Wavelet Descriptor for Closed Curves Detection in Complex Background

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***Abstract*—In the problem of designated a well-defined wavelet describing the closed curve, it is critical that we define a unique starting point for the wavelet representatives of closed curve. In this paper, we design a one-dimensional periodized wavelet transformation for closed curves detection in complex background. The uniqueness property facilitates the quantitative analysis of the unique properties of the one-to-one mapping between the changes of one-dimensional discrete periodized wavelet transformation and the starting point of original sample close curve. We propose uniqueness wavelet descriptor (UWD) by using the unique properties of a new shape descriptor. Robustness of the UWD in complex background is analyzed. By enhancing local shape feature, some experiments show adaptive property of the UWD for ideal starting point determination. Our experiments show that the UWD can provide an optimal pattern classification in complex background.**

***Index Terms*—Discrete periodized wavelet transform, features extraction, wavelet descriptor, Complex Background**

I. INTRODUCTION

Transforming shape information into an array of numbers is a crucial step in shape boundary representation. It has been applied in many applications of pattern recognition (PR) and computer graphics [2]–[5]. These numbers may denote a string of shape features [6]–[10] or shape primitives [11]–[16]. Numerical shape features are usually preferred in statistical approaches, while shape primitives are useful for structural analysis. In this paper, we will concentrate on the problem of extracting numerical shape features. For PR applications, it is often required that these features possess the properties of invariance and uniqueness. For instance, the Fourier descriptors (FD's) and moments have been shown to meet this requirement [6]–[9]. Although the FD has been widely used as shape features, it suffers from the incapability of characterizing local variations of shapes.

Recently, based on applying the one–dimensional (1-D) discrete periodized wavelet transformation (DPWT) to the con- tour, a new multi-scale description method was proposed for boundary representation [17], [18]. The description, which uses normalized DPWT coefficients to characterize shapes, is called the wavelet descriptor (WD).

Manuscript received Oct 15, 2009; revised Nov 15, 2009; accepted Dec 25, 2009.

A merit of the WD is that it can provide a hierarchical PR system with global features in coarser resolution levels and more detailed local features in finer resolution levels. However, DPWT coefficients usually vary with selected starting points (in contrast with the FD's). In other words, the representation is not invariant. Consequently, it is not guaranteed that DPWT coefficients remain unchanged under shape rotation. To overcome this problem, Kashi et al. [18] presented a combined Fourier-wavelet approach to describe planar closed curves. This approach uses the FD technique to select the starting point of a shape contour. Based on the use of the magnitude function of the contour, Li and Kuo [19] proposed another simplified method for defining a unique start point of the contour. Although these methods provide a procedure to select a unique start point for computing DPWT coefficients, the resulting point is usually not optimized in some sense, e.g., in noise resistance.

By utilizing periodized wavelets [1] and orthonormal bases with dilation factor two, an inherent uniqueness property existing in the 1-D DPWT is investigated in this study. By combining the signs of the sums of the wavelet coefficients in all resolution levels to form a combination vector, it is found that there exists one-to-one mapping between the circularly shifted sequences of the originally sampled data and the combination vectors. This one-to-one mapping is called the uniqueness property of the 1-D DPWT. This uniqueness property means that for a no periodic finite 1-D signal function defined in $L^2([0,1])$, there exists only one set of 1-D DPWT coefficients corresponding to each starting point. Applying the uniqueness property to planar closed curve representation, a new shape descriptor called uniqueness wavelet descriptor (UWD) is proposed. The UWD has the important advantage of being resolution independent. For PR applications, the UWD provides an alternative to select the desirable start point which corresponds to the optimal shape features having the best robustness against input noise. However, for finding the desirable start point, a full search in each class (known pattern) is required because only the features of coarser levels are used.

In terms of Fourier representation, the uniqueness property is proven. As an extension to regular (or symmetric) shape representation, the regular shape is defined by 2^α copies of a no periodic part in $L^2([0,1])$, where $\alpha \in Z$. And the uniqueness property is generalized. The generalized theorem indicates that the uniqueness property exists in the nonperiodic part of the regular shape. In order to find the desirable UWD, a fast binary

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doi:10.4304/jcp.5.11.1723-1730

search algorithm is presented. Several experimental results of stability analysis, local characteristic enhancement, and an example are given to illustrate the use of the UWD for PR applications. This paper is a generalized work of [20] in which the special case is discovered. The uniqueness property is also valid in the case of using the biorthogonal basis.

The theoretical background of the 1-D DPWT is briefly reviewed in Section II. The inherent uniqueness property of the 1-D DPWT and the application for planar curve representation are described in Section III. By considering the periodicity of shape, the generalized uniqueness property is presented in Section IV. Several properties of the UWD and an example for pattern recognition are demonstrated in Section V. Finally, Section VI includes some discussions and the conclusions.

II. ONE-DIMENSIONAL DISCRETE PERIODIZED WAVELET TRANSFORM

For finite signals, it was shown that the periodized wavelets based on the compactly supported wavelets [1] can provide the inverse DPWT process with adequate octave band components for deriving the perfect reconstruction result. In this section, we will briefly review the theory of the 1-D DPWT. The periodized wavelets are expressed by the Fourier representation.

The periodized wavelets can be defined by the wavelets with reasonable decay. Let $\phi(x)$ denote a scaling function in the space $L^2(R)$, where R denotes the set of real numbers. The function $\phi(x)$ can be any smoothing function whose integral is equal to one and converges to zero at infinity. The set of functions

$$\{\phi_{j,n}(x) = 2^{-j/2}\phi(2^{-j}x-n);\ j,n\in Z\} \tag{1}$$

derived from dilations and translations constitute an orthonormal basis for $L^2(R)$. For fixed j, let V_j be the closed subspace with basis $\{\phi_{j,n}(x), n\in Z\}$ and referred to as the multiresolution approximation subspace. For a given scaling function $\phi(x)$, there exists a corresponding wavelet function $\psi(x)$ such that the set of functions

$$\{\psi_{j,n}(x) = 2^{-j/2}\psi(2^{-j}x-n);\ j,n\in Z\}$$

also forms an orthonormal basis for $L^2(R)$. For fixed j , let W_j be the closed subspace with basis $\{\phi_{j,n}(x), n\in Z\}$ and referred to also as the multiresolution approximation subspace. For the 1-D DPWT, the periodized scaling and wavelet functions are defined in $L^2(R)$ as

$$\widehat{\phi}_{j,n}(x) = \sum_{l\in Z}\phi_{j,n}(x+l) \tag{3}$$

$$\widehat{\psi}_{j,n}(x) = \sum_{l\in Z}\psi_{j,n}(x+l) \tag{4}$$

where $j\le 0$ and n=0,1,…,2^{-j}-1 . In the space $L^2([0,1])$, the subspaces $\widehat{V}_j$, for all $j\le 0$, satisfy the embedding property that … $\widehat{V}_{-2}\supset\widehat{V}_{-1}\supset\widehat{V}_0$. We have $\widehat{V}_{j-1}=\widehat{V}_j\oplus\widehat{W}_j$, where $\oplus$ denotes the direct sum. Both $\widehat{V}_j$ and $\widehat{W}_j$ are all finite-dimensional in the space $L^2([0,1])$. The set of function $\{\widehat{\psi}_{0,0}\}\bigcup\{\widehat{\phi}_{j,n};\ j\in Z^-, n=0,...,2^{-j}-1\}$ forms an orthonormal basis for $L^2([0,1])$, where Z^-={0,-1,-2,-3,…}.

By the embedding property, the periodized scaling and wavelet functions in can be expressed in $\widehat{V}_j$ terms of a linear combination of the bases in $\widehat{V}_{j-1}$ given by

$$\widehat{\phi}_{j,n}(x) = \sum_{m=0}^{N-1}\widehat{h}[(m-2n)_N]\widehat{\phi}_{j-1,m}(x) \tag{5}$$

$$\widehat{\psi}_{j,n}(x) = \sum_{m=0}^{N-1}\widehat{g}[(m-2n)_N]\widehat{\phi}_{j-1,m}(x) \tag{6}$$

Where $N=2^{-(j-1)}$,$(m-2n)_N$ denotes the residual of (m-2n) mod N and

$$\widehat{h}[(m-2n)_N] = \int_0^1\widehat{\phi}_{j,0}(t)\widehat{\phi}_{j-1,(m-2n)_N}(t)dt \tag{7}$$

$$\widehat{g}[(m-2n)_N] = \int_0^1\widehat{\psi}_{j,0}(t)\widehat{\phi}_{j-1,(m-2n)_N}(t)dt \tag{8}$$

Both of the two discrete-time filters $\widehat{h}[(m)_N]$ and $\widehat{g}[(m)_N]$ are periodic sequences with period $2^{-(j-1)}$. The two filters have the relationship of $\widehat{g}[(n)_N]=(-1)^n\widehat{h}[(-n+1)_N]$. From (5) and (6), the Fourier transforms of and are $\widehat{\phi}_{j,0}(t)$ and $\widehat{\psi}_{j,0}(t)$ are

$$\phi_j(k) = \int_0^1\widehat{\phi}_{j,0}(t)e^{-i2\pi kt}dt = 2^{1/2}G_0(k)\phi_j(\frac{k}{2}) \tag{9}$$

$$\psi_j(k) = \int_0^1\widehat{\psi}_{j,0}(t)e^{-i2\pi kt}dt = 2^{1/2}G_1(k)\phi_j(\frac{k}{2}) \tag{10}$$

Where $G_0(k)=\sum_{n=0}^{N-1}\widehat{h}[n]e^{-i2\pi k(n/N)}$ and $G_1(k)=\sum_{n=0}^{N-1}\widehat{h}[n](-1)^{(1-n)}e^{-i2\pi k((1-n)/N)}$. $G_0(k)$ is the discrete Fourier transform of $\widehat{h}[n]$. By the Parseval's relation, the two discrete-time filters can also be represented by

$$\widehat{h}[n] = 2^{1/2}\sum_{k=-\infty}^{\infty}\phi_j[k]e^{i2\pi k(n/N)}\phi_j^*[\frac{k}{2}] \tag{11}$$

$$\widehat{g}[n] = 2^{1/2}\sum_{k=-\infty}^{\infty}\phi_j[k]e^{i2\pi k((1-n)/N)}\phi_j^*[\frac{k}{2}]e^{in\pi} \tag{12}$$

Where $\phi_j^*[k/2]$ denotes the complex conjugate of $\phi_j[k/2]$.

From the theorem of multi resolution analysis [21], a finite signal function defined by $f(x)\in\widehat{V}_{j-1}$ can be approximately represented as

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$$f(x) \approx \sum_{m=0}^{2^{-(j-1)}-1} S_{j-1}[n]\hat{\phi}_{j,n}(x) = \sum_{n=0}^{2^{-j}-1} S_j[n]\hat{\phi}_{j,n}(x) + \sum_{n=0}^{2^{-j}-1} D_j[n]\hat{\psi}_{j,n}(x) \quad (13)$$

Note that the term $\sum_{n=0}^{2^{-j}-1} S_j[n]\hat{\phi}_{j,n}(x)$ can be further decomposed recursively. The two coefficients S_j[n]and D_j[n] correspond to the projections of f(x) onto the

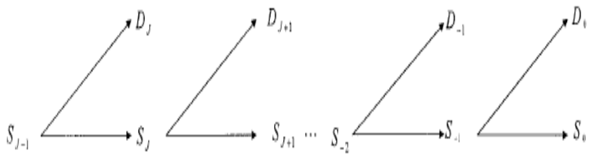

Fig. 1. Recursive pyramid decomposition process of the 1-D DPWT (J < 0).

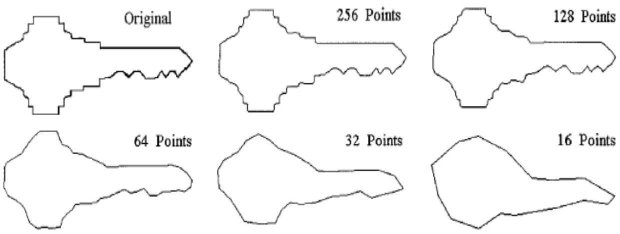

Fig. 2. Illustration of the multi-resolution wavelet coefficients S [n] of a key shape for j = -9; -8; -7 ,- 6; -5, and -4.

subspaces $\hat{V}_j$ and $\hat{W}_j$,respectively. That is

$$S_j[n] = \left\langle f(u), \hat{\phi}_{j,n}(u) \right\rangle_{[0,1]} = \sum_{m=0}^{N-1} \hat{h}[(m-2n)_N] S_{j-1}[m] \quad (14)$$

$$D_j[n] = \left\langle f(u), \hat{\psi}_{j,n}(u) \right\rangle_{[0,1]} = \sum_{m=0}^{N-1} \hat{g}[(m-2n)_N] S_{j-1}[m] \quad (15)$$

Equations (14) and (15) called the 1-D DPWT can be performed by a recursive pyramid algorithm shown in Fig. 1. Substituting (11) and (12) into (14) and (15) yields

$$S_j[n] = 2\sum_{k=-\infty}^{\infty} G_0[k] \,|\, \phi_j[\frac{k}{2}] \,|^2 \sum_{m=0}^{N-1} S_{j-1}[m] \, .e^{i2\pi k(m/N)} e^{-i2\pi k(2n/N)} \quad (16)$$

$$D_j[n] = 2\sum_{k=-\infty}^{\infty} G_0[k] \,|\, \phi_j[\frac{k}{2}] \,|^2 \sum_{m=0}^{N-1} S_{j-1}[m] \, .e^{-i2\pi k((m-2n)/N)} e^{im\pi} e^{-i2n\pi} \quad (17)$$

In (13), sine $\hat{V}_{j-1} = \hat{V}_j \oplus \hat{W}_j$, it implies

$$S_{j-1}[m] = \sum_{n=0}^{2^{-j}-1} \hat{h}[(m-2n)_N] S_j + \sum_{n=0}^{2^{-j}-1} \hat{g}[(m-2n)_N] D_j[n] \quad (18)$$

Where $0 \le m \le N-1$. Equation (18) is called the inverse 1-D DPWT.

III. Boundary Representation Using The Uniqueness Wavelet Descriptor

By tracing a two-dimensional (2-D) object along its boundary, the closed shape contour of the object can be represented by a 1-D periodic signal function. The uniqueness property of the 1-D DPWT as well as the proposed UWD for the periodic signal function representation is detailed described in this section.

A closed planar curve with parametric coordinates (x(s),y(s)) can be represented by the radius r(s) as

$$r(s) = \sqrt{(x(s)-x_C)^2 + (u(s)-y_C)^2} \quad (19)$$

where s is the normalized arc length and (x_C,y_C) denotes the centroid of the curve. From (13), the 1-D signal function $r(s) \in \hat{V}_J$ for J<0 can be approximately represented by

$$r(s) \approx \sum_{n=0}^{2^{-J}-1} S_J[n]\hat{\phi}_{J,n}(s) = r_T(s) + \sum_{j=J+K}^{J+1} d_j(s) \quad (20)$$

where K is defined to satisfy $J+1 \le J+K \le 0$. The two signal functions r_T(s) defined by $\sum_{n=0}^{2^{-(J+K)-1}} S_{J+K}[n]\hat{\phi}_{j+K,n}(s)$ and d_j(s) defined by $\sum_{n=0}^{2^{-j-1}} D_j[n]\hat{\psi}_{j,n}(s)$ denote the approximate representations of the curve in the coarsest resolution level and the detailed signal in the resolution level, respectively. For shape representation, the coefficients S_J[n] are normalized such that $\sum_{n=0}^{2^{-J}-1} S_J[n] = 1$ and the wavelet coefficients D_j[n] are referred to as the WD. To demonstrate the characteristics of the coefficients S_j[n] in different resolution levels, an example is illustrated in Fig. 2. Each key pattern in Fig. 2 is derived by directly applying the linear interpolation function instead of using the scaling function $\hat{\phi}_{j,n}$. Fig. 3 demonstrates the amplitudes of Sj[n]and Dj[n] in different resolution levels. In (19) and (20), it is obvious that the radius denotation is insensitive to the variances of translation, rotation and scaling. But the coefficients D_j[n] will vary if the contour is sampled from different starting points. Some methods using geometrical characteristics like the principal axis and specific points [5], [10], [19], as well as the Fourier descriptor technique [18] can be used to derive a unique start point. However, these approaches usually do not guarantee that the selected starting point provides the optimal DPWT coefficients which have strong robustness against input noise in practical PR applications. Alternatively, the problem of defining a unique start point can be effectively solved by applying the uniqueness property of the 1-D DPWT.

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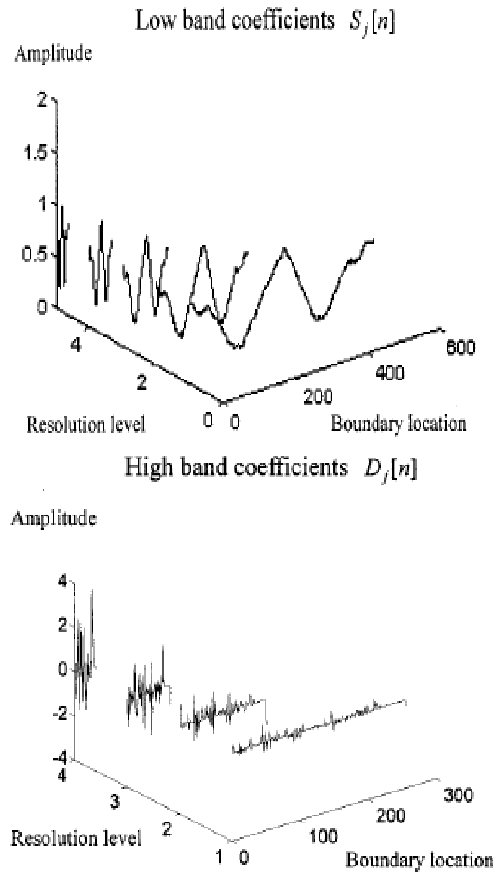

Fig. 3. Three-dimensional exposure of the multi resolution coefficients $S_j[n]$ and $D_j[n]$ of the key shape shown in Fig. 2.

To find the desirable starting point o a 2-D closed curve, a fast binary-search algorithm is described as follows.

Step 1) Assign initial values, i.e., set $\lambda = 0, j = J-1$ and assign a value to the stop_level of the regular shape.

Step 2) Define the desired combination vector $\overline{C_k}$.

Step 3) Decompose the sequence $S^{(\lambda)}_{|J-1|}$ until the stop_level.

Step 4) Increase j and check if the sign of Q_j does not meet the definition of $c_j \in \overline{C}_K$ or j=stop_level. If the condition is satisfied, go to the next step. Else go to Step 4.

Step 5) Let $\lambda = 2^{j-J}$ and go to Step 3.

IV. EXPERIMENTAL RESULTS

In this section, the starting-point shift invariance of the UWD is explored. In order to illustrate the adaptability property of starting point selection, several experiments of local shape characteristic enhancement are also demonstrated. It will be shown that it is able to obtain the robust WD features for PR applications.

A. Starting Point Shift Invariance of the UWD

For planar closed curve recognition, invariance of starting-point shift is an important property usually required for shape features. The UWD can provide desirable features having this invariance property. After choosing a desired combination vector (CV), a unique representation of the curve is determinable in terms of the corresponding UWD. To illustrate the starting-point shift invariance of the UWD, a key pattern with four different rotation angles are shown in Fig. 4. Their results are compared with the original case (i.e., with rotation angle=0^0). In each pattern, the small circle denotes the starting point corresponding to the combination vector $\overline{C}_0$. Fig. 4 shows that the starting points of $\overline{C}_0$ are identical in the four cases. The corresponding low bands UWD in difference resolution levels are shown in Fig. 5. In this experiment, only quantization error is considered and there are several points of deviation in the finest resolution level. In general, the deviation will be significant in noisy environment. For a given $\overline{C}_K$, the deviation will be over the range of points, where is the number of originally sampled data and denotes the finest stable level.

B. Choosing the UWD Based on Local Shape Characteristic Enhancement

Since the uniqueness property of the UWD is based on Q_j that can be regarded as the difference between two specially defined areas of the approximated shape in V_{j-1}, the UWD can be used to enhance some geometric characteristic of the shape in several resolution levels. Three examples are demonstrated in Figs. 6–8, in which the vertical axis denotes the low band coefficients ($S_{-4}[n]$) of j=-4 and the horizontal axis denotes the index n.

In Fig. 6, we choose the maximum values of Q0 and Q_{-1}. The maximum Q_0 indicates that the shape can be partitioned into two parts from the corresponding start point. The two parts have the maximum area difference, i.e.,

($\sum_{n=0}^{7} S^{(0)}_{-4}[n] - \sum_{n=8}^{15} S^{(0)}_{-4}[n]$)>($\sum_{n=0}^{7} S^{(l)}_{-4}[n] - \sum_{n=8}^{15} S^{(l)}_{-4}[n]$) for l=1,…15, where l=0 denotes the corresponding start point. The maximum Q_{-1} indicates that the shape can be partitioned into four parts from the starting point with areas A_0, A_1, A_2 and A_3, where $A_i = \sum_{n=4_i}^{4_i+3} S^{(0)}_{-4}[n]$ and the UWD has the geometric characteristic of the maximum $[(A_0+A_2)-(A_1+A_3)]$. In Fig. 7, we search the starting point by which the UWD has the most sensitivity of area difference in resolution levels j=-1 and -2. In Fig. 8, we consider to search the UWD which has the properties of reflection invariance and maximum distance between the two classes in the UWD feature space. The basis to form the feature space is defined by $S^{(l)}_{-4}[n]$ for n=0, 1, 2,…15. This requirement can be met by maximizing the values of $(S^{(l)}_{-4}[0] + S^{(l)}_{-4}[8] - S^{(l)}_{-4}[4] - S^{(l)}_{-4}[12]$ and $(Q_{-3,O} + Q_{-3,R})$, where $Q_{-3,O}$ and $Q_{-3,R}$ denote the values of Q_{-3} of the original and the reflected patterns, respectively. Note here that it is not enough to determine a unique start point if only one level's shape

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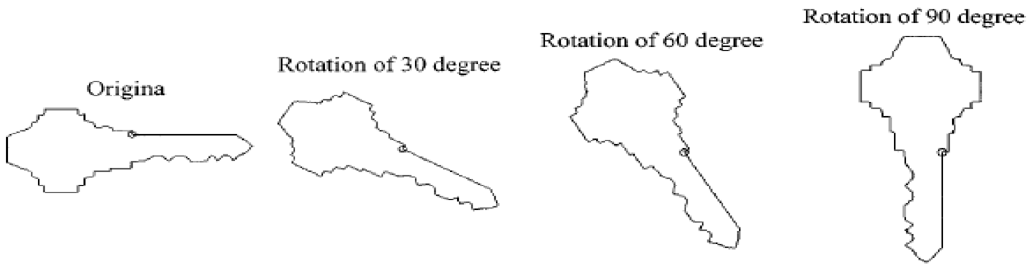

Fig. 4. Illustration of rotation invariance and starting point shift invariance of the UWD.

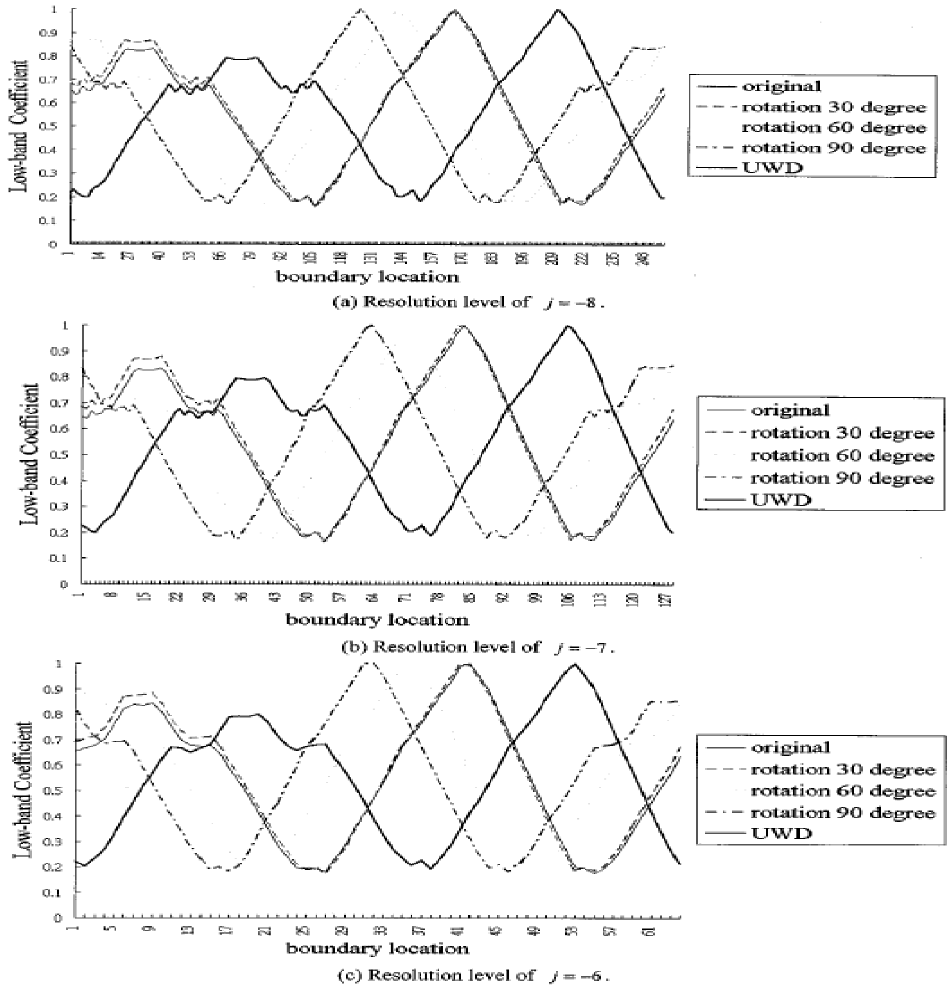

Fig. 5. Low band coefficients of the UWD of the rotated patterns shown in Fig. 4: (a) resolution level of j = -8; (b) resolution level of j = -7; and (c) resolution level of j = -6

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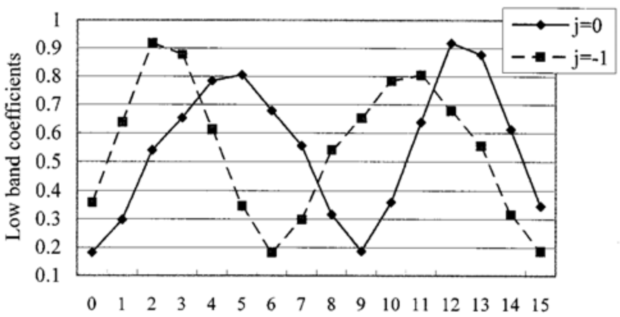

Fig. 6. UWD corresponding to the starting point of maximum Q.

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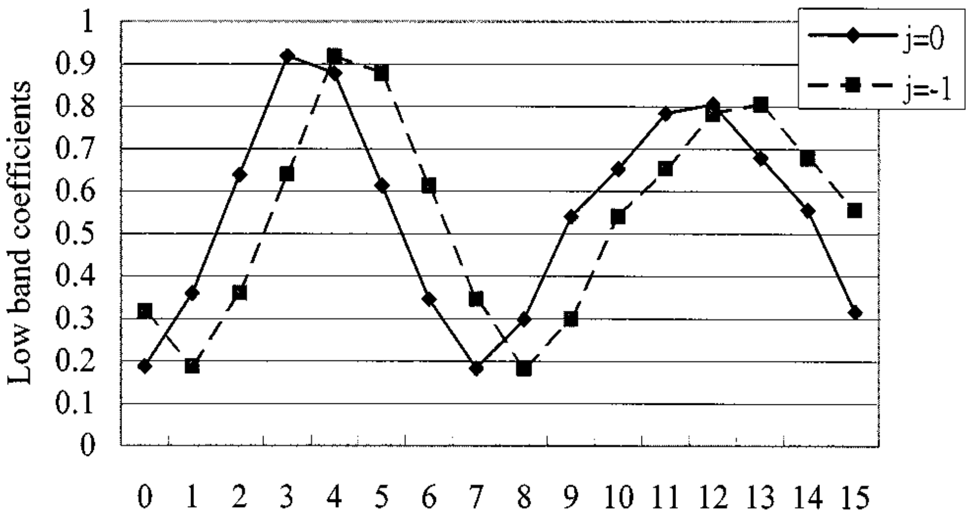

Fig. 7. UWD corresponding to the start point maximizing $\Delta Q_j = Q_j^{\tau+1} - Q_j^{\tau}$.

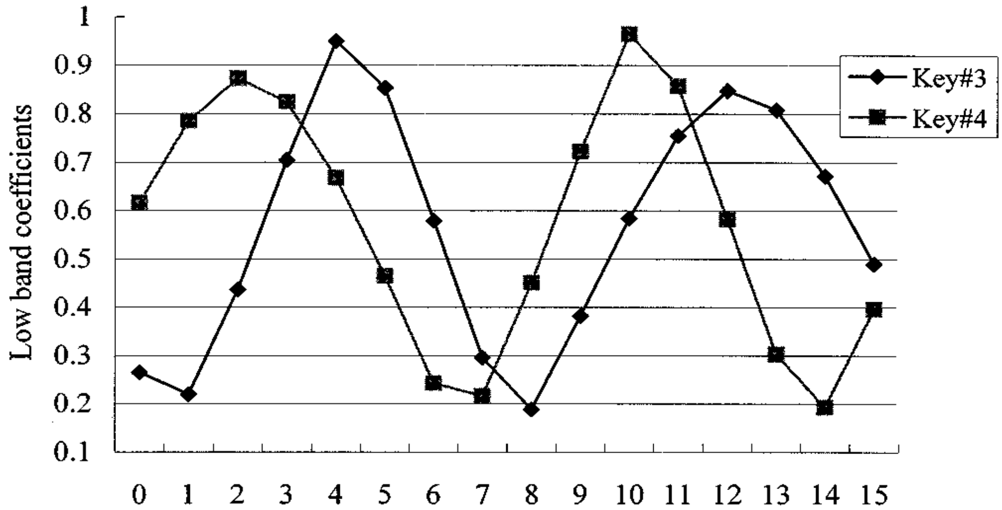

Fig. 8. UWD of two key patterns corresponding to the same CV having maximum distance between classes.

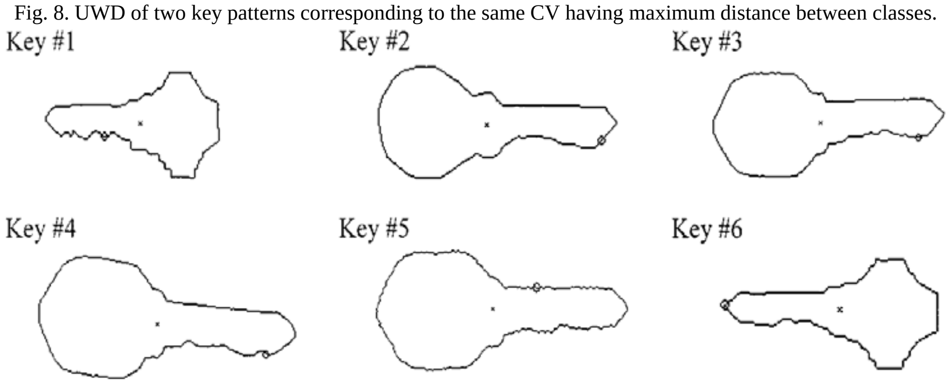

Fig. 9. Six test keys for pattern recognition

TABLE I COMPARISON OF THE FD METHOD AND THE UWD METHOD IN CLASSIFYING THE SIX KEYS. THE MATCHING RATE IS IN %. SNR = 5.451 908 dB. THE STARTING POINT FOR THE UWD IS $\overline{C}_1$

Unknown input	Classified Classes											
	KEY #1		KEY #2		KEY #3		KEY #4		KEY #5		KEY #6	
	FD	UWD	FD	UWD	FD	UWD	FD	UWD	FD	UWD	FD	UWD
KEY #1	88.27	99.88	0.00	0.00	0.04	0.00	0.35	0.00	11.33	0.05	0.00	0.08
KEY #2	0.00	0.58	38.58	87.33	12.76	2.72	2.77	7.72	0.43	0.39	45.44	1.27
KEY #3	0.02	2.00	41.70	4.92	20.60	76.05	6.62	4.11	1.92	5.28	29.14	7.64
KEY #4	0.72	0.22	22.90	8.97	32.87	6.19	22.88	81.38	16.14	1.78	4.50	1.47
KEY #5	1.08	1.55	24.38	0.00	29.52	0.00	21.78	0.00	16.62	91.11	6.62	7.34
KEY #6	0.00	2.47	10.77	0.00	1.12	12.50	0.11	0.05	0.01	7.41	87.98	77.58

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TABLE II MATCHING RESULTS IN THE SAME EXPERIMENTS OF TABLE I WITH THE C AS THE STARTING POINT

Unknown input	Classified Classes					
	KEY #1	KEY #2	KEY #3	KEY #4	KEY #5	KEY #6
KEY #1	89.47	0.00	0.00	0.00	0.23	10.30
KEY #2	0.00	77.41	2.59	16.62	0.39	2.98
KEY #3	0.80	5.06	73.19	7.02	6.64	7.30
KEY #4	0.08	18.80	7.34	69.75	1.80	2.23
KEY #5	0.33	0.00	0.31	0.58	91.61	7.17
KEY #6	12.69	4.42	6.06	0.08	7.64	69.11

characteristic is concerned. To define a unique start point, the chosen CV should involve the last one-point level (Q_0). In exploring the properties of the UWD, the three examples are by no means complete. However, the three examples clearly illustrate the adaptability property of the UWD and the method of selecting the desirable start point of a shape contour.

C. Pattern Recognition Using the UWD

One major goal of planar closed curve representations is to use them for PR applications. An experiment of key shape recognition is demonstrated in this section. The reason of choosing the key shape is that most keys are similar in entire boundary shape but have differences in local shape. In this experiment, we did not concentrate on finding the wavelet function to obtain the best matching performance, but focused on using the UWD and exploring the robustness of the UWD against input noise. The 6-tap Daubechies' filter was used. The shapes of the six test keys are shown in Fig. 9. The boundary of each key was sampled into N=1024 points. Then random noise was added to sampled data with the mean of the SNR about 5.451 908 dB. Only the Q_j in the three levels j=0,-1,-2 were used for starting point determination. The six high band DPWT coefficients of j=-1,-2 were used as the shape features. We maximized the value $\sum_{j=-2}^{0}|Q_j|$ and selected the corresponding CV as the desired start point. It means that under the same CV, most of the key shapes have the property of maximum $\sum_{j=-2}^{0}|Q_j|$. With this property, we can obtain the Q_j which possesses maximum value of the SNR. Meeting this requirement, $\overline{C}_1$ can be found as the desired starting point. The classification method is based on the simplest minimum distance discrimination function. Since for a given CV, each shape will involve 128 [i.e., $N/2^{-(j-1)}$] sets of UWD features, full search was used for finding the minimum distance in each class. To compare with the FD method in which 16 coefficients, excluding the DC coefficients, are used as the shape features, the results of over 2000 experiments are shown in Table I. This result clearly shows that the UWD yields more stable and accurately matching results, even in noisy environments. On the contrast, if one chooses the $\overline{C}_3$ as the starting point in the same experiment, then the performance of matching result will be decreased as shown in Table II. This example is by no means complete. But the example clearly reveals that the recognition performance of the WD approach depends on the starting point. The UWD can provide an alternative to find the optimal shape features which have best recognition performance.

V. DISCUSSIONS AND CONCLUSIONS

Using the periodized wavelet with dilation factor two, an inherent uniqueness property of the 1-D DPWT is developed in this paper. The uniqueness property describes the one-to-one mapping between the circular shift numbers of a finite 1-D signal and the sets of 1-D DPWT coefficients. By this means, each sampled point during the non periodic part of a finite 1-D signal can be chosen as the starting point by which a unique set of 1-D DPWT coefficients are obtained. By applying the uniqueness property of the 1-D DPWT for boundary representation, the UWD was proposed to characterize shape. The UWD provides an adaptive method to select the starting point of the contour, which is optimized in some sense, e.g., in the sense of noise resistance and reflection invariance. To find the desirable UWD, a fast binary search algorithm was also proposed. Exploring the influence of the periodicity of the 1-D signal, the generalized uniqueness property of the 1-D DPWT was also developed. Based on the generalized uniqueness property, the UWD can also be used for shape regularity measurement. Several experiments have been conducted to illustrate the use of the UWD for local shape characteristic enhancement and for PR applications. The PR experiment shown in this paper reveals that the matching performance of the WD approach usually depends on the stating points. Nevertheless, the UWD provides a feasible method to find the desired start point which leads to the best matching performance. Although only the Daubechies' wavelets are demonstrated in all the experiments, the uniqueness property of the 1-D DPWT also exists in using the compactly supported wavelets seen in [17]. As the sampling period approximates to zero, the uniqueness property exists. It implies that the uniqueness property is resolution independent. Besides, because the uniqueness property is based on the orthonormality relations of the two-channel filter banks [22], it has been explored that the uniqueness property also exists in the case of using the biorthogonal bases.

Since the uniqueness property of 1-D DPWT is based on high band coefficients, in the work of PR, only the values of Q in several coarser levels can be used for starting point determination. This implies that each class must involve $N/2^{-(j-1)}$ sets (or clusters) of UWD features for representing the shape, where j denotes the finest stable level. Consequently, a full search over all clusters is required to match an unknown input to each class. However, the full search is time consuming. For

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decreasing the matching time, the clustering technique, such as nearest-neighbor classification [4] and vector quantization [23], can be applied to reduce the clusters of each class.

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