

Dynamic Investment under Asymmetric Information

Ruxing Xu and Dan Wu

Department of Mathematics, China Jiliang University, Hangzhou, China

Email: {rxing, wudan}@gmail.com

Shenghong Li

Department of Mathematics, Zhejiang University, Hangzhou, China

Email: shli@zju.edu.cn

Abstract—This paper develops a tractable real options framework to analyze the effects of asymmetric information on firms' investment decisions when firms issue equity to finance investment. We assume that firm insiders exactly know the firms' growth prospects, but outside investors do not know. Our analysis shows that, under equity financing, the corporate insiders can signal their private information to outside investors using the timing of investment and avoid selling underpriced equity. It is demonstrated that informational asymmetry significantly erodes the option value of waiting to invest and leads firms with good growth prospects to speed up investment. Comparative static analysis shows that the model is consistent with the available empirical evidence.

Index Terms—adverse selection, asymmetric information, financing, investment timing, signaling

I. INTRODUCTION

The problem of raising external funds by issuing securities under asymmetric information is one of the main issues of contemporary corporate finance. When there is informational asymmetry between firm insiders and outside investors, firms raising external capital to fund new investment project face an adverse selection problem (see, e.g., [1]). Since outside investors are unable to distinguish between high-quality and low-quality firms if no signal is given, low-quality firms can issue securities that mimic those offered by high-quality firms, resulting in overvalued securities for low-quality firms and undervalued securities for high-quality firms. How high-quality firms can signal their quality to outside investors has been the subject of considerable research.

When a firm has a potential nonnegative net present value (NPV) project that requires outside funds, Reference [2] demonstrates that in the presence of informational asymmetry, the firm with underpriced stock may prefer debt to equity financing because of the lower information costs associated with debt issues. However, sometimes equity issues can be more attractive than debt issues even for firms with ample debt capacity. Reference [3] demonstrates that issuing new equity has positive effect on stock price when allowing for the realistic possibility of potential projects with negative NPV.

Another example is [4], which report that small high-growth firms do not behave according to the pecking order hypothesis, as opposed to large firms. This result is consistent with the evidence reported in [5].

In this study, we consider the problem of the timing of investment and financial decisions when firm insiders know more about the quality of an investment project than potential investors. Specifically, suppose that there are two types of firms — good and bad — and that good firms have an investment opportunity with higher growth prospects than do bad firms. To undertake valuable investment projects, both types of firms need external funds to cover the sunk costs. We assume that firms can issue equity to raise the funds. While firm insiders know the growth prospects of their own project, outside investors cannot distinguish among them. Market value, therefore, must reflect average project quality. In a pooling equilibrium, all firms invest at the same investment threshold and issue the same number of equity shares compared to the perfect information benchmark, under which both types of firms choose different investment thresholds and issue fairly priced claims. In such a context, bad firms may be tempted to mimic good firms and issue overpriced securities, leading to a decrease in good firms' value and an increase in good firms' investment costs. Accordingly, good firms have an incentive to differentiate from bad ones and signal their higher growth prospects to outside investors. As we show in the paper, they can do so by speeding up investment (lowering investment threshold) and imposing some mimicking costs on bad firms.

The present paper relates to several articles in the literature. Reference [6] develops a separating Nash equilibrium signaling model in which firms use the debt levels and dividends to convey information to the market regarding the variance of their underlying cash flow. Their analysis is extended by [7] to account for two sources of asymmetric information: the mean and the variance of the cash flow. Reference [8] studies the investment timing decision under incomplete information. Specifically, they assume that the value of an investment project evolves as an arithmetic Brownian motion. Owners of the project cannot know the growth prospects of the project before investment. They show that the

optimal investment region is characterized by a continuous and nondecreasing boundary in the value-belief state space. Reference [9] examines the investment timing decision in a decentralized firm under asymmetric information and moral hazard. Our paper differs from theirs in following respects. First, assuming that outside investors do not know firms' growth prospects, we only focus on insider-outsider conflicts. Second, we consider that firms can issue equity to undertake the investment project for the reasons discussed above. Third, our contribution shows that firms' investment behavior under informational asymmetry differs substantially from that of the standard real options model with perfect information.

The remainder of the paper is organized as follows. Section II introduces information asymmetry between firm insiders and outside investors and other assumptions the model is based on. In section III, we study the effects of asymmetric information on investment timing decisions when firms cover capital outlay by issuing equity, and demonstrate that good firms can separate from bad ones by speeding up investment appropriately. Section IV analyzes the impact of the various parameters of the model on investment policy. Conclusion is presented in Section V.

II. ASSUMPTIONS

Throughout the paper, the model is based on the following assumptions. Capital markets are perfect with no transaction costs. There is a simple tax structure that only includes corporate tax denoted by κ . The default-free term structure is flat with a constant rate r , at which investors may lend and borrow freely. We consider a set of infinitely-lived firms, each of which has monopoly rights to a risky investment project and chooses when to invest in the project. Investment is irreversible and entails sunk costs $I > 0$. The firms have no cash, and must issue equity to cover the sunk costs. If they issue equity, we assume that inside stockholders are passive so that the new issue goes to a different group of investors, and management acts in the interests of old stockholders by maximizing the intrinsic value of existing shares when making investment decisions. The project, once completed, will produce a perpetual continuous cash flow shock X_t , which evolves as an arithmetic Brownian motion defined on a probability space (Ω, F, P) :

$$dX_t = \mu dt + \sigma dW_t, \tag{1}$$

where μ and σ denote the drift and diffusion parameters which are assumed to be positive constants. W_t is a standard Wiener process under a risk-neutral probability measure P .

There are two types of firms in the market: good and bad, meaning that μ takes only two values: μ_1, μ_2 with $\mu_1 > \mu_2$ and $P(\mu = \mu_1) = p > 0$, which is common knowledge. The values of state variable X_t and diffusion parameter σ are perfectly observed by all agents. Firm insiders know their firm's type, i.e., the true value of μ ,

but outside investors do not know. Meanwhile, the outside investors cannot infer the true value of μ , because it is impossible to distinguish perfectly between the relative contributions of the unknown drift μt and diffusion components σW_t to the instantaneous variations of the value X_t (for details, see [8]). Since potential investors cannot distinguish between firms with different growth prospects in the absence of a signal, bad firms are tempted to mimic good firms and issue overpriced securities, leading to a reduction in the value of good firms. As a result, good firms may try to separate from bad ones by changing the timing of the investment and imposing some mimicking costs on them. Next we will analyze the impact of asymmetric information on equilibrium investment strategies when firms finance the capital outlay by issuing equity.

III. SIGNALING EQUILIBRIUM UNDER EQUITY FINANCING

A. Investment Timing under Complete Information

We first consider that inside stockholders and outside investors have full information about the growth prospects of the firms' investment projects, and the firms finance the capital outlay I by issuing common equity, which serves as a benchmark case. The method presented here is based on the real options approach ([10]).

In the paper good firms are labeled as 1 and bad firms labeled as 2. By index i we refer to firms of an arbitrary type. Denote old equity value of type i firm before and after investment by $V_i^-(X)$ and $V_i^+(X)$ respectively. Inside stockholders of type i firm choose the investment threshold x_i to maximize the firm value. Then optimal investment time τ_i can be defined as

$$\tau_i = \inf\{s > t : X_s = x_i\}. \tag{2}$$

Firm value denoted by $V_i(X)$ is interpreted as the value of a claim on the entire after-tax cash flow $(1 - \kappa)X$, so we have

$$\begin{aligned} V_i(x_i) &= E_{\tau_i}^P \left[\int_{\tau_i}^{\infty} (1 - \kappa) X_s e^{-r(s - \tau_i)} ds \right] \\ &= (1 - \kappa) \left(\frac{x_i}{r} + \frac{\mu_i}{r^2} \right). \end{aligned} \tag{3}$$

We assume that the initial capital structure of each firm consists of only one share of common equity. Upon investing at time τ_i , inside stockholders issue $n_i(x_i)$ new shares to finance the investment cost I . Then the break-even constraint can be represented as follows:

$$\frac{n_i(x_i)}{1 + n_i(x_i)} V_i(x_i) = I. \tag{4}$$

After investment, the inside stockholders have a fraction $1/[1 + n_i(x_i)]$ of the firm value, i.e.,

$$V_i^+(X) = \frac{1}{1 + n_i(x_i)} V_i(X). \tag{5}$$

Since the firms do not produce any cash flow when X is below the investment threshold x_i , the value of equity

shares before investment $V_i^-(X)$ satisfies the differential equation:

$$\mu_i \frac{\partial V_i^-}{\partial X} + \frac{1}{2} \sigma^2 \frac{\partial^2 V_i^-}{\partial X^2} = rV_i^-, X < x_i. \quad (6)$$

The solution of the equation is given by

$$V_i^-(X) = A_{i1} e^{\beta_{i1} X} + A_{i2} e^{\beta_{i2} X}, \quad (7)$$

where A_{i1} and A_{i2} are constants to be determined by appropriate boundary conditions, and β_{i1} , β_{i2} are roots of the equation $\mu_i \beta + \frac{1}{2} \sigma^2 \beta^2 - r = 0$ which are given by

$$\beta_{i1} = \frac{-\mu_i + \sqrt{\mu_i^2 + 2\sigma^2 r}}{\sigma^2} > 0$$

and

$$\beta_{i2} = \frac{-\mu_i - \sqrt{\mu_i^2 + 2\sigma^2 r}}{\sigma^2} < 0.$$

Taking into account the boundary conditions

$$\lim_{X \rightarrow -\infty} V_i^-(X) = 0 \quad (8)$$

and

$$\lim_{X \rightarrow x_i} V_i^-(X) = V_i^+(x_i), \quad (9)$$

we can find A_{i1} and A_{i2} . The optimal investment level x_i is obtained by invoking the smooth pasting condition

$$\frac{\partial V_i^-}{\partial X} \Big|_{X=x_i} = \frac{\partial V_i^+}{\partial X} \Big|_{X=x_i}. \quad (10)$$

Solving the optimization problem above, we get the following proposition.

Proposition 1 *When firms finance the capital outlay I by issuing common equity under complete information, we have*

1) *The optimal investment threshold is given by*

$$x_i = \frac{1}{\beta_{i1}} - \frac{\mu_i}{r}. \quad (11)$$

2) *The number of new equity shares is given by*

$$n_i(x_i) = \frac{I}{(1-\kappa)\left(\frac{x_i}{r} + \frac{\mu_i}{r^2}\right) - I}. \quad (12)$$

3) *The value of old equity before investment is given by*

$$V_i^-(X) = \frac{1-\kappa}{1+n_i(x_i)} \left(\frac{x_i}{r} + \frac{\mu_i}{r^2}\right) e^{\beta_{i1}(X-x_i)}. \quad (13)$$

4) *The value of old equity after investment is given by*

$$V_i^+(X) = \frac{1-\kappa}{1+n_i(x_i)} \left(\frac{X}{r} + \frac{\mu_i}{r^2}\right). \quad (14)$$

Since $\mu_1 > \mu_2$, from Proposition 1 we easily have

$$x_1 < x_2, n_1(x_1) < n_2(x_2). \quad (15)$$

The equations in (15) show that, under complete information both types of firms choose different investment thresholds and issue fairly priced claims. However, this is not the case when there is asymmetric information between inside stockholders and outside investors, which will be explained in the next subsection.

B. Investment Timing in the Pooling Equilibrium

Now we consider the impact of asymmetric information on investment decisions. Since $\mu_1 > \mu_2$, we

have $V_1(X) > V_2(X)$. When outside investors do not know whether they face good firms or not, bad firms have an incentive to mimic good ones and sell overpriced equity. Accordingly, there is a pooling equilibrium, in which all firms invest at the same investment threshold and issue the same number of equity shares to finance the capital outlay. Cash flow shock X in the pooling equilibrium is modeled as

$$dX_t = \bar{\mu} dt + \sigma dW_t, \quad (16)$$

where $\bar{\mu} = p\mu_1 + (1-p)\mu_2$.

Denote by \bar{x} the optimal investment threshold of the pooled firm, and its optimal investment timing τ can be defined as

$$\tau = \inf\{s > t : X_s = \bar{x}\}. \quad (17)$$

Then the present value of the pooled firm $V(X)$ at the time of investment is given by

$$\begin{aligned} V(\bar{x}) &= E_\tau^P \left[\int_\tau^\infty (1-\kappa) X_s e^{-r(s-\tau)} ds \right] \\ &= (1-\kappa) \left(\frac{\bar{x}}{r} + \frac{\bar{\mu}}{r^2} \right). \end{aligned} \quad (18)$$

The value of pooled old equity before investment denoted by $V^-(X)$ satisfies the differential equation

$$\bar{\mu} \frac{\partial V^-}{\partial X} + \frac{1}{2} \sigma^2 \frac{\partial^2 V^-}{\partial X^2} = rV^-, X < \bar{x}. \quad (19)$$

The solution of equation is given by

$$V^-(X) = B_1 e^{\beta_1 X} + B_2 e^{\beta_2 X}, \quad (20)$$

where B_1, B_2 are constants to be determined, and β_1, β_2 are roots of the equation $\bar{\mu} \beta + \frac{1}{2} \sigma^2 \beta^2 - r = 0$ which are given by

$$\beta_1 = \frac{-\bar{\mu} + \sqrt{\bar{\mu}^2 + 2\sigma^2 r}}{\sigma^2} > 0$$

and

$$\beta_2 = \frac{-\bar{\mu} - \sqrt{\bar{\mu}^2 + 2\sigma^2 r}}{\sigma^2} < 0.$$

We denoted by $n(\bar{x})$ the number of new shares outstanding after issuing. Then the break-even constraint on average can be represented as follows:

$$\frac{n(\bar{x})}{1+n(\bar{x})} V(\bar{x}) = I. \quad (21)$$

Taking into account the following boundary conditions

$$\lim_{X \rightarrow -\infty} V^-(X) = 0 \quad (22)$$

and

$$\lim_{X \rightarrow \bar{x}} V^-(X) = \frac{V(\bar{x})}{1+n(\bar{x})}, \quad (23)$$

we can get B_1 and B_2 . The optimal investment level \bar{x} is obtained by invoking the smooth pasting condition

$$\frac{\partial V^-}{\partial X} \Big|_{X=\bar{x}} = \frac{1}{1+n(\bar{x})} \frac{\partial V}{\partial X} \Big|_{X=\bar{x}}. \quad (24)$$

Solving the optimization problem above yields the following proposition.

Proposition 2 *In a pooling equilibrium in which firms of both types invest at the same threshold and issue the same number of new shares, we have*

1) *The optimal investment threshold \bar{x} is given by*

$$\bar{x} = \frac{1}{\beta_1} - \frac{\bar{\mu}}{r}. \quad (25)$$

2) *The number of new shares $n(\bar{x})$ is given by*

$$n(\bar{x}) = \frac{I}{(1-\kappa)(\frac{\bar{x}}{r} + \frac{\bar{\mu}}{r^2}) - I}. \quad (26)$$

3) *The value of pooled old equity before investment is given by*

$$V^-(X) = \frac{1-\kappa}{1+n(\bar{x})} \left(\frac{\bar{x}}{r} + \frac{\bar{\mu}}{r^2} \right) e^{\beta_1(X-\bar{x})}. \quad (27)$$

Since $\mu_1 > \bar{\mu} > \mu_2$, we easily have

$$x_1 < \bar{x} < x_2, \quad n_1(x_1) < n(\bar{x}) < n_2(x_2). \quad (28)$$

The equations in (28) show that, asymmetric information increases (respectively decreases) the investment threshold selected by good firms (respectively bad firms) compared to the case of perfect information. As a result, asymmetric information causes good firms to delay investment and bad firms to speed up investment in a pooling equilibrium. If good firms finance the project by issuing common equity, they have to issue underpriced equity which dilutes the value of existing equity, and thereby are hurt by the presence of the bad firms. At the same time, bad firms can increase the value of existing equity by issuing overpriced equity at investment threshold \bar{x} .

Now we show how the value of existing equity shares of good firms changes when they invest at the threshold \bar{x} under informational asymmetry. In the pooling equilibrium, the value of good firms' equity before investment denoted by $V_1^{p-}(X)$ satisfies the differential equation

$$\mu_1 \frac{\partial V_1^{p-}}{\partial X} + \frac{1}{2} \sigma^2 \frac{\partial^2 V_1^{p-}}{\partial X^2} = rV_1^{p-}, \quad X < \bar{x}. \quad (29)$$

The solution of equation is given by

$$V_1^{p-}(X) = C_1 e^{\beta_1 X} + C_2 e^{\beta_2 X}, \quad (30)$$

where C_1, C_2 are constants to be determined.

When good firms finance the capital outlay, they have to raise $n(\bar{x})$ new shares. After investment, the inside stockholders have a fraction $1/[1+n(\bar{x})]$ of the firm value. Then the value of good firms' old equity after investment denoted by $V_1^{p+}(X)$ is given by

$$V_1^{p+}(X) = \frac{1}{1+n(\bar{x})} V_1(X). \quad (31)$$

We can get C_1, C_2 by considering the following boundary conditions

$$\lim_{X \rightarrow -\infty} V_1^{p-}(X) = 0 \quad (32)$$

and

$$\lim_{X \rightarrow \bar{x}} V_1^{p-}(X) = V_1^{p+}(\bar{x}). \quad (33)$$

Solving the problem above yields the following lemma.

Lemma 3 *In a pooling equilibrium, where both types of firms issue $n(\bar{x})$ new shares and invest at the threshold \bar{x} under asymmetric information, we have*

1) *The value of good firms' equity before investment is given by*

$$V_1^{p-}(X) = \frac{1-\kappa}{1+n(\bar{x})} \left(\frac{\bar{x}}{r} + \frac{\mu_1}{r^2} \right) e^{\beta_{11}(X-\bar{x})}. \quad (34)$$

2) *The value of good firms' inside shares after investment is given by*

$$V_1^{p+}(X) = \frac{1-\kappa}{1+n(\bar{x})} \left(\frac{X}{r} + \frac{\mu_1}{r^2} \right). \quad (35)$$

3) *The value of bad firms' inside shares after investment is given by*

$$V_2^{p+}(X) = \frac{1-\kappa}{1+n(\bar{x})} \left(\frac{X}{r} + \frac{\mu_2}{r^2} \right). \quad (36)$$

According to Proposition 1 and Lemma 3, good firms delay investment and suffer a significant loss because of asymmetric information. Thus good firms may try to separate by imposing some mimicking costs on bad firms. Next we will show that they can do so by changing the timing of the investment, which makes mimicking good firms an unprofitable strategy for bad firms.

C. Investment Timing in a Signaling Equilibrium

Now we examine whether there exists a signaling equilibrium, under which good firms signal their growth prospects to outside investors by speeding up investment. When deciding whether to pool or not, bad firms make a trade-off between the overpricing of the shares and the reduction in option value of waiting due to the change in investment policy. On the other hand, good firms can signal their private information to outside investors by lowering the investment threshold, which reduces the value of the pooled firm at the time of investment, and hence the benefits of pooling for bad firms. We want to find out if there exists an investment threshold such that good firms find it profitable to invest and bad firms find it unprofitable to mimic. To determine whether there exists a signaling equilibrium, we first check the incentive compatibility constraint of bad firms. If bad firms mimic good firms, they need to issue $n(\bar{x})$ shares to finance the capital outlay. The value of the inside equity shares after investment is $V_2^{p+}(X)$ given in (36), which reflects the fact that the cross subsidization reduces the investment costs for bad firms. Instead of mimicking good firms, bad firms can follow their first best strategy under perfect information by waiting to invest before the cash flow shock reaches the threshold x_2 . The option value of waiting before investment under perfect information is given in (13) when $i = 2$, i.e.,

$$V_2^-(X) = \frac{1-\kappa}{1+n_2(x_2)} \left(\frac{x_2}{r} + \frac{\mu_2}{r^2} \right) e^{\beta_{21}(X-x_2)}. \quad (37)$$

The condition that bad firms are indifferent between pooling and waiting to issue equity and invest under perfect information is given by

$$V_2^{p+}(X) = V_2^-(X). \quad (38)$$

Since $n(\bar{x}) < n_2(x_2)$, when cash flow shock X reaches the threshold x_2 we have

$$V_2^{P+}(x_2) > V_2^-(x_2). \tag{39}$$

If $X = -\frac{\mu_2}{r}$, then $V_2^{P+}(-\frac{\mu_2}{r}) = 0$ and $V_2^-(\frac{\mu_2}{r}) > 0$. So we have

$$V_2^{P+}(-\frac{\mu_2}{r}) < V_2^-(\frac{\mu_2}{r}). \tag{40}$$

Because $V_2^{P+}(X)$ and $V_2^-(X)$ are monotonically increasing functions of X , there exists a unique x^* ($x^* < x_2$), which makes

$$V_2^{P+}(x^*) = V_2^-(x^*). \tag{41}$$

Figure 1¹ describes the trade-off made by bad firms when deciding whether to pool or not. In this figure, the solid line represents the pooled value of the inside shares after investment, as described by equation (36). The dashed curve and the dash-dot line respectively represent the value of the inside shares before and after investment under perfect information. The investment threshold x_2 maximizes bad firms' option value of waiting to invest. At the threshold x^* , bad firms find it unprofitable to mimic. As a result, good firms can separate from bad firms when good firms invest at $X < x^*$, assuming that the incentive compatibility constraint of good firms is satisfied. Otherwise, good firms are better off pooling with bad firms.

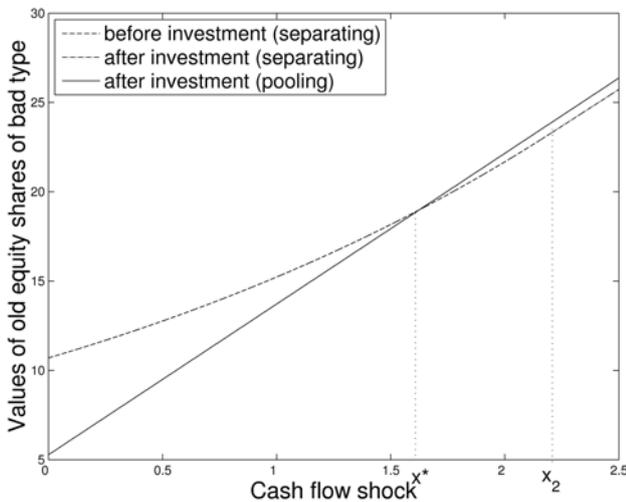


Figure 1. Incentive compatibility constraint of bad firms with equity financing

Now we check the incentive compatibility constraint of good firms. The condition that good firms are indifferent between investment early under perfect information and pooling is

$$V_1^+(X) = V_1^{P-}(X), \tag{42}$$

where $V_1^+(X)$ is given in (14) when $i=1$, and $V_1^{P-}(X)$ is given in (34). Since $n_1(x_1) < n(\bar{x})$, when cash flow shock X reaches the threshold \bar{x} we have

$$V_1^+(\bar{x}) > V_1^{P-}(\bar{x}). \tag{43}$$

If $X = -\frac{\mu_1}{r}$, then $V_1^+(-\frac{\mu_1}{r}) = 0$ and $V_1^{P-}(-\frac{\mu_1}{r}) > 0$. So we have

$$V_1^+(-\frac{\mu_1}{r}) < V_1^{P-}(-\frac{\mu_1}{r}). \tag{44}$$

Because $V_1^+(X)$ and $V_1^{P-}(X)$ both are monotonically increasing functions of X , there exists a unique x_{min} ($x_{min} < \bar{x}$), which makes

$$V_1^+(x_{min}) = V_1^{P-}(x_{min}). \tag{45}$$

Figure 2 describes the trade-off made by good firms when they decide whether to separate from bad firms. In this figure, the dash-dot curve and the dotted line respectively represent the option value of waiting to invest and the value of inside equity shares after investment in a pooling equilibrium. The dashed curve and the solid line respectively represent the value of inside equity shares before and after investment in the perfect information benchmark. The threshold x_1 maximizes the good firms' option value of waiting to invest. The threshold x_{min} represents the lowest value of the cash flow shock X such that good firms prefer to separate rather than pool with bad firms. If $x^* > x_{min}$, we have

$$V_1^+(x^*) > V_1^{P-}(x^*), \tag{46}$$

and it is better for good firms to separate from bad firms at $X = x^*$.

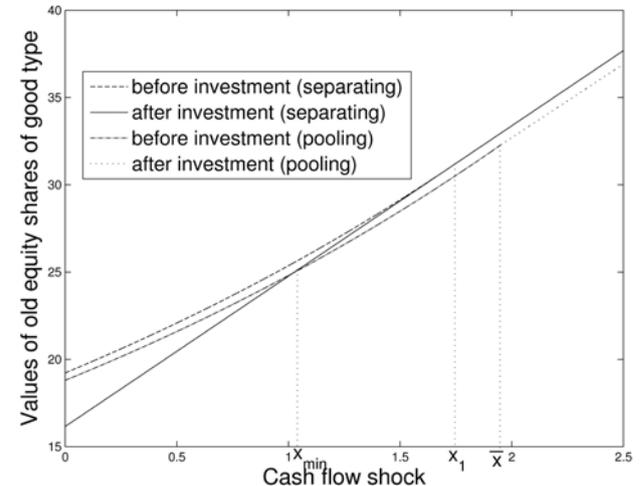


Figure 2. Incentive compatibility constraint of good firms with equity financing

Using the incentive compatibility constraints of both types, we can establish the following results:

Proposition 4 When financing capital outlay by issuing equity under asymmetric information, good firms can separate from bad firms by investing the first time that cash flow shock X reaches the threshold x^* ($x^* \leq x_1$), which satisfies the equation (41), i.e.,

$$x^* = \frac{1 + n(\bar{x})}{1 + n_2(x_2)} \left(x_2 + \frac{\mu_2}{r} \right) e^{\beta_{21}(x^* - x_2)} - \frac{\mu_2}{r} \tag{47}$$

¹In Figure 1 and 2, the parameters for computation are as follows: $p = 0.5$, $\mu_1 = 0.15$, $\mu_2 = 0.05$, $\sigma = 1$, $r = 0.08$ and $I = 5$.

and satisfies $x^* \geq x_{min}$, where x_{min} satisfies the equation (45), i.e.,

$$x_{min} = \frac{1+n_1(x_1)}{1+n(\bar{x})} \left(\bar{x} + \frac{\mu_1}{r} \right) e^{\beta_1(x_{min}-\bar{x})} - \frac{\mu_1}{r}. \quad (48)$$

In the signaling equilibrium, good firms speed up investment when $x^* < x_1$, and bad firms invest at their first best investment threshold x_2 . If $x^* < x_{min}$, good firms are better off pooling with bad firms, and the selected investment policy is as in Proposition 2.

According to Proposition 4, there exists an investment threshold $x^* > x_{min}$ that satisfies the non-linear equation (47) such that good firms can separate from bad firms by investing at that threshold. The investment strategy eliminates the underpricing associated with a pooling equilibrium and triggers a reduction in the costs of investment for good firms.

IV. COMPARATIVE STATIC ANALYSIS

In this section, we analyze the impact of the various parameters of the model on investment strategy. The figures below plot the separating threshold x^* (solid curve) and the indifferent threshold of good firm x_{min} (dashed curve) as functions of the fraction of good firms in the economy p , growth rate of the cash flow shock μ_1 , volatility rate of the cash flow shock σ , and interest rate r . In the figures, we use the following base case parameter values: $p = 0.5$, $\mu_1 = 0.15$, $\mu_2 = 0.05$, $\sigma = 1$, $r = 0.08$ and $I = 5$.

From Figure 3 and 4, it can be seen that an increase in the fraction of good firms in the economy or the growth rate of good firms' cash flow shock leads to a decrease in the separating investment threshold, because an increase in p or μ_1 leads to an increase in the benefits of mimicking. Accordingly, good firms need to increase the cost of mimicking for bad firms and can do so by speeding up investment (thereby reducing the pooled value at the time of investment). As the fraction of good firms in the economy is below p^* or the growth rate of good firms' cash flow shock rises above μ^* , the cost of pooling for good firms increases, and the signaling equilibrium dominates the pooling equilibrium for good firms.

Since the option value of waiting to invest increases with the volatility of the cash flow shock, an increase in volatility leads to an increase in the separating investment threshold and the indifferent threshold of good firms as shown in Figure 5.

From Figure 6, an increase in interest rate leads to an increase in the benefits of mimicking and hence a decrease in the separating investment threshold. When interest rate rises above r^* , the cost of pooling for good firms decreases, and good firms are better off pooling with bad firms.

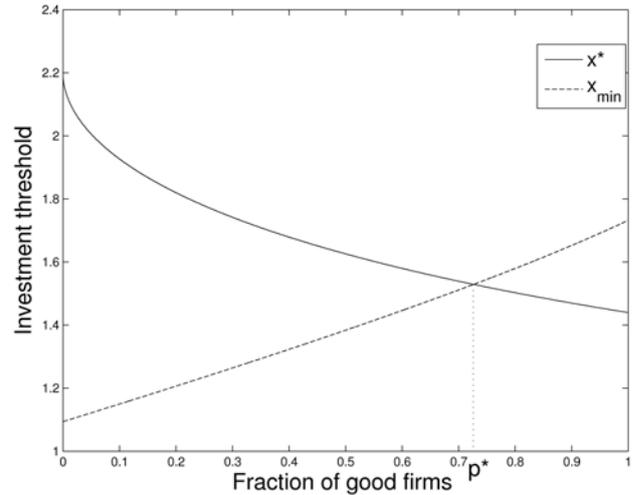


Figure 3. The effects of p on Separating threshold x^* and the indifferent threshold x_{min}

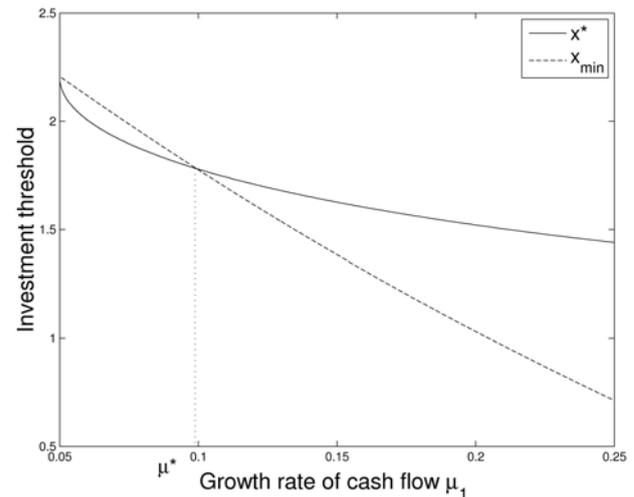


Figure 4. The effects of μ_1 on Separating threshold x^* and the indifferent threshold x_{min}

V. CONCLUSION

Asymmetric information is a common feature of market interactions. This paper examines the impacts of asymmetric information on investment decisions in a continuous-time framework when external funds are needed to cover the capital outlay. By speeding up investment, good firms can signal their private information about the firms' growth prospects to outside investors and avoid selling an underpriced equity. Changing the timing of investment is a costly action for good firms, because it can lead to a significant erosion of the value of the option of waiting to invest. Comparative static analysis demonstrates the impacts of the various parameters of the model on investment strategy, which is consistent with the available empirical evidence. The model can be easily extended to the case where firms finance the capital expenditure using debt and equity.

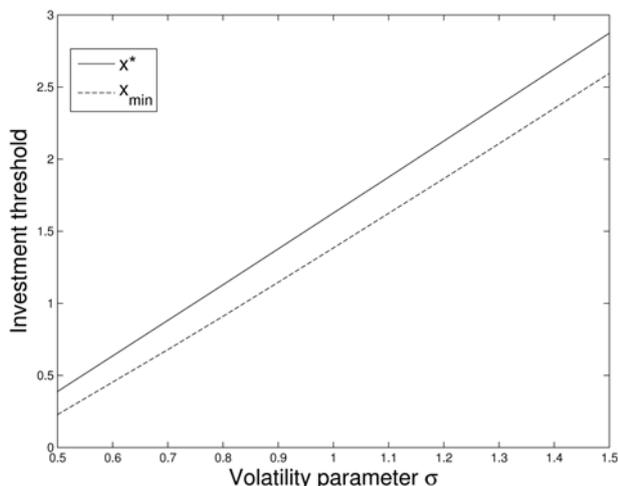


Figure 5. The effects of σ on the threshold x^* and the threshold x_{min}

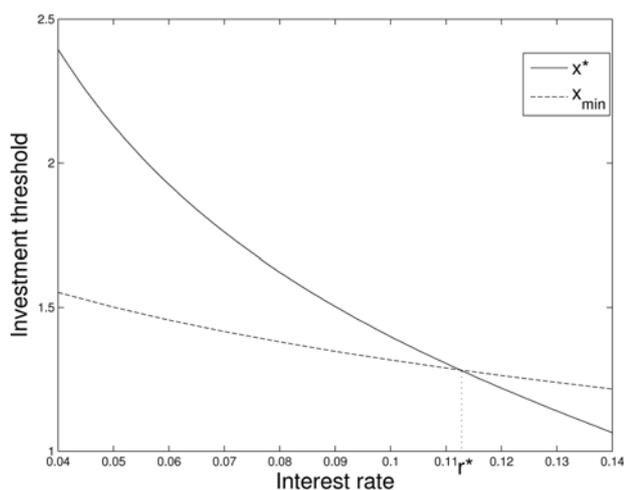


Figure 6. The effects of r on Separating threshold x^* and the indifferent threshold x_{min}

ACKNOWLEDGEMENT

The authors wish to express their gratitude to Shuguang zhang and Xueqiao Du for continuing support during this work. This work was supported in part by the Key Grant Project of Chinese Ministry of Education (NO.309018) and the National Natural Science Foundation of China (No. 10601051).

REFERENCES

[1] G. Akerlof, "The Market for Lemons: Qualitative Uncertainty and the Market Mechanism," *Quarterly Journal of Economics*, vol. 84, no. 3, pp. 488–500, 1970.
 [2] S. C. Myers and N. S. Majluf, "Corporate financing and investment decisions when firms have information that investors do not have," *Journal of Financial Economics*, vol. 13, no. 2, pp. 187–221, 1984.
 [3] J. Cooney and A. Kalay, "Positive information from equity issue announcements," *Journal of Financial Economics*, vol. 33, no. 2, pp. 149–172, 1993.

[4] M. Z. Frank and V. K. Goyal, "Testing the pecking order theory of capital structure," *Journal of Financial Economics*, vol. 67, no. 2, pp. 217–248, 2003.
 [5] M. T. Leary and M. R. Roberts, "The pecking order, debt capacity, and information asymmetry," unpublished.
 [6] I. E. Brick, M. Frierman, and Y. K. Kim, "Asymmetric Information Concerning the Variance of Cash Flows: The Capital Structure Choice," *International Economic Review*, vol. 39, no. 3, pp. 745–761, 1998.
 [7] G. Y. Luo, I. Brick, and M. Frierman, "Strategic Decision Making of the Firm Under Asymmetric Information," *Review of Quantitative Finance and Accounting*, vol. 19, no. 2, pp. 215–237, 2002.
 [8] J. Decamps, T. Mariotti, and S. Villeneuve, "Investment timing under incomplete information," *Mathematics of Operations Research*, vol. 30, no. 2, pp. 472–500, 2005.
 [9] S. R. Grenadier and N. Wang, "Investment timing, agency, and information," *Journal of Financial Economics*, vol. 75, no. 3, pp. 493–533, 2005.
 [10] A. Dixit and R. Pindyck, *Investment under uncertainty*. Princeton University Press Princeton, NJ, 1994.
 [11] L. S. Klein, T. J. O'Brien, and S. R. Peters, "Debt vs. Equity and Asymmetric Information: A Review," *The Financial Review*, vol. 37, no. 3, pp. 317–349, 2002.
 [12] J. Tirole, *The theory of corporate finance*. Princeton, NJ: Princeton University Press, 2006.
 [13] S. Tsai, "Information asymmetry and corporate investment decisions: A dynamic approach," *Financial Review*, vol. 43, no. 2, pp. 241–271, 2008.
 [14] D. Feldman, "Incomplete information equilibria: Separation theorems and other myths," *Annals of Operations Research*, vol. 151, no. 1, pp. 119–149, 2007.
 [15] G. Pawlina and P. Kort, "Investment under uncertainty and policy change," *Journal of Economic Dynamics and Control*, vol. 29, no. 7, pp. 1193–1209, 2005.
 [16] B. Lambrecht and W. Perraudin, "Real options and preemption under incomplete information," *Journal of Economic Dynamics and Control*, vol. 27, no. 4, pp. 619–643, 2003.
 [17] R. Novy-Marx, "An Equilibrium Model of Investment Under Uncertainty," *Review of Financial Studies*, vol. 20, no. 5, pp. 1461–1502, 2007.
 [18] M. Bank and J. Lawrenz, "Informational asymmetry between managers and investors in the optimal capital structure decision," unpublished.
 [19] J. Decamps and T. Mariotti, "Investment timing and learning externalities," *Journal of Economic Theory*, vol. 118, no. 1, pp. 80–102, 2004.
 [20] E. Fama and K. French, "Testing trade-off and pecking order predictions about dividends and debt," *Review of Financial Studies*, vol. 15, no. 1, pp. 1–33, 2002.
 [21] J. Riley, "Silver Signals: Twenty-Five Years of Screening and Signaling," *Journal of Economic Literature*, vol. 39, no. 2, pp. 432–478, 2001.
 [22] P. Bolton and X. Freixas, "Equity, Bonds, and Bank Debt: Capital Structure and Financial Market Equilibrium under Asymmetric Information," *Journal of Political Economy*, vol. 108, no. 2, pp. 324–351, 2000.
 [23] K. Lofgren, T. Persson, and J. Weibull, "Markets with Asymmetric Information: The Contributions of George Akerlof, Michael Spence and Joseph Stiglitz," *Scandinavian Journal of Economics*, vol. 104, no. 2, pp. 195–211, 2002.

Ruxing Xu was born in Bengbu, China, in 1976. He is currently a Ph.D. candidate at Zhejiang University, Hangzhou,

China. He received his MS degree in computational mathematics from Hefei University of Technology in 2004.

He is currently a lecturer at China Jiliang University. His research interests include real option theory, optimal investment and credit derivatives pricing.

Dan Wu was born in Taian, China, in 1979. She received her PhD degree in applied mathematics from Zhejiang University, Hangzhou, China, in 2009.

She is currently a lecturer at China Jiliang University. Her research interests include watermark and CAGD.

Shenghong Li was born in Hunan, China, in 1956. He received his PhD degree in applied mathematics from Zhejiang University, Hangzhou, China, in 1995.

He is currently a PhD Candidate Supervisor and a Professor of Operation research at Zhejiang University. His current research interests include real option theory and credit derivatives pricing.