was showed as the formula (11).

$$ T = \frac{2 \pi}{\Delta \omega} . \quad (11) $$

Although there was variable in formula (10), because of $a << \gamma_c$, so $\lambda(t)$ was very small, its changes had small influence on the system, this could be ignored.

When $\Delta \omega$ is smaller, the intermittent chaos phenomenon is easy identification. Not only it could determine whether a detecting signal was exist, but also it could get frequency difference between external signal and reference driving force by formula (11) according to the tested the cycle T of system intermittent Chaotic status. From this way, we also could achieve measurement of weak signal’s frequency. But the result showed that when $\Delta \omega$ was more, the occurrence speed of the status change was very quickly, the system was difficult to hold steady periodic status for a long time. So it was hard to identify the rule of the intermittent chaos phenomenon. It meant that the Duffing oscillator phase has strong immune ability to periodic interference with large frequency difference.

VI. THE INFLUENCE ON SIGNAL RECEPTION WITH CHANGES OF THE PHASE

Regarding formula (9), if $\Delta \omega = 0$, namely expresses that the detecting signal’s frequency is just same as the reference frequency of reference driving force. Shows as formula (12).

$$ \gamma(t) = \sqrt{\gamma_r^2 + 2\gamma_c a \cos \varphi + a^2} . \quad (12) $$

In this case, when the phase difference between the external detecting signal and the reference driving force satisfied the following formula (13):

$$ \frac{\sqrt{\gamma_r^2 + 2\gamma_c a \cos \varphi + a^2} \geq \lambda_c}{} . \quad (13) $$

Namely:

$$ |\varphi| \leq \arccos \left( \frac{\gamma_r^2 - \gamma_c^2 - a^2}{2\gamma_c a} \right) . \quad (14) $$

That was to say the phase difference between the external detecting signal and the reference driving force was in the range of formula (14), only then, the system had the possibility to jump from the chaotic status to the large-scale periodic status. In addition, the system always stayed at the chaotic status.

From formula (14), we could discover that, in the case of other condition invariable, the bigger the detecting signal amplitude was, the bigger the permission phase angle was, the lower the request to testing equipment was. The experimental confirmation result was as followed:

$$ f = 600Hz $$

$$ \gamma_c = 0.8674 $$

$$ \gamma_r = 0.8670 $$

$$ a = 0.0010 $$

$$ |\varphi| \leq 0.369 \pi $$

The experimental result was consistent with the result of calculation.
VII. THE SIGNAL RECEIVING

The signal’s receiving mainly includes establishment and judgment of the large-scale periodic status.

A. Large-scale periodic status establishment

Towards the situation that there was small frequency difference between the detecting signal’s frequency and the reference signal’s frequency, according to formula (9), we could know that the large-scale periodic status must appear periodically. But towards the situation that received signal’s frequency is same as reference signal’s frequency, according to analysis of formula (12), the large-scale periodic status would appear only under given phase. The results indicated that we could solve this problem by the phase shifting circuit array. To confirm its effectiveness, we designed the phase shifting circuit in Fig 10 and then it was tested.

Suppose: \[ F_r = 0.827 \sin(1200\pi t) \]

When: \[ F_i = 0.001\sin(1200\pi t + \frac{\pi}{3}) \]

Or \[ F_i = 0.001\sin(1200\pi t - \frac{5\pi}{6}) \]

When: \[-\frac{5\pi}{6} \leq \phi \leq \frac{\pi}{3}\]

all of these could detect signal effectively, and proved the feasibility of the phase shift method.

B. Large-scale periodic status judgment

The traditional large-scale periodic status’s judgment mainly makes use of phase diagram change and so on, its structure is complex, and difficult to realize. The research results indicated that when the system turns up the large-scale periodic status, \(x\) or \(x'\) is single frequent signal and when the system turns up the chaotic status, \(x\) or \(x'\) is signal including many low frequency components. Utilized this characteristic, we designed a kind of low-pass filter circuit, showed in Fig 10, its structure is simple, easy to realize, has solved this problem effectively. The judgment result shows as Fig 19–20.

According to the difference of established modes of the large-scale periodic status, the detection modes could be divided into the same frequency detection mode (large-scale periodic status expressing 1 and constant chaos status expressing 0) and the alien frequency detection mode (intermittent chaos status express 1 and constant chaos status expressing 0). Both could be used to the downhole acoustic telemetry system. Although the same frequency detection mode had high efficiency, is suitable for situation that transmission rate is high, but its structure was complex and the cost is high; However the structure of the alien frequency detection mode is simple, easy to realize, and the practical value is high.
VIII. CONCLUSION

The results show that, Duffing oscillator weak signal detection technology for downhole acoustic telemetry system is entirely feasible. Information on downhole acoustic telemetry system transmitted digitally encoded 2ASK modulation using acoustic technology, "1" with a certain length of time that single-frequency sine acoustic wave; "0" with a certain length of standby time. Therefore, as long as the weak sine acoustic wave signals were detected can be realized the effective data transmission. Through theoretical analysis and analog circuit simulation results show that, Duffing oscillator has a unique sensitivity for a single frequency sinusoidal signal buried in the noise and suppresses random noise at the same time. Specially, its structure with alien frequency receiving mode is simple, but it had good detecting effect, Just right for using this feature, by determining whether the validity of the single-frequency sinusoidal acoustic signals, downhole acoustic telemetry system can be realized.

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Xuanchao Liu was born in Xi’an of Shanxi province, China, on february 1960. Bachelor, graduated from Electronic Science and Technology University.

He works in Xi’an Shiyou university, researches and teaches in measurement technology and instruments disciplines.

Xiaolong Liu was born in Baoji of Shanxi province, China, on march 1981. Master, graduated from Xi’an Shiyou University.

He works in Xi’an Sitan Petroleum Instruments Co., researching field is precision instruments and machinery.