A Fast Image Thresholding Method Based on Chaos Optimization and Recursive Algorithm for Two-Dimensional Tsallis Entropy

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Abstract: **The two-dimensional (2-D) maximum Tsallis entropy method often gets ideal segmentation results, because it not only takes advantage of the spatial neighbor information with using the 2-D histogram of the image, but also has some flexibility with a parameter. However, its time-consuming computation is often an obstacle in real time application systems. In this paper, a fast image thresholding method based on chaos optimization and recursive algorithm for 2-D Tsallis entropy is presented. Firstly, improve the traditional chaos optimization algorithm(COA) so that it can get global solution with lower computation load, then propose a recursive algorithm with the stored matrix variables, finally combine the improved COA and the recursive algorithm to reduce much computational cost in the process of solving the 2-D maximum Tsallis entropy problem. Experimental results show the proposed approach can get better segmentation performance and has much faster speed.**

Index Terms: **Image thresholding, Two-dimensional (2-D) Tsallis entropy, Chaos optimization algorithm (COA), Recursive algorithm**

I. Introduction

Image segmentation denotes a process by which a raw input image is partitioned into non-overlapping regions such that each region is homogeneous and the union of two adjacent regions is heterogeneous. Since the segmentation is considered by many authors to be an essential component of any image analysis system, that problem has received a great deal of attention; in the literature, many techniques have been developed for image segmentation [1]. Thresholding is a popular tool for image segmentation for its simplicity, especially in the fields where real time processing is needed. In 2006, Sahoo & Arora [2] proposed a thresholding method based on 2-D Tsallis-Havrda-Charvάt entropy, which was based on the entropic thresholding method proposed in their earlier paper [3]. It extended a method due to Pavešić and Ribarić [4] and Portes de Albuquerque et al [5]. This method behaves well when applied to quite a few real-world and synthetic images. However, because of the 2-D histogram, it gives rise to the exponential increment of computation time. So Qiao and Wu [6] employed maximum inter-class entropy and recursive algorithm to enhance segmentation performance and to reduce some time cost, and Du and Shi [7] used 2-D maximum entropy method based on the particle swarm optimization (PSO) in which the PSO raised the computing speed. Although these methods got good segmentation results with less computational time, but they are not too ideal yet. Chaos is a bounded unstable dynamic behavior, which exhibits sensitive dependence on initial conditions and includes infinite unstable periodic motions [8]. Optimization algorithms based on the chaos theory are search methodologies that differ from any of the existing traditional stochastic optimization techniques. Due to the non-repetition of chaos, it can carry out overall searches at higher speeds than stochastic ergodic searches that depend on probabilities. So, in this paper a fast image thresholding method based on the chaos optimization algorithm (COA) for 2-D Tsallis entropy is presented, which uses maximum 2-D Tsallis entropy criteria, not only improves the traditional COA, but also combines the improved COA and recursive algorithm with the stored matrix variables to greatly reduce computational cost and to select the optimal threshold value, and finally enhances segmentation performance.

The paper is organized as follows: In Section 2, 2-D Tsallis entropy is presented. Section 3 explains the improved chaos optimization algorithms. Section 4 describes the recursive algorithm with the stored matrix variables in detail and image segmentation based on the proposed method. In Section 5, experimental results on some images are presented. Conclusions are drawn in Section 6.

II. The 2-D Tsallis entropy method

A..Tsallis – Havrda – Charva't entropy

Let $P = (p_1, p_2, \cdots, p_n) \in \Delta_n$, where

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$$\Delta_n = \{(p_1, p_2, \cdots, p_n) \mid p_i \geq 0, i = 1,2,\cdots,n, n \geq 2, \sum_{i=1}^{n} p_i = 1\}$$

is a set of discrete finite n-ary probability distributions. Havrda and Charvát [9] defined entropy of degree α as

$$H_n^\alpha = \frac{1}{1-2^{1-\alpha}}[1 - \sum_{i=1}^{n} p_i^\alpha].$$

Independently Tsallis [10], proposed a one parameter generalization of the Shannon entropy as:

$$H_n^\alpha(P) = \frac{1}{\alpha - 1}[1 - \sum_{i=1}^{n} p_i^\alpha] \quad (1)$$

where α is a real positive parameter not equal to one.

Both these entropies essentially have the same expression except the normalizing factor. This entropy became often used in statistical physics after the seminal work of Tsallis.

B.2-D Tsallis entropy

Let $f(m,n)$ be the gray value of the pixel located at the point (m,n). In a digital image $\{ f(m,n) \mid m \in \{1,2,\cdots,M\}, n \in \{1,2,\cdots,N\} \}$ of size $M \times N$, let the histogram be $h(x)$ for $x \in \{0,1,2,\cdots,255\}$. For the sake of convenience, we denote the set of all gray levels $\{0,1,2,\cdots,255\}$ as G. Global threshold selection methods usually use the gray level histogram of the image. The optimal threshold is determined by optimizing a suitable criterion function obtained from the gray level distribution of the image and some other features of the image.

In order to compute the 2-D histogram of a given image we proceed as follows. Calculate the average gray value of the neighborhood of each pixel. Let $g(x,y)$ be the average of the neighborhood of the pixel located at the point (x,y). The average gray value for the 3×3 neighborhood of each pixel is calculated as

$$g(x,y) = \frac{1}{9}\sum_{i=-1}^{1}\sum_{j=-1}^{1} f(x+i, y+j)$$

While computing the average gray value, disregard the two rows from the top and bottom and two columns from the sides. The pixel's gray value $f(x,y)$ and the average of its neighborhood $g(x,y)$ are used to construct a 2-D histogram using: $h(i,j) = \text{Prob}(f(x,y) = i \text{ and } g(x,y) = j)$, where $i, j \in G$, for a given image, there are several methods to estimate this density function. One of the most frequently used methods is the method of relative frequency. The normalized histogram is approximated by using the formula

$$p_{ij} = \frac{n_{ij}}{M \times N}$$

where $M \times N$ denotes the image size, and n_{ij} denotes the number of a pixel whose gray value equals i and local average value equals j. The 2-D histogram plane can be describe as Fig. 1: where the area 1 and 2 denote objects and background respectively, and 3 and 4 denote edges and noise. So a threshold vector (t,s), where t is a threshold for pixel intensity and s is another threshold for the pixel average of pixels, should be determined to divide them. According to the maximum Tsallis entropy principle, the determined threshold vector should make area 1 and 2 have maximum Tsallis information.

Our normalization is accomplished by using a posteriori class probabilities, $p_1(t,s)$ and $p_2(t,s)$. Thus according to equation (1), the Tsallis entropies associated with object and background distributions are given by:

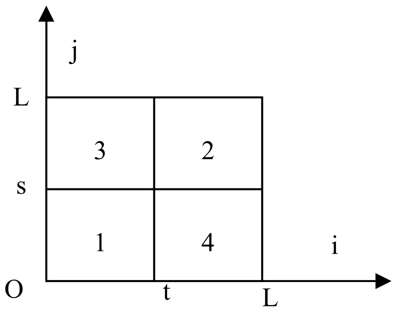


Figure 1. The 2-D histogram plane

$$H_b^\alpha(t,s) = \frac{1}{\alpha-1}[1 - \sum_{i=0}^{t}\sum_{j=0}^{s}(\frac{p(i,j)}{p_1(t,s)})^\alpha]$$
$$= \frac{1}{\alpha-1}[1 - \frac{1}{(p_1(t,s))^\alpha}\sum_{i=0}^{t}\sum_{j=0}^{s}(p(i,j))^\alpha]$$
$$H_w^\alpha(t,s) = \frac{1}{\alpha-1}[1 - \sum_{i=t+1}^{255}\sum_{j=s+1}^{255}(\frac{p(i,j)}{p_2(t,s)})^\alpha]$$
$$= \frac{1}{\alpha-1}[1 - \frac{1}{(p_2(t,s))^\alpha}\sum_{i=t+1}^{255}\sum_{j=s+1}^{255}(p(i,j))^\alpha]$$

where $$p_1(t,s) = \sum_{i=0}^{t}\sum_{j=0}^{s} p(i,j). \quad (2)$$

$$p_2(t,s) = \sum_{i=t+1}^{255}\sum_{j=s+1}^{255} p(i,j)$$

If we consider that a physical system can be decomposed into two statistical independent subsystems A and B, the probability of the composite system is $p^{A+B} = p^A p^B$, then the Tsallis entropy of the system follows the non-additivity rule:

$$H_T^\alpha(A+B) = H_T^\alpha(A) + H_T^\alpha(B) + (1-\alpha)H_T^\alpha(A)H_T^\alpha(B)$$

Thus, in this paper we use:

$$\Phi_\alpha(t,s) = H_b^\alpha(t,s) + H_w^\alpha(t,s) + (1-\alpha)H_b^\alpha(t,s)H_w^\alpha(t,s)$$

as a criterion function. We obtain our optimal threshold pair $(t^*(\alpha), s^*(\alpha))$ by maximizing the above criterion function $\Phi_\alpha(t,s)$.

III. THE IMPROVED CHAOS OPTIMIZATION ALGORITHMS

Chaos theory is recognized as very useful in many engineering applications. An essential feature of chaotic systems is that small changes in the parameters or the starting values for the data lead to vastly different future behaviors, such as stable fixed points, periodic oscillations, bifurcations, and ergodicity. These behaviors can be analyzed based on Lyapunov exponents and the attractor theory. Details about analysis of chaotic behavior can be found in [11].

The application of chaotic sequences can be an interesting alternative to provide the search diversity in an optimization procedure. Due to the non-repetition of chaos, it can carry out overall searches at higher speeds than stochastic ergodic searches that depend on probabilities. A novel chaotic approach is proposed here based on logistic map: $z_{k+1} = 4z_k(1-z_k)$

Many unconstrained optimization problems with continuous variables can be formulated as the following functional optimization problem.

Find X to minimize $f(\mathbf{x}), \mathbf{x} = [x_1, x_2, \cdots, x_n]$

Subject to $x_i \in [L_i, U_i],\ i = 1,2,\cdots,n$, where f is the objective function, and $\mathbf{x}$ is the decision solution vector consisting of n variables $x_i \in \Re^n$ bounded by lower (L_i) and upper limits (U_i).

The improved chaotic search procedure based on logistic map [12] can be illustrated as follows:

Output:

x^* : Best solution from current run of chaotic search;

f^* : Best objective function (minimization problem);

Algorithm:

Step 1: Initialize the number of the chaos search $M1$, the number of the second chaos search iteration $M2$, iterative constant j_0 and initial value of chaos variables $0 < z_i < 1$ (i=1,2, ⋯, n) which have small differences, Set k=1, $k' = 1, j = 1$ and $f^* = \infty$.

Step 2: The first carrier wave. Map chaos variables $z_i(k)(i = 1,2,\cdots,n)$ into the variance range of optimization variables by the following equation:

$$x_i(k) = L_i + z_i(k)(U_i - L_i)$$

Step 3: Rough search. Compute $f(k) = f(x_i(k))$, where $f(k)$ is an objective function. Set $x_i^* = x_i(0)$, $f^* = f(0)$; if $f(k) \le f^*$, then $f^* = f(k)$, $x_i^* = x_i(k)$.

Step 4: If $k < M1$, then $k = k+1$, and go to Step3, else stop the first chaos search process.

Step 5: The second carrier wave. Map chaos variables $z_i(k)(i = 1,2,\cdots,n)$ into the variance range of optimization variables by the following equation:

$$x_i(k') = x_i^* + \lambda_i(z_i(k') - 0.5)$$

where $\lambda_i = (U_i - L_i)/(j + j_0)$

Step 6: Precise search. Compute $f(k') = f(x_i(k'))$, if $f(k') \le f^*$, then $f^* = f(k')$, $x_i^* = x_i(k')$.

Step 7: if $k' < M1$, then $k' = k' + 1$, and go to Step 6.

Step 8: if $j < M2$, then $j = j + 1$, and go to Step 6, else stop the second chaos search process.

IV. IMAGE SEGMENTATION BASED ON THE PROPOSED METHOD

A..The recursive algorithm

In this section, a fast recursive algorithm is introduced.

Let
$$w_1(t,s) = \sum_{i=0}^{t}\sum_{j=0}^{s}(p(i,j))^{\alpha} \quad (3)$$

$$w_2(t,s) = \sum_{i=t+1}^{255}\sum_{j=s+1}^{255}(p(i,j))^{\alpha}$$

According to formula (1) and (2), the Tsallis entropies may be: $H_b^{\alpha}(t,s) = \frac{1}{\alpha-1}[1 - \frac{w_1(t,s)}{(p_1(t,s))^{\alpha}}]$

$$H_w^{\alpha}(t,s) = \frac{1}{\alpha-1}[1 - \frac{w_2(t,s)}{(p_2(t,s))^{\alpha}}]$$

In the above method, for each pair of (t,s), $w_1(t,s)$ and $p_1(t,s)$ are calculated from (0, 0) to (t,s), it takes too much repetitive calculation to every one of $w_1(t,s)$ and $p_1(t,s)$ .The above equations show that $w_1(t,s)$ is the accumulation of $(p(i,j))^{\alpha}$ at the limits of {(i, j)| i∈[0, t], j∈[0, s]}in the 2D histogram, so the evaluation of $w_1(t,s)$ is limited within the rectangle area at the lower left-hand of the point (t,s). Figure 2 show the 2D plane. It can be seen that the evaluation limits of $w_1(t,s-1)$ is the shadow of positive oblique lines and that of $w_1(t-1,s-1)$ is the shadow of crossed lines.

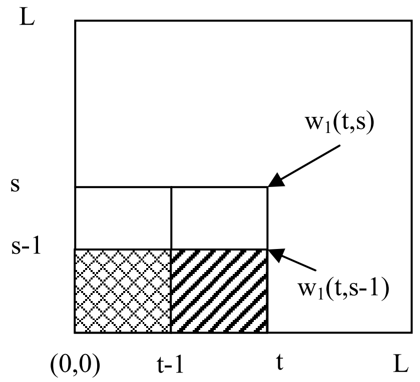


Figure 2. The evaluation limits of $w_1(t,s)$

According to equation (3), the formula for calculating $w_1(t,s)$ can be rewritten in recursive form

$$w_1(t,0) = w_1(t-1,0) + (p(t,0))^{\alpha} \quad (4)$$

$$w_1(t,s) = \sum_{i=0}^{t}\sum_{j=0}^{s-1}(p(i,j))^{\alpha} + \sum_{i=0}^{t}(p(i,s))^{\alpha}$$

$$= w_1(t,s-1) + \sum_{i=0}^{t-1}(p(i,s))^{\alpha} + (p(t,s))^{\alpha}$$

$$= w_1(t,s-1) + w_1(t-1,s) + w_1(t-1,s-1) + (p(t,s))^{\alpha} \quad (5)$$

Let s be the outer circular recursive variant and t be the inner circular recursive variant. When increasing from t-1 to t, $w_1(t-1,0)$ is calculated in previous t circle; when increasing from s-1 to s, $w_1(t-1,s)$ and $w_1(t-1,s-1)$ are the data generated in the previous t circle. $w_1(t,s-1)$ is generated in the last s step of current s circle. Hence, equations (4) and (5) translate equation (3) to the recurring formula. Create an array of 256×256 called w. If $w(1,1) = w_1(0,0)$, t from 0 to 255 and s from 0 to 255, the value of every element in the array w is got in this way. A lot of experiments prove the following equation:

$$w_2(t,s) = w(256,256) - w(t+1,256) - w(256,s+1) + w_1(t,s)$$

The proof is as follows:

$$\sum_{i=0}^{255}\sum_{j=0}^{255}(p(i,j))^{\alpha} = \sum_{i=0}^{255}[\sum_{j=0}^{s}(p(i,j))^{\alpha} + \sum_{j=s+1}^{255}(p(i,j))^{\alpha}] = \sum_{i=0}^{255}\sum_{j=0}^{s}(p(i,j))^{\alpha} + \sum_{i=0}^{255}\sum_{j=s+1}^{255}(p(i,j))^{\alpha}$$

$$= \sum_{i=0}^{255}\sum_{j=0}^{s}(p(i,j))^{\alpha} + \sum_{i=0}^{t}\sum_{j=s+1}^{255}(p(i,j))^{\alpha} + \sum_{i=t+1}^{255}\sum_{j=s+1}^{255}(p(i,j))^{\alpha}$$

$$\sum_{i=0}^{255}\sum_{j=0}^{255}(p(i,j))^{\alpha} = \sum_{i=0}^{255}\sum_{j=0}^{s}(p(i,j))^{\alpha} + \sum_{i=0}^{t}\sum_{j=s+1}^{255}(p(i,j))^{\alpha} + \sum_{i=0}^{t}\sum_{j=0}^{s}(p(i,j))^{\alpha} - \sum_{i=0}^{t}\sum_{j=0}^{s}(p(i,j))^{\alpha} + \sum_{i=t+1}^{255}\sum_{j=s+1}^{255}(p(i,j))^{\alpha}$$

$$= \sum_{i=0}^{255}\sum_{j=0}^{s}(p(i,j))^{\alpha} + \sum_{i=0}^{t}\sum_{j=0}^{255}(p(i,j))^{\alpha} - \sum_{i=0}^{t}\sum_{j=0}^{s}(p(i,j))^{\alpha} + \sum_{i=t+1}^{255}\sum_{j=s+1}^{255}(p(i,j))^{\alpha}$$

$$\sum_{i=t+1}^{255}\sum_{j=s+1}^{255}(p(i,j))^{\alpha} = \sum_{i=0}^{255}\sum_{j=0}^{255}(p(i,j))^{\alpha} - \sum_{i=0}^{255}\sum_{j=0}^{s}(p(i,j))^{\alpha} - \sum_{i=0}^{t}\sum_{j=0}^{255}(p(i,j))^{\alpha} + \sum_{i=0}^{t}\sum_{j=0}^{s}(p(i,j))^{\alpha}$$

Owing to $w_2(t,s) = \sum_{i=t+1}^{255}\sum_{j=s+1}^{255}(p(i,j))^{\alpha}$, $w(256,256) = \sum_{i=0}^{255}\sum_{j=0}^{255}(p(i,j))^{\alpha}$, $w(256,s+1) = \sum_{i=0}^{255}\sum_{j=0}^{s}(p(i,j))^{\alpha}$

$$w(t+1,256) = \sum_{i=0}^{t}\sum_{j=0}^{255}(p(i,j))^{\alpha}, \quad w_1(t,s) = \sum_{i=0}^{t}\sum_{j=0}^{s}(p(i,j))^{\alpha}$$

Hence,

$$w_2(t,s) = w(256,256) - w(t+1,256) - w(256,s+1) + w_1(t,s)$$

Here, each value of $w_1(t,s)$ may also be obtained through w, that's to say, $w_1(t,s) = w(t+1,s+1)$. So once recurring, two values, $w_1(t,s)$ and $w_2(t,s)$ may be taken, which cut off repetitive computation.

In the same manner, create an array of 256×256 called p, the every value of the array p is obtained through equation (2), then $p_1(t,s) = p(t+1,s+1)$,

$$p_2(t,s) = p(256,256) - p(t+1,256) - p(256,s+1) + p_1(t,s)$$

In the above process $p_2(t,s)$ is not approximated, while in the traditional 2-D histogram, since the area 3 and area 4 as shown in Fig.1 contain information about edges and noise alone, they are ignored in the calculation, $p_2(t,s)$ is usually computed approximately, $p_2(t,s) \approx 1 - p_1(t,s)$ so that the optimum threshold could not be obtained precisely sometimes.

B. Combining the improved COA and the maximum 2-D Tsallis entropy method

Considering the recursive algorithm and the COA together for the 2-D maximum Tsallis entropy method, we set the threshold t and s as the search variables, and criterion function $\Phi_{\alpha}(t,s)$ as the objective function to guide the search. After getting the 2-D histogram of the image, we adopt the COA procedure to search the optimum result of (t^*,s^*). Then, the image can be segmented according to the value of (t^*,s^*). On the basis of the above, the fast segmentation based on 2-D Tsallis entropy and the COA is implemented as follows:

Step 1: Input an image and calculate p_{ij}

Step 2: Create two matrixes w and p, and every element value of them is set using the above recursive form respectively, then $p_1(t,s)$, $w_1(t,s)$ and $w_2(t,s)$ may be gotten with w and p.

Step 3: Apply the improved COA to search t and s to get the optimal value.

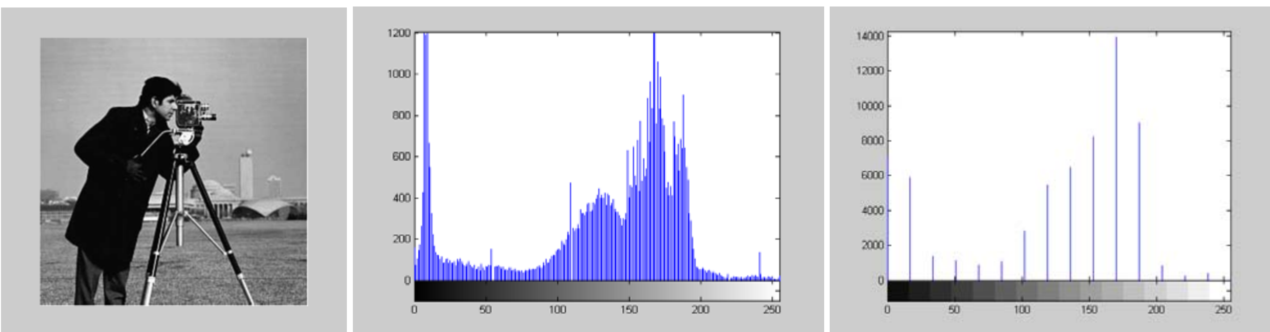
Step 4: Segment the image with the threshold value.

Ⅴ. EXPERIMENTAL RESULTS

In this section, we discuss the experimental results obtained using the proposed method, Qiao & Wu's and Du & Shi's, and some relative things. This discussion includes the choice of the optimal threshold and the presentation of the optimal threshold values of some real-world images. All our experiments are run on the Intel(R) AT/AT Compatible with CPU 1.8G and 256M DDR RAM and MATLAB 6.5. Four classic images for segmentation are illustrated in Fig. 3, 4, 5 and 6 in this paper. Fig.3 (a), Fig.4 (a), Fig. 5 (a) and Fig. 6 (a) are four original images, Owing to the segmentation performance sensitive to the images' brightness and contrast, in order to repeat the experiments to testify the method in this paper in other occasions the four selected image's histograms in sixteen bins are shown in Table Ⅰ and in Fig.3 (c), Fig.4 (c), Fig. 5 (c) and Fig. 6 (c), where a bin is simply a subdivision of the intensity. Fig.3 (e), Fig.4 (e), Fig. 5 (e) and Fig. 6 (e) are the thresholded images with 2-D maximum inter-class cross entropy methods, with some details lost; and Fig.3 (f), Fig.4 (f), Fig. 5 (f) and Fig. 6 (f) are the thresholded images using 2-D maximum entropy, which did not segment the background and the objects well. Fig.3 (d), Fig.4 (d), Fig. 5 (d) and Fig. 6 (d) are the segmentation images using the proposed method when α=0.65, these segmented images are better than those of other two thresholding methods in visual effect. Table Ⅱ lists the optimal threshold values that are found for these images in four different methods, where the fourth method is an exhaustive search method based on Tsallis entropy that can get the global and best solution, but costs the most time. Table Ⅲ displays the limits of two variables t and s, tl and sl stand for the lower limits of t and s respectively, and tu and su the upper limits of t and s. Nq, Nd and Np represent the computing times of the objective function for three methods based on Tsallis entropy in the recurring method, in the method based on the PSO, and in the proposed method based on COA respectively. It shows that the proposed method has least computing times. Table Ⅳ lists the computing time for three methods based on Tsallis entropy, which shows that the proposed method has lower time cost, the three-twentieths second approximately, than only any of the recurring, which was adopted by Qiao & Wu, only the PSO employed in the Du & Shi's approach, and the COA, because, for example, while the objective function in the proposed method is implemented by 174 times, it is by near 256×256 times in the method with only the recurring, and the method with only the COA takes too much repetitive calculation in the kits image experiment. On the grounds that optimization algorithms based on the chaos theory are search methodologies that differ from any of the existing traditional stochastic optimization techniques, due to the non-repetition of chaos, that it can carry out overall searches at higher speeds than stochastic ergodic searches that depend on probabilities, the times taken by only the improved COA is less than that by only the PSO, the computing times by the PSO is 270 compared with 174 by the improved COA in Kits segmentation. However, Table Ⅳ also shows only COA or only recurring does not work well in the 2-D histogram entropy thresholding, for example, in the Cameraman segmentation experiment, only COA needs 29.12 seconds, only recurring needs 2.52 seconds, and the proposed method, which combines the improved COA and the recurring with the stored matrix variables, need only 0.151 second, which decreases $2.52/0.151\approx16.7$ times compared with only the recurring adopted in the Qiao & Wu's method and is $0.151/29.12\approx0.52$ percent of the time taken by only the COA.

TABLE I. THE HISTOGRAMS OF THREE IMAGES IN SIXTEEN BINS

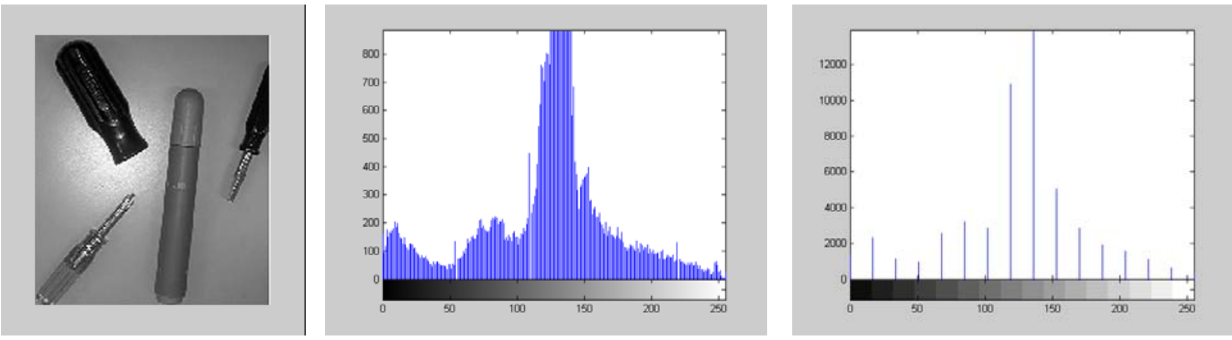
Images	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
Cameraman.	7077	5946	1428	1155	908	1092	2846	5515	6490	8267	13991	9063	877	304	446	131
Kits	1405	2370	1179	983	2635	3252	2904	10935	17288	5086	2870	1964	1613	1132	667	293
Card	47	1120	9449	4680	827	1114	1935	1995	1677	6389	19112	11506	1553	962	376	2794
Columbia	910	7275	8701	12056	8269	6150	4457	4079	3781	3314	2298	1738	1046	806	565	91



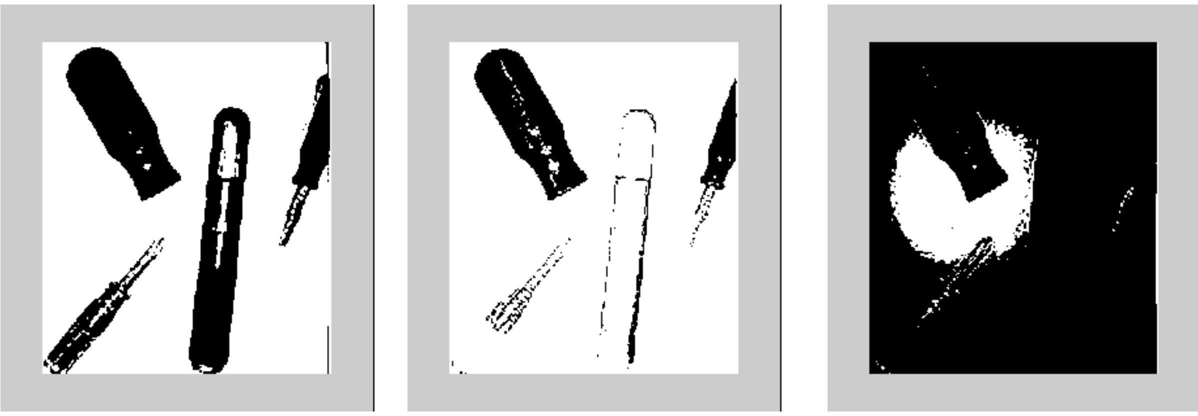
(a) (b) (c)

(d) (e) (f)

Figure 3. Cameraman image, its gray level histograms and the thresholded images.

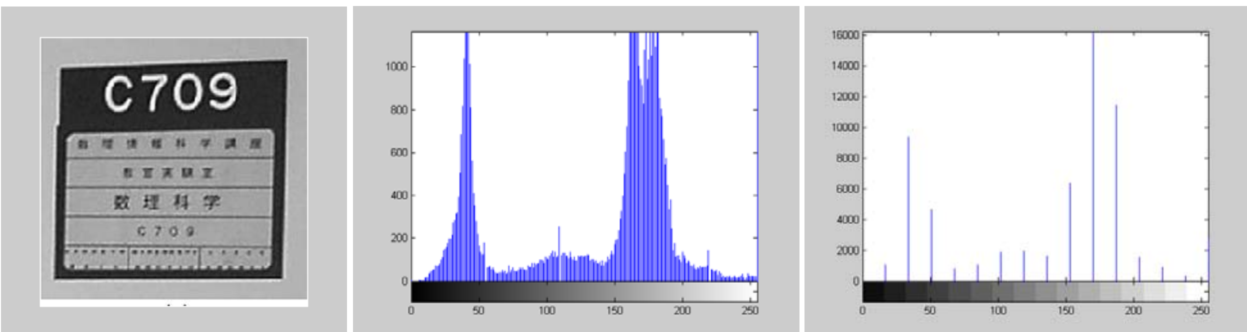


(a) (b) (c)



(d) (e) (f)

Figure 4. Kit image, its gray level histograms and the thresholded images.



(a) (b) (c)

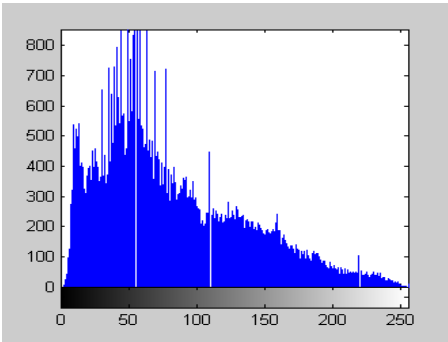
(d)

(e)

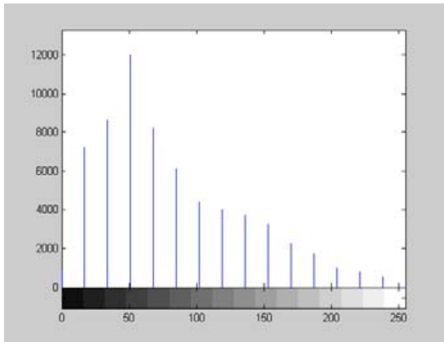
(f)

Figure 5. Card image, its gray level histograms and the thresholded images.

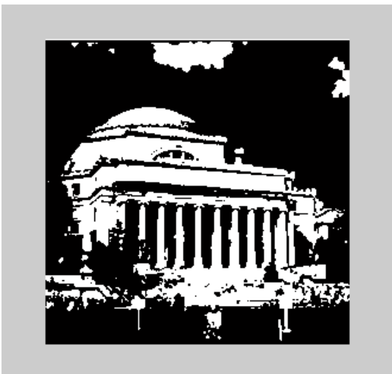
(a)



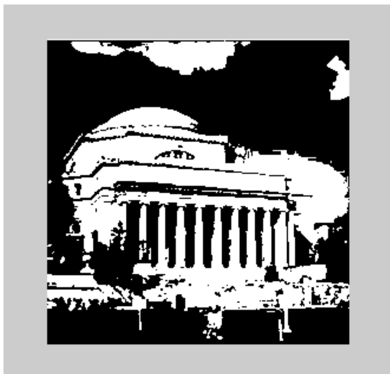
(b)



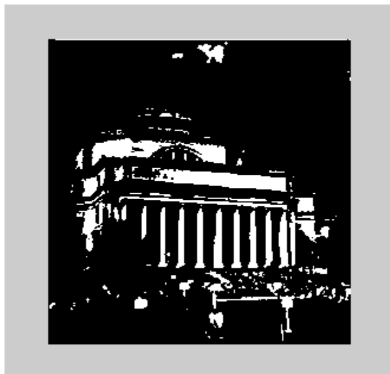
(c)



(d)



(e)



(f)

Figure 6. Columbia image, its gray level histograms and the thresholded images.

TABLE II. (T*, S*) FOR FOUR METHODS

Test images	Methods			
	Qiao & Wu's	*Du & Shi's*	*The proposed*	*The exhaustive*
Cameraman.	(59,82)	(195,152)	(105,111)	(102,104)
Kits	(80,91)	(165,108)	(107,105)	(104,107)
Card	(94,102)	(152,149)	(136,133)	(139,138)
Columbia	(83,100)	(140,123)	(100,101)	(101,102)

TABLE III. THE LIMITS AND THE COMPUTING TIMES OF THE OBJECTIVE FUNCTION FOR THREE IMAGES

Test images	The limits of t and s				The times		
	sl	*su*	*tl*	*tu*	*Nq*	*Nd*	*Np*
Cameraman.	2	249	0	255	63488	370	174
Kits	1	248	0	255	63488	370	174
Card	14	255	0	255	61952	370	174
Columbia	6	246	0	255	61696	370	174

TABLE IV. COMPUTING TIME FOR THREE METHODS BASED ON TSALLIS ENTROPY

Test images	Methods/s		
	Only recurring	*Only COA*	*The proposed*
Cameraman.	2.52	29.12	0.151
Kits	2.51	27.35	0.151
Card	2.49	23.31	0.149
Columbia	2.48	22.46	0.148

Ⅵ. CONCLUSION

The 2-D histogram entropy thresholding such as Tsallis entropy is a good method to do image segmentation, but it gives rise to the exponential increment of computation time in image segmentation, so we have presented an approach based on chaos optimization and recursive algorithm for 2-D Tsallis entropy. The contributions of this work include:

- Once recurring and being put in a matrix, two values, such as $w_1(t,s)$ and $w_2(t,s)$ may be taken through the matrix, which gets rid of repetitive computation.
- Based on the properties of ergodicity, stochastic property and "regularity" of chaos, the chaos optimization method can get global solution with low computational load, and it reduces more computational cost to combine the improved COA and the recursive algorithm to segment images for 2-D histogram. So the proposed approach is effective and has promising future. Especially, the proposed approach is full of importance in the system where real time processing is needed. What's more, the proposed method in this paper may be applied to the other 2-D histogram thresholding methods.
- Although there are a lot of examples of applying COA to the continuous optimization problem, there is relative less in the discrete fields. The maximum Tsallis entropy problem is an example of discrete problem. Owing to the success of the proposed method in this paper, we can see the capability of COA to deal with discrete problems is also promising.
- The image segmentation method proposed in this paper not only reduces much computational time owing to combining chaos optimization and recursive algorithm with the stored matrix variables, but it also get ideal performance taking advantage of the 2-D Tsallis entropy. The experimental results show that the visual effect by this method outperforms those by Qiao & Wu's method using cross entropy and Du & Shi's with 2-D maximum entropy method based on PSO in segmentation images.

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