Research on Programming Algorithm of Trajectory for Hypersonic Vehicles Based on Particle Swarm Optimization

Chuanfeng Li[1,2]
[1]Department of Control Science and Engineering, Huazhong University of Science and Technology, Wuhan, China
[2]Department of Computer and Information Engineering, Luoyang Institute of Science and Technology, Luoyang, China
Email: lichuanfeng@smail.hust.edu.cn

Yongji Wang, Lingling Tang and Zongzhun Zheng
Department of Control Science and Engineering, Huazhong University of Science and Technology, Wuhan, China
Email: {wangyjch@mail.hust.edu.cn, tanglingling@smail.hust.edu.cn, zzzsiemen@163.com}

***Abstract*—Aiming at the sensitivity to initial value and long computation time spent on iteration and programming the reference trajectory in reentry trajectory optimization for hypersonic vehicles, we propose a trajectory programming algorithm, which is based on drag acceleration profile. First of all, models of motion in reentry process of vehicle are built and an independent variable is introduced for optimization to reduce the difficulty of iterative computation. Then the optimal control problem of trajectory programming is simplified as one-dimensional searching problem including longitudinal and lateral parts. Subsequently, the tracking controller is designed for tracking the drag acceleration profile, where the particle swarm optimization is adopted in order to optimize the gain coefficient of tracking controller, from which a good tracking accuracy is obtained. Simulation results reveal that the obtained reentry trajectory presented by this paper can save the subsequently optimization iteration time and approach the best trajectory, which shows that this rational algorithm has great engineering value in practical application.**

***Index Terms*—hypersonic vehicles; constraint processing; drag acceleration profile; trajectory tracking; particle swarm optimization**

I. INTRODUCTION

The reentry of hypersonic vehicles indicates a process like this: when it gets back to earth, the Earth's atmosphere is utilized as a kind of natural resource to consume its huge kinetic energy and potential energy in the form of thermal energy and at last it re-entry to the earth surface. At the same time, compression and friction between vehicle and atmosphere transform a part of energy into heat energy around, and the heat energy partly passes back to the vehicle by convection and radiation. In this whole process the vehicle suffers severe aerodynamic heating and overload[1].

Supported by the National Natural Science Foundation of China (No.60674105), the Scientific Research Cultivation Project of Ministry of Education (20081383), and the Spaceflight Support Foundation (2008).

The optimization of reentry trajectory is done by designing an optimized trajectory and making the vehicle move along it[2], which can alleviate the aerodynamic heating of the vehicle during the reentry process as well as greatly reduce the pressure in designing heat-resistance, furthermore, it can well reduce the overload of the vehicle during the process, hence raise the payload weight and reduce the flight cost. The accuracy of landing points can be guaranteed by the optimal design in which it not only enhance the safety of the vehicle, but make horizontal landing and repeat-pass possible. Reentry trajectory optimization has long been a focus of research in this class of vehicles. Direct numerical treatment of the problem as an optimal control problem is a prevalent approach, given the nonlinearity of entry dynamics and stringent constraints on a typical entry trajectory. A few recent samples of literature in this category are shown in [3,4].

The optimization of reentry trajectory can be described as a nonlinear optimal control problem which is subject to some control constraints, terminal constraints and process constraints. It is difficult to find accurate analytic solutions of optimal control variables because of complicated nonlinear model characteristic. The direct method for trajectory optimization is focused in recent research, but it requires enormous iterative time to search the optimal control variables and its corresponding optimal trajectory[5]. In addition, it is very sensitive to the initial position of the reference trajectory because the quality of the reference trajectory will affect convergence and accuracy in subsequent optimization process. Therefore, in order to guarantee the iterative efficiency, the reference trajectory should be as close to the optimal solution as possible[6,7]. Because the flight control system is mostly depends on aerodynamics, so the trajectory optimization problem is actually to design or search a guidance law. We take the angle of attack α and the bank angle σ as control variables in order to minimize a certain performance index.

Based on constructing non-dimensional dynamic model, this paper calculates the reentry corridor of drag

doi:10.4304/jcp.5.7.1003-1010

acceleration vs. velocity according to the constraint conditions. Among the corridor, the reference profile of drag acceleration can be decided by one-dimensional search method, and then a reference trajectory can be planned quickly by designing tracking controller, which can increase the iterative efficiency in subsequent optimization algorithm as well as enhance the autonomy of vehicles when applied to on-line planning.

II. MOTION MODEL AND ANALYSIS OF VEHICLE

A. Motion Model of Vehicle

We study the reentry trajectory of vehicle under the assumptions of the following conditions.

1) The Earth is a uniform sphere which rotates around itself, and its self-rotation rate keeps invariant.

2) The atmosphere is stationary relative to the Earth and it is even in the same altitude.

3) Vehicle can be regarded as a particle that is unpowered and controllable, and its mass is constant in reentry process.

4) The sideslip angle of vehicle is zero, that is, the lateral force F_z=0.

5) The influence of the coriolis acceleration and implicated acceleration caused by the Earth's self-rotation should be considered in motion equation.

From the assumptions above and according to theoretical mechanics and kinematics principle[8,9], we can deduce the derivation of kinematics equations of hypersonic vehicle. The 3DOF equations of motion through dimensionless method are given by

$$\begin{cases} \dfrac{dR}{d\tau}=V\sin\gamma \\ \dfrac{d\theta}{d\tau}=\dfrac{V\cos\gamma\sin\psi}{R\cos\phi} \\ \dfrac{d\phi}{d\tau}=\dfrac{V\cos\gamma\cos\psi}{R} \\ \dfrac{dV}{d\tau}=-D-\left(\dfrac{\sin\gamma}{R^2}\right)+\omega^2R\cos\phi(\sin\gamma\cos\phi-\cos\gamma\sin\phi\cos\psi) \\ \dfrac{d\gamma}{d\tau}=\dfrac{1}{V}[L\cos\sigma+\left(V^2-\dfrac{1}{R}\right)\left(\dfrac{\cos\gamma}{R}\right)+2\omega V\cos\phi\sin\psi+ \\ \qquad \omega^2R\cos\phi(\cos\gamma\cos\phi+\sin\gamma\cos\psi\sin\phi)] \\ \dfrac{d\psi}{d\tau}=\dfrac{1}{V}[\dfrac{L\sin\sigma}{\cos\gamma}+\dfrac{V^2\cos\gamma\sin\psi\tan\phi}{R}-2\omega V(\tan\gamma\cos\psi\cos\phi-\sin\phi)+ \\ \qquad \dfrac{\omega^2}{\cos\gamma}\sin\psi\sin\phi\cos\phi] \end{cases} \tag{1}$$

where, the dimensionless parameters of geocentric vector R, velocity of vehicle V, time τ and rotational angular velocity of the Earth ω represent the average vector of the earth R_0, $\sqrt{g_0R_0}$, $\sqrt{R_0/g_0}$, $\sqrt{g_0/R_0}$ respectively. g_0 represents the sea level gravitational acceleration. θ and ϕ represent geographical longitude and latitude respectively. γ represents flight path angle, which is the angle between velocity vector and the local horizontal level; ψ represents heading angle, which is the angle between local longitude line and the projection of horizontal plane of velocity vector, along the clockwise rotation toward north, its value is positive. m represents the mass of vehicle, dimensionless drag acceleration and lift acceleration are given by

$$D=\frac{C_D\rho v^2S_r}{2mg_0} \qquad L=\frac{C_L\rho v^2S_r}{2mg_0} \tag{2}$$

B. Analysis of Motion Model

The control law of attack angle can be obtained by qualitative analysis of reentry process. According to flight mission, angle of attack can be preselected[10]. Commonly, the curve of attack angle will change with flight velocity. Then, we can decide the reentry trajectory by deciding the control law of bank angle σ only.

The lift-to-drag ratio is equal to the ratio of lift coefficient and drag coefficient quantitatively, which reveals basic aerodynamic characteristics of vehicle. The lift coefficient and drag coefficient are generally obtained according to the aerodynamic force data table with interpolation technique, and they are a function of attack angle and mach number. In this paper, the lifting reentry vehicle was studied and the change of lift-to-drag ratio L/D with different mach number is plotted in Fig.1.

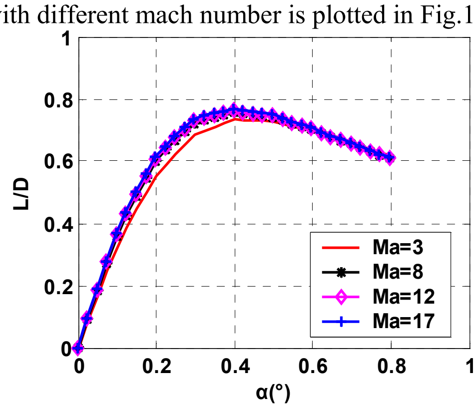

Figure 1. The relation of lift-to-drag ratio and angle of attack in different mach number.

The heating rate constraints of aircraft surface should be taken into account firstly because the vehicle has extreme velocity in initial reentry stage, so the maximum angle of attack is chosen at initial stage, for this reason, we select 10 degrees in this paper. However, in order to ensure that the voyage can be reached in intermediate stage of flight, the position of attack angle should be set at maximum lift-to-drag ratio. In reentry terminal, the angle of attack is set by 4 degree in order to satisfy the requirement of terminal accuracy. By determining these features of attack angle profile which not only ensuring the vehicle has enough flight ability, but making the constraints of heating rate to meet requirements.

The size of bank angle directly affects the overload, heating rate and voyage of vehicle and its symbol has influence on lateral performance index. For the control law of bank angle, it needs to be determined from longitudinal programming and lateral planning. On the one hand, the size of bank angle is determined by the longitudinal programming in reentry process to ensure the requirements of longitudinal voyage. On the other hand, the symbol is determined by lateral planning to meet the requirements of cross-range and angle of lateral. This is a more appropriate formulation for the entry optimization problem since the initial and terminal conditions are given at determinate energy values, whereas time plays no role.

III. TRAJECTORY PROGRAMMING ALGORITHM

A. Longitudinal Trajectory Programming

Generally speaking, the reentry trajectory programming can be divided into the longitudinal profile programming and lateral profile programming. In this paper, we simplify those algorithms and turn them into the two independent parameter optimization problems of one-dimensional search[11].

Due to severe aerodynamic heating, overload and dynamic pressure problem of the reentry process have to be considered when the hypersonic vehicle has a flight voyage of around one thousand kilometers and long flight time in the reentry process. In another aspect, the Quasi-Equilibrium Glide Condition(QEGC) must be taken into account which in order to ensure that the vehicle flies smoothly in reentry process of the trajectory which is a non-strict constraint[12]. This means that we don't order it meets the requirement very strictly, especially in the last phase of the flight. The main function of these constraints takes place in equilibrium gliding stage of the reentry process.

1) Aerodynamic Heating Rate Constraint

In reentry process, the friction between the vehicle and the atmosphere will leads to severe aerodynamic heating problem, and the surface temperatures of different parts of the vehicle are different. The effect of aerodynamic heating of the critical heating region in the vehicle head must be considered as it is relatively serious. On the other hand, it is complex to compute the effect of friction on the temperature of vehicle surface, because the temperature of the surface is decided by many other factors like the material properties of the vehicle and the thermal conversion capacity. So the limits on the surface temperature can be transferred to the constraint for the aerodynamic heating rate in engineering practice, that is to say, we only take the total heating rate of the vehicle surface into account and it has nothing to do with the absorptive heat of the surface:

$$Q_s \le Q_{s\max} \tag{3}$$

where Q_s is determined by the following formula[13]:

$$Q_s = k(\frac{\rho}{\rho_0})^{0.5} V^{3.25} \tag{4}$$

where k is a constant coefficient which determined by the curvature radius of the vehicle head. ρ_0 is the atmosphere density of sea level. ρ is the present inflow of the atmosphere density and it is gained by approximate fitting[14].

$$\rho = \rho_0 e^{-\frac{(r-R_0)}{h_s}} \tag{5}$$

where h_s is scalar height coefficient. The heating rate is as a function of height, and the constraint to heating rate can be transformed into the constraint to height by substituting (3) and (4):

$$R \ge 1 + \frac{h_s}{R_0} \ln \frac{k^2 V^{6.5}}{Q_{smax}^2} \tag{6}$$

The drag acceleration is given by

$$D \le \frac{C_D S_r}{2mg_0} (\frac{Q_{smax}^2}{k^2 V^{4.5}}) \tag{7}$$

2) Overload Constraint

In 3DOF reentry trajectory, the overload constraint quantity is the maximum overload and it depends on structural strength of the vehicle, the overload range of on-flight equipment and the overload capacity of man who can bear. The total overload constraint satisfies:

$$\sqrt{(L^2 + D^2)} \le n_{max} \tag{8}$$

By (2) and (5), the R can be restricted as

$$R \ge 1 + \frac{h_s}{R_0} \ln \frac{\rho_0 R_0 V^2 S_r \sqrt{(C_L{}^2 + C_D{}^2)}}{2n_{max} mg_0} \tag{9}$$

The constraint to the drag acceleration can be transformed into

$$D \le \frac{n_{max} C_D}{\sqrt{C_L{}^2 + C_D{}^2}} \tag{10}$$

3) Dynamic Pressure Constraint

There should be some restrictions on the dynamic pressure in order to reduce the weight of actuator. The maximum dynamic pressure restriction depends on the strength of defending heat materials for aircraft surface and the aerodynamic hinge moment that augments with the change of dynamic pressure[15]. And it should satisfy:

$$q \le q_{max} \tag{11}$$

where

$$q = \frac{1}{2} \rho (VV_0)^2 \tag{12}$$

Equation (13) can be derived by substituting (5)

$$R \ge 1 + \frac{1}{R_0 \beta} \ln \frac{\rho_0 R_0 g_0 V^2}{q_{max}} \tag{13}$$

According to the relationship between dynamic pressure and the velocity as well as the relationship between velocity and drag acceleration, we can conclude the constraint on the drag acceleration:

$$D \le \frac{C_D q_{max} S_r}{mg_0} \tag{14}$$

4) The Quasi-Equilibrium Glide Condition

Under normal circumstances, there will be some oscillation and bounce for reentry trajectory of vehicle in the reentry process and they should be avoided. The ideal reentry trajectory should have no bounce and its flight path angle should change smoothly, which means $\gamma \approx 0$ and $d\gamma/dt \approx 0$. According to (1) and if we ignore the Earth's rotation, then we will conclude that the balance of gliding constraint should satisfy:

$$L\cos\sigma + (V^2 - \frac{1}{R})\frac{1}{R} = 0 \tag{15}$$

Through the calculation under constraints, the upper and lower boundaries of drag acceleration which satisfy constraints can be obtained under every velocity, and form the reentry corridor of drag acceleration and velocity[16].

If we adopt proper drag acceleration curve in reentry corridor, then the process constraints mentioned above will be certainly satisfied. With the consideration of the vehicle characteristics studied in this paper, and the simplification of the profile of five stage drag acceleration applied in US Space Shuttles, we assume that reference drag acceleration profile is composed by two curves. As shown in Fig.2, the variation of the initial drop stages is represent by the first stage starting from D_1, the second curve represents the Quasi-Equilibrium Glide stage, D_3 is the terminal point of gliding phase. Suppose that the relationship between drag acceleration and velocity is a linear curve, and then we have

$$D = D_2 + \frac{D_3 - D_2}{v_3 - v_2}(v - v_2) \tag{16}$$

In the entire drag acceleration profile, the height of state variables such as the initial reentry point, velocity are known, the height of the reentry terminal point and velocity have ideal value and corresponding range. Therefore, the initial drag acceleration D_1 and the terminal point D_3 can be obtained by pre-computation, and the only point need to be identified is the drag acceleration D_2 at the transition point from the initial drop phase to Quasi-Equilibrium Glide stage.

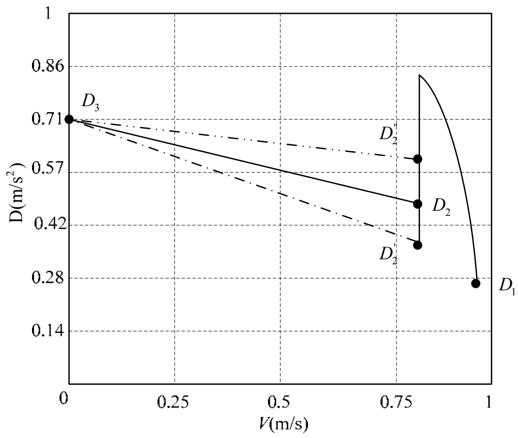

Figure 2. Schematic diagram of drag acceleration profile.

In the initial drop stage, the density of atmosphere is low and the lift of the vehicle is small, we can not perform effective control through the angle of attack and bank angle. The main constraint condition is maximum heating rate, so we can set the bank angle to be zero or a small constant to ensure the satisfaction for the heating rate constraint. Through repeated iteration and integration in this section and when they meet:

$$\left| \frac{dR}{dV} - \left(\frac{dR}{dV} \right)_{QEGC} \right| < \delta \tag{17}$$

That is, the point is corresponding upper bound of D_2.

In Fig.2, when the value of D_2 is set to be between D_{low} and D_{up}, the maximum D_{up} is corresponding to the largest average drag acceleration and the smallest longitudinal voyage, on the contrary, minimum D_{down} is corresponding to the smallest average drag acceleration and the largest longitudinal voyage. The average drag acceleration goes down when the D_2 decreases, reflecting the characteristic of strict monotone decrease, that is, if the flight capacity is sufficient the voyage conditions can be ensured by the value D_2 between D_{low} and D_{up}, and the iterative relations is given by

$$D_2^{i+1} = (D_{low}^{i+1} + D_{up}^{i+1})/2 \tag{18}$$

where

$$\begin{cases} D_{low}^{i+1} = D_2^i,\ D_{up}^{i+1} = D_{up}^i & (s_{togo} > 0) \\ D_{up}^{i+1} = D_2^i,\ D_{low}^{i+1} = D_{low}^i & (s_{togo} < 0) \end{cases} \tag{19}$$

Here, S_{togo} is the longitudinal voyage error between the vehicle and ideal target points.

In the course of searching D_2, we need to track reference profile of drag acceleration to get the variation of control parameters for bank angle and all state variables of the vehicle, and the control parameters of bank angle satisfies the following condition:

$$\cos\sigma = \frac{(L/D)_R}{L/D} \tag{20}$$

From the aforesaid reference drag acceleration profile, the $(L/D)_R$ can be computed as

$$(\frac{L}{D})_R = \frac{L}{D}\cos\sigma = \frac{\frac{1}{D_R}(\frac{dD}{d\tau})_R^2 - (\frac{d^2D}{d\tau^2})_R - \frac{4D_R}{v^2}(\frac{dV}{d\tau})^2 + d}{\frac{D_R^2}{h_s}(1 + \frac{2gh_s}{V^2})} + \frac{(g - \frac{V^2}{R})}{D_R} \tag{21}$$

where the value of drag acceleration and derivative of time can be obtained through the reference drag acceleration, however, the derived model has modest simplification. As it is difficult to track for the drag acceleration profile only through $(L/D)_R$, so we add the lift-drag ratio $\Delta(L/D)$ in our computation:

$$\cos\sigma = \frac{(L/D)_R + \Delta(L/D)}{(L/D)} \tag{22}$$

To guarantee the tracking effect, the PD controller is introduced to determine the $\Delta(L/D)$:

$$\Delta(L/D) = K_1(D_R - D) + K_2(\dot{D}_R - \dot{D}) \tag{23}$$

The adjusting of controller parameters will affect the feasible trajectory accuracy of programming, so we can obtain it through experimental method and fixed-coefficient method. The former relies on expertise, and the latter has huge calculation quantity, in this work we adopt the intelligent optimization search algorithm: PSO, which can search more quickly with suitable control parameters. There will be detailed analysis and elaboration in next section.

B. Lateral Trajectory Programming

Under the circumstance of not making the bank angle reverse in reentry process, the heading of vehicle will generates larger shift and higher terminal deviation because the lifting re-entry vehicle takes the bank angle as the control parameter[17]. The lateral trajectory characteristic of vehicle is determined by the symbol of bank angle σ, therefore, in the final trajectory programming, we need to determine the symbol of bank angle in reentry process. The symbol for bank angle will be reversed if the heading angle error and instruction value exceed a certain limit in reentry process, among which the point where bank angle symbol changes is called reversal point of bank angle. The lateral programming should satisfy two requirements of flight, one is the terminal heading angle ψ and the other is cross-range of vehicle.

The reversal point strategy of bank angle of vehicle can use searching strategy of single inversion point and can also adopt searching strategy of multiple reversal point. If we adopt the single reversal point and meet the requirements above, then the reversal point of bank angle can be found in theory, but it need to consider the longitudinal characteristic at the same time which will become more complicated to solve. Therefore, considering the lateral characteristic, we can search reversal point of bank angle to meet the requirements above by using the searching strategy of two reversal points of bank angle with a short iteration step.

First of all, we predefine the first reversal point V_1 and make it in the starting point of Quasi-Equilibrium Glide stage, then adjust the second reversal point V_2 which has a clear change relationship with heading angle. The heading angle has a positive deviation from the precision bound if the searching point is less than its ideal position, likewise, it has a negative deviation from the precision bound if the searching point is more than its ideal position. So we can adjust the position of reversal point through the derivation state of current heading angle until it gets the lateral trajectory to meet the heading angle ψ. However, the combination of two inversion process can not satisfy the cross-range requirements of reentry process because the first inversion point of vehicle is given in advance.

For that reason, it needs to adjust the first reversal point V_1. Since the inversion position given at first is maximum position of velocity in the Quasi-Equilibrium Glide stage of vehicle, so we can make the inversion position delay shifting new lateral change state of vehicle according to the fixed step. After the satisfaction for cross-range, the heading angle ψ will derivate from the requirements again and it needs to be adjusted iteratively. Finally, we will find two reversal points can meet the requirements for cross-range and heading angle through iteration and adjustment again and again.

The flow chart of the programming algorithm is shown as in Fig.3.

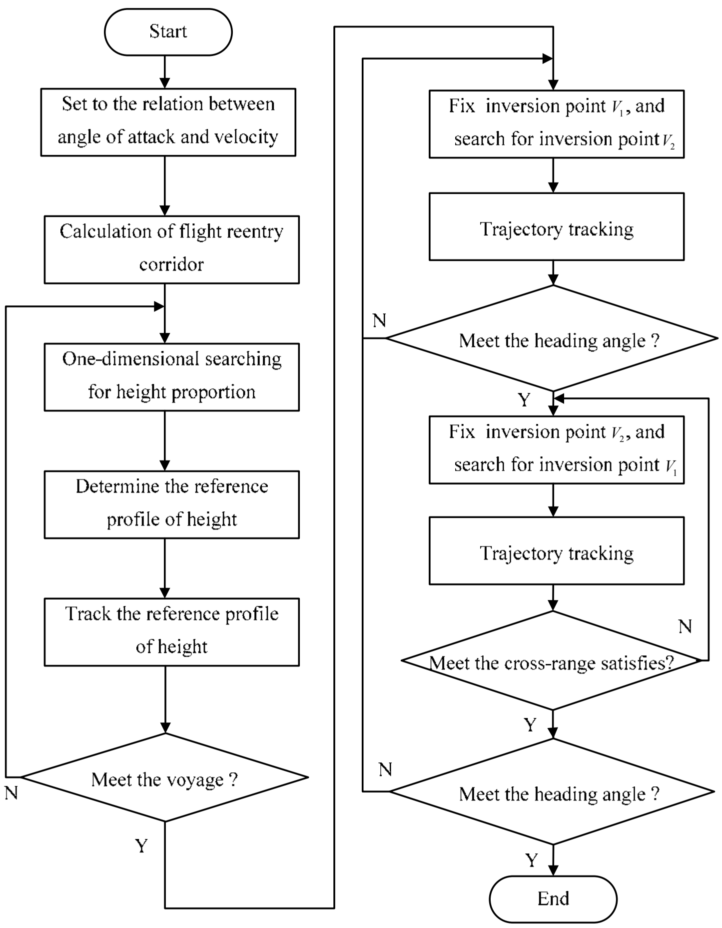

Figure 3. Flowchart of trajectory programming strategy.

IV. TRAJECTORY TRACKING CONTROLLER WITH PSO ALGORITHM

A. Priciple of Particle Swarm Optimization

The particle swarm optimization(PSO) is a population based stochastic optimization technique developed by Dr.Eberhart and Dr.Kennedy in 1995, inspired by social behavior of bird flocking or fish schooling[18]. The principle of standard particle swarm optimization algorithm can be described as follows:

Every feasible solution is called a “particle” in optimization problems, and a large number of feasible solutions constitute the particle population. Each individual updates its own position constantly by obtaining the search experience of its own and other particles of the population, and they will eventually tend to the same point which is the optimal solution of the solution space. Of course, a large-scale iterative is required if they converge to the same point completely, so we take a certain radius of the particle population converge or maximum iteration time as the termination judgment condition[19].

We suppose the search space of optimization problem is D-dimensional and the number of particles is N. The position of the ith particle can be represented as a vector $X_i = (x_{i1}, x_{i2}, \ldots, x_{iD})$ and the rate of the position change(velocity) for particle i is represented as the vector $V_i = (v_{i1}, v_{i2}, \ldots, v_{iD})$, where $1 \le d \le D$ and $1 \le i \le N$. The best previous position (the position giving the best fitness value) of the ith particle is recorded and represented as $P_i = (p_{i1}, p_{i2}, \ldots, p_{iD})$.The position of the best particle among all particles in the population is represented by the symbol P_g or *gBest*. The position and its change rate of every individual updates continuously according to the following equations:

$$\begin{aligned} v_{id}(t+1) = w \times v_{id} + c_1 \times rand() \times (p_{id}(t) - x_{id}(t)) \\ + c_2 \times rand() \times (p_{gd}(t) - x_{id}(t)) \end{aligned} \tag{24}$$

$$x_{id}(t+1) = x_{id}(t) + v_{id}(t+1) \tag{25}$$

where, c_1 and c_2 are positive constants which are called acceleration factors. They determine the dependent degree on individual information and group information. The function rand () is used to generate random numbers between 0 and 1 to ensure a certain degree of diversity for searched solutions. w represents the inertia factor while the d-dimensional of the change scope of location and the change scope of velocity are [$-x_{dmax}$ x_{dmax}] and [$-v_{dmax}$ v_{dmax}] respectively, which can becomes symmetry through moving horizontally. We will get the bound values if the x_{id} and v_{id} are beyond the bounds. The initial velocity and position of PSO can be generated randomly according to the position and velocity bound, or set by the expert experience.

Some optimization parameters have relatively mature common value throughout the optimization process, such as the acceleration factor can be set as $c_1 = c_2 = 2$ and the inertial factor is set as $w=1$. However, among the adjustable parameters, the maximum speed of particles v_{max} affects the searching efficiency of algorithm greatly, especially when there is no inertial factor for the original PSO and it will affect the initialized situation of PSO. The value of v_{max} can be obtained by simulation and debugging in algorithm implementation, but over accuracy is not necessary. The scale of population also has certain effect for the algorithm in that if the number are small, then they will fall into local optimal solution[20], on the other hand, the larger the number of the particles, the slower the convergence speed, which thus lost the advantage of rapidity of PSO.

At present, the most popular method is time-varying weight method and this adjustment method is a weights strategy with the linear reduced number of iterations which is presented by Shi[21]. Its effectiveness has been verified in a large number of applications, that is

$$\omega = \omega_1 + \frac{(\omega_0 - \omega_1)}{N}(N - n) \tag{26}$$

where, N is the maximum iteration number set by PSO algorithm, n represents the current iteration, ω_0 and ω_1 are initial and ultimate inertia weight respectively.

B. Selection of Optimization Parameters

After the reference profile of drag acceleration of vehicle has been obtained, we need to track the profile in order to obtain the control parameter of bank angle, in which the PD controller is adopted. The particle swarm optimization algorithm can be used to optimize the parameters of the tracking controller because it is a function optimization problem in essence [22].

First of all, the solution space of parameter application problem should be mapped to population of PSO, moreover, the particle can be set to a two-dimensional real number vector, and the first dimension is the proportional coefficient of controller while the second dimension is the differential coefficient of controller. Here, it is a continuous particle optimization problem. In this optimization program, we select 30 particles, and the maximum displacement and velocity rely on the problem itself. Considering the proportional relation of control parameters and error, and it is reasonable to set the searching range of maximum displacement varies between 0 and 0.1. In this paper, we limit the maximum of particle between 0 and 0.01 in order to find better optimal solution.

Secondly, the problem is about how to select the fitness function. In order to ensure the good performance in the whole tracking process, we take the sum of square error of instruction height and tracking height as the performance index, and at the same time in order to ensure the constraint of smooth shift for control quantity, we also consider adding the constraint of it for the fitness function, that is

$$f = \sum_{i=1}^{n} (H_{com}^i - H^i)^2 + \sum_{i=1}^{n-1} (\sigma^{i+1} - \sigma^i)^2 \tag{27}$$

where, n represents the total steps of flight simulation, that is, the error value needs to be accumulated in each step of simulation. Therefore, aiming at one computation of fitness function, the whole flight process requires to be calculated for one time in order to obtain the entire flight state.

V. SIMULATION AND RESULTS ANALYSIS

The programming algorithm presented in this paper is implemented in generating entry trajectory with a certain vehicle data. The terminal constraint of target point and satisfying condition are shown in table 1.

TABLE I. TERMINAL CONSTRAINTS OF REFERENCE TRAJECTORY

	ΔR (m)	**$\Delta\theta$ (°)**	**$\Delta\phi$ (°)**	**ΔV (m/s)**	**$\Delta\gamma$ (°)**	**$\Delta\psi$ (°)**
Constraint requirement	±500	±0.05	±0.05	±20	-3~0	±2
Simulation results	10.4	0.045	0.019	2.7	-1.51	-1.08

From Fig.4 to Fig.8, some curve of parameter variables are given in reentry process of vehicle, which obtained by the programming algorithm presented in this paper.

The tracking performances of height are plotted in Fig.4 and Fig.5 respectively by PSO algorithm and LQR method. The PSO algorithm has better tracking performance for height by comparison.

With respect to the tracking controller parameters of trajectory programming problem for drag acceleration profile, which is also optimized by PSO algorithm and the optimization results are very similar to one-dimensional searching method by trail-and-error. From Fig.6 and Fig.7, we can see the tracking performance has no large difference with two methods. But the PSO algorithm is a random search method using computer and dependency on human is little. However, the one-dimensional searching method by trail-and-error need excess practical experience, so the PSO algorithm has certain advantages. Fig.8 denotes the optimization curve of bank angle using PSO and the control variable changes smoothly and meets the control constraints well.

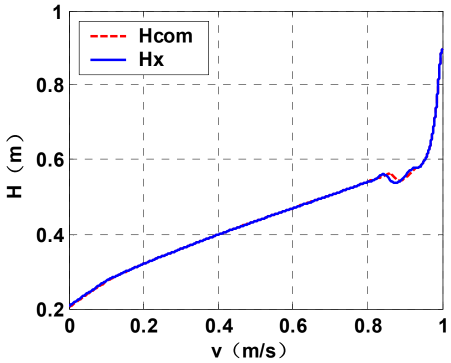

Figure 4. Tracking curves for height using PSO algorithm.

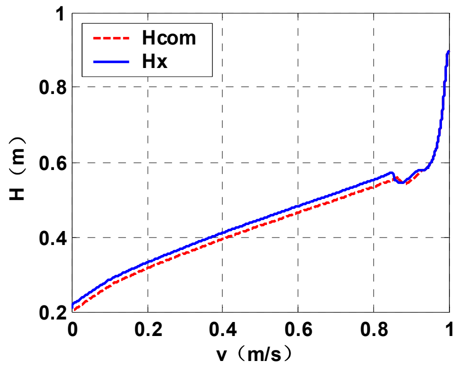

Figure 5. Tracking curves for height using LQR method.

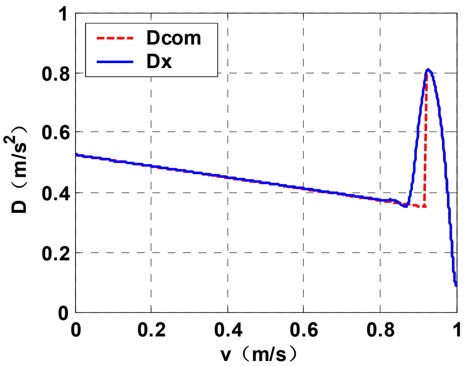

Figure 6. Tracking curves of drag acceleration using PSO algorithm.

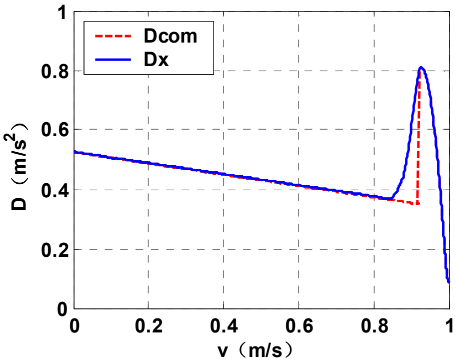

Figure 7. Tracking curves of drag acceleration using one-dimensional searching.

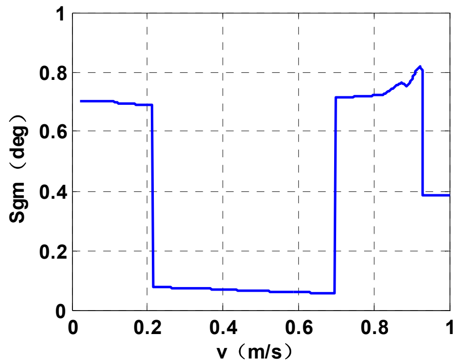

Figure 8. Optimization curve of bank angle using PSO.

Simulation results have proved that the constraint conditions of terminal height, longitude, latitude, bank angle and heading angle are well satisfied in the course of flight which adopted by the trajectory programming algorithm in this article. Furthermore, the presented algorithm can satisfy the constraints of heating rate, overload, and dynamic pressure in reentry process of vehicle and the reentry trajectory is smooth as well as easy to control.

VI. CONCLUSIONS

This paper introduces the study of programming algorithm of reentry trajectory on a certain hypersonic gliding vehicle model. The ultimate aim is planning an entry reference trajectory as initial value to advance the speed of entry trajectory optimization. Equations of motion are normalized and an independent variable is introduced for optimization to reduce the difficulty of iterative computation. On the basis of analyzing the profile of drag acceleration vs. velocity, programming trajectory was simplified as one-dimensional searching problem from longitudinal and lateral in two ways, which can meet the requirements in iteration time and accuracy in generating trajectories and beneficial to the subsequent optimization iteration, especially, the particle swarm optimization algorithm has been adopted to improve the accuracy of trajectory tracking. Simulation results show the effectiveness by the programming algorithm above, and it has certain engineering application value in trajectory programming for vehicles.

ACKNOWLEDGMENTS

We are grateful for the support of the National Nature Science Foundation of China, No. 60674105, the scientific research cultivation project of Ministry of Education, No. 20081383 and the 2008 Spaceflight Support Foundation.

REFERENCES

[1] H.Y,Zhao, *Reentry Dynamics and Guidance of Vehicles*, Changsha:National University of Defense Technology Press,first edition,1997, pp.1–22.

[2] J.A.Leavitt, K. D.Mease, “Feasible Trajectory Generation for Atomospheric Entry Guidance,” *Journal of Guidance, Control, and Dynamics*, vol.30, no.2,pp.473–481,Mar.2007.

[3] C.Jansch,A.Markl, “Trajectory Optimization and Guidance for a Hermes-Type Reentry Vehicle,” In *Proceedings of AIAA Guidance, Navitgation, and Control Conference*, New Orleans, LA, pp.543–553, Aug.1991.

[4] J.D.Schierman, J.R.Hull, “In-flight Entry trajectory Optimization for Reusable Launch Vehicles,” In *Proceeding of AIAA Guidance, Navigation, and Control Conference and Exhibit*, San Francisco, California, Aug.2005.

[5] R. J.Vanderbei, “Case Studies in Trajectory Optimization: Trains, Planes, and Other Pastimes,” *Optimization and Engineering*, vol. 2, no. 2, pp. 215–243, Jun.2001.

[6] K.P.Bollino,I.M.Ross and D.D.Doman,“Optimal Nonlinear Feedback Guidance for Reentry Vehicle,” In *Proceeding of AIAA Guidance, Navigation, and Control Conference and Exhibit*, Keystone,Volorado, pp.563–582, Aug.2006.

[7] P.Lu, “Regulation about time-varying trajectories: precision entry guidance illustrated,”*Journal of Guidance, Control, and Dynamics*,vol.22,no.6, pp.784-790,Nov.1999.

[8] N.X.Vinh, A.Busemann and R.D.Culp, *Hypersoinic and Planetary Entry Flight Mechanics*, Ann Arbor, MI: The University of Michigan Press, 1980.

[9] P.Lu, John M.Hanson, “Entry Guidance for the X-33 Vehicle,” *Journal of Spacecraft and Rockets*, vol.35, no.3,pp. 342-349, May.1998.

[10] J.M.,Hanson, R.Jones, and D.Krupp, “Advanced Guidance and Control Methods for Reusable Launch Vehicle: Test Results,” In *Proceedings of AIAA Guidance , Navigation, and Control Conference and Exhibit*, Monterey, California, Aug.2002.

[11] D. W.Miles, S. M.Rock, “Real-time Dynamic Trajectory Optimization,” In *Proceedings of AIAA Guidance, Navigation, and Control Conference*, San Diego, CA Jul.1996

[12] M.G.Wang, Q.Tang and T.G.Pei, “Fast optimization of constrained reentry trajectory”, *57th International Astronautical Congress*, Valencia, Spain, Oct.2006.

[13] Z.Shen, P.Lu, “On-Board generation of three-dimensional constrained entry trajectories,” In *Proceedings of AIAA Guidance , Navigation, and Control Conference and Exhibit*, Monterey, California, Aug.2002.

[14] Justus, C. G., Johnson, D. L. *The NASA/MSFC Global Reference Atomospheric Model:1999 Version(Gram-99)*, NASA TM209630, May, 1999.

[15] R. J.Vanderbei, “Case Studies in Trajectory Optimization: Trains, Planes, and Other Pastimes,” *Optimization and Engineering*,vol. 2, no. 2, pp. 215–243, Jun.2001.

[16] K. D.Mease, D. T.Chen and P.Teufel, “Reduced-Order Entry Trajectoy Planning for Acceleration Guidance,” *Journal of Guidance, Control, and Dynamics*, vol.25, no.2, pp.257–266, Mar.2002.

[17] Z.Shen, P.Lu, “Dynamic lateral entry guidance logic,” *Journal of Guidance, Control and Dynamics*, vol.27, no.6, pp.949–959,Nov.2004.

[18] J.Kennedy, R.C.Eberhart, “Partical Swarm Optimization”. In *Proceedings of the IEEE International Conference on Neural Networks*, pp.1942–1948, 1995.

[19] M.Clerc, J.Kennedy, “The Partical Swarm: Explosion, Stability, and Convergence in a Mutiple-dimensional Complex Space,” *IEEE Transactions on Evolutionary Computation*, vol.6, no.1, pp.58–73, Feb.2002.

[20] F.van den Berh,A.P.Engelbrecht, “A cooperative approach to particle swarm optimization”.*IEEE Transaction on Evolutionary Computation*, vol.8, no.3, pp.225–239, Jun. 2004.

[21] Y.H.Shi,R.C.Eberhart, “A modified particle swarm optimizer,” In *Proceedings of IEEE International Conference on Evolutionary Computation*, Anchorage, AK, pp.69–73, May.1998.

[22] A.A.A.EI-Gammal, A.A.EI-Samahy, “A modified design of PID controller for DC motor drives using Particle Swarm Optimization PSO,” *International Conference on Power Engineering, Energy and Electrical Drives*, Lisbon, Portugal, pp.419–424,Mar.2009.

Chuanfeng Li is a Ph.D. candidate of Huazhong University of Science and Technology of China. He received B.S. degree in Electronic and Information Engineering College from Henan University of Science and Technology in 2000, and received M.S. degree in Department of Control Science and Engineering from Huazhong University of Science and Technology in 2007. His current research interests include guidance, control and trajectory optimization on aircrafts.

Yongji Wang is a Professor of Electrical Engineering. He has received his MSc and the PhD in Automation from HUST in 1984 and 1990, respectively. He has guided many PhD research scholars. He is a Member of IEEE, USA; Standing Member of Council of Electric Automation Committee of Chinese Automation Society and Member of Council of Intelligent Robot Committee of Chinese Artificial Intelligence Society. He is an Area Editor (Asia and Pacific) of International Journal of Modelling, Identification and Control. His main interest is in intelligent control and autonomous mobile robots and he has done research in neural network control, predictive control, adaptive control and most recently, vehicles trajectory optimization, guidance and control system design.

Lingling Tang received his Bachelor’s degree in Automation in 2007 from Wuhan University of Technology, China, and his Master’s degree in Control Science and Engineering in 2009 from Huazhong University of Science and Technology, China. His research interests include trajectory optimization on aircrafts and communication software development.

Zongzhun Zheng received his Bachelor’s degree in Automation in 2005 from Fuzhou University, China, and his Master’s degree in Control Science and Engineering in 2007 from Huazhong University of Science and Technology, China. He is currently working towards the PhD. degree in Control Theory and Control Engineering at Huazhong University of Science and Technology. His research interests are in vehicle guidance, control and trajectory optimization.