A Novel Diversity-Controlled Genetic Algorithm for Optimization of BIBO Stable Digital IF Filters Over CSD Multiplier Coefficient Space

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Abstract— **This paper presents a novel diversity-controlled (DC) genetic algorithm (GA) for the optimization of digital Intermediate Frequency (IF) filters over the (finite-precision) canonical signed-digit (CSD) multiplier coefficient space. This optimization exploits the bilinear-lossless-discrete-integrator (bilinear-LDI) lattice digital filter design approach for the realization of the required infinite-precision seed digital IF filter chromosome. A look-up table (LUT) approach is proposed to ensure that the finite-precision CSD digital IF filter chromosomes generated in the course of DCGA optimization are guaranteed to be bounded-input bounded-output (BIBO) stable. The salient feature of DCGA optimization is that it permits external control over the population diversity (i.e. the parent selection pressure) to achieve a high convergence speed. This feature is illustrated through the application of the proposed DCGA optimization to the design of a pair of practical digital IF filters satisfying different design specifications. It is observed that, for both digital IF filter designs, the DCGA optimization results in around an order of magnitude improvement in the convergence speed as compared to a conventional GA optimization.**

Index Terms— **Digital IF Filters, Diversity-Controlled Genetic Algorithms, Bilinear-LDI Lattice Digital Filters, BIBO Stability, Canonical Signed-Digit Numbers, Finite-Precision Digital Filter Optimization.**

I. INTRODUCTION

Intermediate frequency (IF) filters find a variety of applications in modern communication systems, for example in superheterodyne receivers in radio and television broadcast networks, and the American Mobile Telephone System digital AMPS (IS-54) [1] cellular communication. Other wireless digital communication systems such as CDMA (IS-95) also make use of linear-phase wide-passband IF filters [2].

IF filters are required to satisfy stringent frequency-selective bandpass magnitude frequency-response characteristics. These filters select the signal (frequency-shifted via a local oscillator) associated with the desired channel, while attenuating the signal power levels associated with all the unwanted (adjacent) channels.

Conventionally, IF filters are realized in hardware in terms of a combination of analog operational transconductance amplifiers (OTAs) and capacitors (Cs). In [3], an OTA-C IF filter was designed by employing genetic algorithms (GAs) for the optimization of the corresponding magnitude frequency-response characteristics.

IF filters can also be realized as infinite impulse-response (IIR) digital filters by using the wave-digital lattice [4] or the bilinear-LDI digital lattice design technique [5]. The salient feature of digital IF filters is that they can be integrated with other parts of the overall communication system (e.g. with the constituent A/D converter) on the same chip by employing the emerging CMOS hardware implementation technologies. Moreover, they permit high yields and involve low implementation costs. Therefore, the resulting overall communication system tends to require smaller chip area and entails lower power consumption as compared to the corresponding system incorporating analog IF filters [4].

This paper is concerned with the design and discrete optimization of digital IF filters over the canonical signed-digit (CSD) multiplier coefficient space. Such an optimization problem tends to have a multimodal cost function, calling for built-in internal or external mechanisms for escaping from local optimal solutions in the course of optimization.

The most widely used techniques for the above discrete optimization problems include simulated-annealing (SA) algorithms [6], [7] and GAs.

SA algorithms often require too many cost function evaluations before convergence, leading to a high computational overhead [1].

It is well known that GAs provide a promising approach to solve discrete and multimodal optimization problems due to the fact that they are capable of automatically finding near-optimal solutions while keeping the computational complexity of the optimization at moderate levels [9]. Consequently, they have emerged as an efficient alternative for the optimization of finite impulse-response (FIR) and IIR digital filters [10]. These algorithms encode the digital filter realization into a chromosome, and proceed toward an optimal solution through the evolution of a population of potential candidate chromosomes iteratively from one generation to the next.

In [11], GAs were applied to the optimization of the magnitude frequency-response of a digital IF filters, where the IF filter chromosome was formed by the concatenation of the binary representation of the constituent

[1]Seeker Optimization (SO) [8] is another emerging technique, but is not directly applicable to the optimization of finite-precision digital filters.

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multiplier coefficient values. From a practical point of view, particularly in the hardware implementation of the digital IF filters, there is every incentive to represent the constituent multiplier coefficient values in more computationally-efficient number systems, e.g. signed-power-of-two (SPT) [12] or CSD systems [13], while still satisfying all the other design specifications. This is mainly because of the fact that such number systems permit the representation of multiplier coefficient values in terms of only a few non-zero digits within the coefficient wordlength, thereby reducing the corresponding multiplication operations to few shift and add operations.

In [14], a nonlinear programming approach was developed for the optimization of frequency response-masking (FRM) FIR digital filters over the SPT multiplier coefficient space. In [15], GAs were applied to the same FRM FIR digital filter optimization problem, leading to FIR digital filters with superior magnitude frequency-response characteristics. In addition, it was shown that the use of GAs overcomes the inherent drawback of the hitherto linear programming techniques which require the separate optimization of the constituent FIR digital sub-filters.

It is well known that conventional GAs do not search the solution space robustly since they lack mechanisms through which entrapment at local optimal solutions can be avoided. It was demonstrated in [16] that diversity control (DC) can be applied to help to increase the convergence speed of conventional GAs.

The main principle behind DCGA is to increase the diversity of the population pool through the incorporation of additional non-elite chromosomes based on a pair of external control parameters. In principle, DCGA is capable of finding the global optimal solution provided that no bound is imposed on the constituent number of generations. However, in practical situations, DCGA is set to terminate once all of the design specifications have been satisfied. In such situations, the resulting solution may or may not represent the global optimal solution, but simply a solution that satisfies all of the given design specifications.

Unfortunately, a direct application of DCGA to the optimization of digital IF filters gives rise to two separate problems:

- The first problem arises because of the fact that the operations of crossover and mutation in the course of DCGA optimization may lead to chromosomes that may no longer conform to the CSD number format.
- The second problem, on the other hand, stems from the fact that DCGA optimization may result in a solution chromosome that satisfies the given magnitude frequency-response design specifications, but the corresponding digital IF is not bounded-input bounded-output (BIBO) stable.

In summary, GAs lack inherent mechanisms to ensure that the optimized digital IF filters conform to CSD number format, and that they are BIBO stable.

In [17], the first problem was resolved in the context of the DCGA optimization of FRM [12] FIR digital filters. This was facilitated by generating an indexed look-up table (LUT) of *permissible* CSD multiplier coefficient values (with pre-specified wordlengths and pre-specified maximum number of nonzero digits), and by employing the indices of the resulting multiplier coefficient values (as opposed to their values) to represent FRM FIR digital filter chromosomes. The key point in generating the CSD LUT is to ensure that the constituent indices form a *closed* set under the operations of crossover and mutation (or other similar operations) in the course of the underlying DCGA optimization so as to preserve adherence to the CSD number system.

This paper presents a novel approach to the design and DCGA optimization of the magnitude frequency-response of BIBO stable digital IF filters over the CSD multiplier coefficient space. In this approach, the above mentioned two problems are resolved by resorting to three different LUTs. These LUTs consist of CSD multiplier coefficient values which automatically lead to BIBO stable digital IF filter chromosomes under the operations of crossover and mutation throughout the course of DCGA optimization. In this way, the resulting digital IF filter chromosomes not only conform to the CSD number system, but are also BIBO stable (without any recourse to the process of gene repair).

The proposed DCGA optimization starts from a seed digital IF filter consisting of infinite-precision multiplier coefficient values. The infinite-precision seed digital IF filter is derived from a corresponding analog prototype reference bandpass filter section by using the bilinear-lossless-discrete-integrator (bilinear-LDI) lattice digital filter realization technique [5]. The main advantage of the bilinear-LDI lattice digital filter design approach is that the resulting digital IF filters exhibit exceptionally low passband sensitivity to multiplier coefficient values, while being minimal in the number of digital multipliers and digital unit-delays [5][2]. Then, the infinite-precision multiplier coefficient values are quantized to their closest CSD counterparts (to within the pre-specified wordlength and the pre-specified maximum number of non-zero digits), in order to obtain a corresponding finite-precision seed digital IF filter. The resulting CSD seed digital IF filter is subsequently used to form an initial population pool of CSD digital IF filter chromosomes for DCGA optimization[3].

The remainder of this paper is organized as follows: Section II is concerned with the application of the bilinear-LDI lattice digital filter realization approach to the design of the infinite-precision seed digital IF filter. Section III presents an overview of the DCGA optimization technique. Section IV is concerned with the development of novel BIBO stability constraints for CSD digital IF filters. Section V proposes novel CSD LUTs for ensuring the BIBO stability of digital IF filter chro-

[2]They are also practically minimal in the number of digital adders.

[3]It is important to point out that due to the subsequent DCGA optimization, the initial analog prototype reference filter needs to satisfy the given magnitude-frequency design specifications *only approximately*.

mosomes throughout the course of DCGA optimization. Section VI outlines the digital IF filter design methodology based on DCGA optimization. Section VII presents the application of the proposed DCGA optimization to the design of a pair of digital IF filters for two different design specifications. Finally, section VIII summaries the main conclusions of the paper.

II. Design of Infinite-Precision Seed Digital IF Filter

This section is concerned with the realization of an infinite-precision seed digital IF filter for subsequent DCGA optimization. The infinite-precision seed digital IF filter is obtained *indirectly* by applying the bilinear-LDI lattice digital filter realization technique [5] to a suitable analog prototype reference filter.

The salient features of the bilinear-LDI lattice digital filter realization technique are twofold:

- It employs the LDI frequency transformation
$$s = \frac{1}{T}(z^{\frac{1}{2}} - z^{-\frac{1}{2}}) \tag{1}$$
for digital filter realization, leading to a digital filter incorporating LDIs as the only frequency-dependent elements. Here, s (z) represents the continuous-time (discrete-time) complex frequency-variable, and T represents the sampling period.
- It maintains the bilinear frequency transformation
$$s = \frac{2}{T}\frac{z-1}{z+1} \tag{2}$$
for mapping between the (discrete-time) z-domain and the corresponding (continuous-time) s-domain frequency-response specifications.

The resulting bilinear-LDI digital IF filters themselves have the following desirable features from a hardware realization perspective:

- They are minimal in the number of digital multiplication operations.
- They lend themselves to fast two-cycle parallel digital signal processing speeds.
- They exhibit exceptionally low passband sensitivity to their multiplier coefficient values, permitting small coefficient wordlengths.

In this paper, the analog prototype reference filter is chosen as a cascade combination of three identical passive bandpass filter sections [4]. The individual bandpass filter sections themselves are formed as equally resistively terminated symmetrical lattice reactance two-port networks as shown in Fig. 1, where $Z_1(s)$ and $Z_2(s)$ represent the series arm and lattice arm reactances, respectively.

The bandpass filter section in Fig. 1 possesses a voltage transfer function

$$H_{BP}(s) = \frac{1}{2}\frac{Z_1(s) - Z_2(s)}{1 + Z_1(s) + Z_2(s) + Z_1(s)Z_2(s)} \tag{3}$$

By choosing the bandpass transfer function $H_{BP}(s)$ to be of order six, the series arm and lattice arm reactances

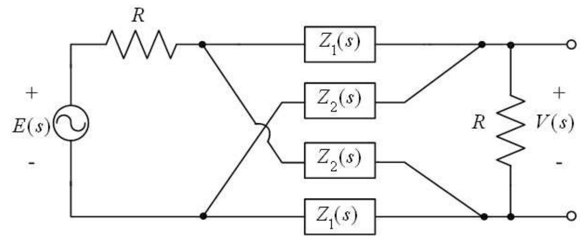

Figure 1. Analog Prototype Reference Bandpass Filter Section

$Z_1(s)$ and $Z_2(s)$ can be realized as shown in Figs. 2a and 2b, respectively, where C_{11} (L_{11}) represents the constituent series capacitor (inductor), and where C_{21} and C_{22} (L_{21} and L_{22}) represent the constituent lattice capacitors (inductors).

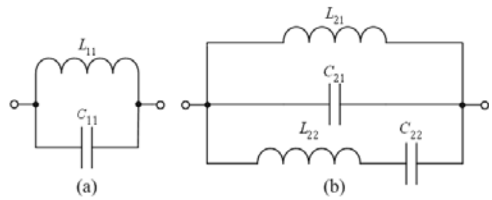

Figure 2. Realization of Reactances $Z_1(s)$ and $Z_2(s)$

It is expedient to decompose the transfer function $H_{BP}(s)$ in Eqn. (3) into a pair of transfer functions $H_1(s)$ and $H_2(s)$ in accordance with

$$H_{BP}(s) = \frac{1}{2}[H_1(s) - H_2(s)] \tag{4}$$

where

$$H_i(s) = \frac{Z_i(s)}{1 + Z_i(s)} \tag{5}$$

for $i = 1, 2$ as shown in Fig. 3. Consequently, the transfer functions $H_i(s)$ can be realized by means of the feedback systems shown in Fig. 4.

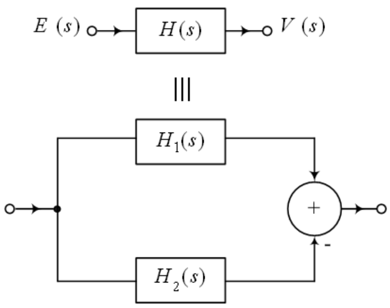

Figure 3. Decomposition of Transfer Function $H(s)$ into Transfer Functions $H_1(s)$ and $H_2(s)$

By employing the bilinear-LDI lattice digital filter realization technique [18], the discrete-time transfer functions

$$\bar{H}_i(z) = H_i(s)|_{s=\frac{2}{T}\frac{z-1}{z+1}} \tag{6}$$

for $i = 1$ and $i = 2$ can be realized in the z-domain as shown in Figs. 5 and 6, respectively.

Moreover, the values of the constituent multiplier coefficients m_{ik_i} (for $k_1 = 1, 2$, and $k_2 = 1, 2, 3, 4$) can be

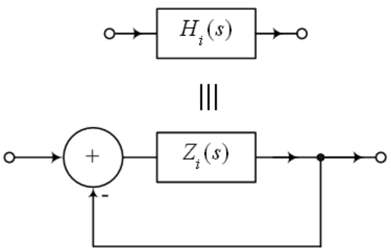

Figure 4. Realization of Decomposed Transfer functions $H_i(s)$ for $i = 1$ and $i = 2$

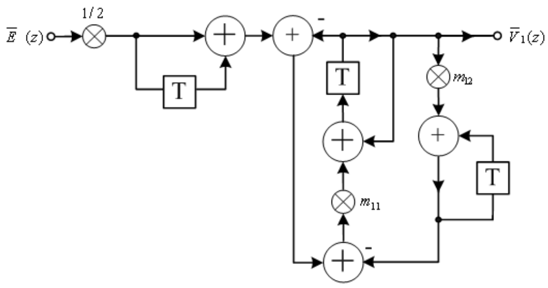

Figure 5. Bilinear-LDI Digital Realization of Transfer Function $H_1(s)$

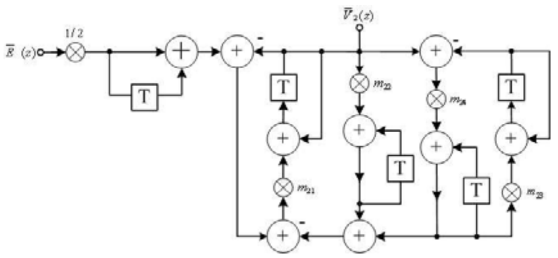

Figure 6. Bilinear-LDI Digital Realization of the Transfer Function $H_2(s)$

obtained in terms of C_{il_i}, L_{il_i} (for $l_1 = 1$ and $l_2 = 1, 2$) as given in Tables I and II.

TABLE I.
INFINITE-PRECISION MULTIPLIER COEFFICIENTS FOR $\tilde{H}_1(z)$

m_{11}	$T/(C_{11} + \frac{T^2}{4L_{11}} + \frac{T}{2R})$
m_{12}	T/L_{11}

The transfer function $\bar{H}_{BP}(z)$ is subsequently realized in accordance with

$$\bar{H}_{BP}(z) = \bar{H}_1(z) - \bar{H}_2(z) \tag{7}$$

$$= H_{BP}(s)|_{s=\frac{2}{T}\frac{z-1}{z+1}} \tag{8}$$

TABLE II.
INFINITE-PRECISION MULTIPLIER COEFFICIENTS FOR $\tilde{H}_2(z)$

m_{21}	$T/(C_{21} + \frac{T^2}{4L_{21}} + \frac{T}{2R} + \frac{C_{22}}{(1+4C_{22}L_{22}/T^2)})$
m_{22}	T/L_{21}
m_{23}	$T(1 + \frac{T^2}{4C_{22}L_{22}})/C_{22}$
m_{24}	$\frac{T}{L_3[(C_{22}+\frac{T^2}{4L_{22}})/C_{22}]^2}$

Finally, the desired digital IF filter is formed to have a transfer function

$$\bar{H}(z) = [\bar{H}_{BP}(z)]^3 \tag{9}$$

by cascading three identical bandpass filter sections each realizing a transfer function $\bar{H}_{BP}(z)$.

The infinite-precision seed digital IF filter obtained above is subsequently replaced by a corresponding finite-precision seed digital IF filter by quantizing the values of the multiplier coefficients m_{ik_i} in Tables I and II to their nearest CSD counterparts. The finite-precision seed digital IF filter is manipulated to form an initial population pool of candidate digital IF filter chromosomes for DCGA optimization.

III. OVERVIEW OF DCGA OPTIMIZATION TECHNIQUE

Broadly speaking, there are two categories of approaches available for the optimization of digital filters, namely, the approaches that are based on the conventional gradient-based (continuous) optimization, and those based on discrete optimization techniques. The latter approaches find particular practical applications in the optimization of digital filters over discrete (i.e. finite-precision) multiplier coefficient space. In this paper, a combination of the two approaches is employed for the discrete optimization of digital IF filters. More specifically, the bilinear-LDI lattice digital filter realization technique [5] is used to obtain a seed digital IF filter over the infinite-precision multiplier coefficient space. This is then followed by the discrete optimization of the resulting digital IF filter so as to satisfy the given design specifications over CSD multiplier coefficient space.

GAs are one of the most promising techniques for finding solutions to discrete optimization problems by emulating biological evolution [9]. In GAs, potential solutions called chromosomes are represented as bit-strings. Beginning with an initial population pool of chromosomes, GAs proceed from one generation to the next towards finding an optimal solution to the optimization problem. In order to form the population pool of chromosomes for the next generation, chromosomes in the current population pool are ranked (by evaluating them based on a suitable fitness function), so as to form a mating pool. These parent pairs subsequently undergo genetic operations of crossover and mutation to form corresponding pairs of offspring. The offspring so produced become members of the next-generation population pool. This process is repeated until the desired solution is reached or a pre-specified stop condition is met .

Unfortunately, the conventional GAs lack mechanisms through which getting trapped at a local optimal solution can be avoided. This entrapment stems from the rapid decline in diversity in the population pool from one generation to the next, rendering the chromosomes in future generations as very similar.

Many methods have been developed to improve the population pool diversity in GAs. In [19], to maintain high diversity, Srinivas proposed the use of adaptive probabilities for crossover and mutation, where the probabilities of crossover and mutation were varied based on the values of the fitness function. In [20], a SA approach was introduced to prevent GAs from premature convergence. Shimodaira [16] proposed DCGA, where a cross generational probability survival selection (CPSS) scheme was used to choose candidate chromosomes for the efficient exploration of the solution space with less likelihood of entrapment at local optimal solutions.

The DCGA approach strikes a balance between the diversity in the population pool on the one hand, and the computational time on the other [16]. The computational time for DCGA is much less than those of the SA in [20], and the variable crossover and mutation rate in [19]. This is a direct consequence of external control over the population pool diversity and the parent selection pressure. This external control is facilitated by incorporating a few non-elite candidate chromosomes in the next-generation population pool to be discussed in the following.

In DCGA, the members of a current population pool $P(t)$ of size N are combined together with their offspring chromosomes to form an enlarged population pool $\hat{P}(t)$ of $2N$ chromosomes, where t represents the generation number ($t = 0$ initially). This is followed by ranking the chromosomes in the enlarged population pool $\hat{P}(t)$ by evaluating their fitness values. Then, the chromosome in $\hat{P}(t)$ with the best fitness value is selected as a member of the next-generation population pool $P(t+1)$. The remaining $N-1$ chromosomes in the population pool $P(t+1)$ are selected from $\hat{P}(t)$ probabilistically based on the relationship:

$$p_s = [(1-c)h/L + c]^{\alpha} \tag{10}$$

where h represents the hamming distance (i.e. the number of bit locations at which a given chromosome is different from another chromosome) between a candidate chromosome and the best chromosome in $\hat{P}(t)$, where L represents the bit-length of the individual chromosomes, and where c and α denote, respectively, the shape coefficient and the exponent parameter. In this way, the selected chromosome with hamming distance h is chosen as a candidate for the next generation population pool if p_s is greater than a locally generated uniform random number between 0 and 1. This selection process is referred to as the CPSS scheme [16].

The CPSS scheme is repeated until N chromosomes are selected for the next generation population pool $P(t+1)$. However, in situations when CPSS leads to less than N chromosomes, the remaining chromosomes are formed by randomly complementing bits in the chromosome with the highest fitness value. By choosing the values of c and α appropriately, CPSS facilitates the desired external control over the diversity of the population pool permitting a rapid convergence to an optimal solution. In the absence of analytical solutions, the values of the shape coefficient c and the exponent parameter α are usually selected through an empirical investigation. This investigation is conducted over suitable range of values for c and α [10].

In accordance with Eqn. 10, chromosomes with large hamming distances from the chromosome with the highest fitness value in the population pool have a high chance of survival. The selection of such chromosomes increases the diversity of the population pool, giving rise to a substantial increase in the convergence speeds [16]. In particular, if the fitness function has a few local optima only, then by using a high value of c and/or small value of α, one can achieve a rapid convergence to an optimal solution [10]. On the other hand, when the fitness function is multimodal, lower values of c and/or higher values of α can be chosen to increase the probability of selecting chromosomes that are different from the chromosome with the highest fitness value [10].

IV. BIBO STABILITY CONSTRAINTS FOR CSD DIGITAL IF FILTERS

This section is concerned with the development of appropriate constraints for the guaranteed BIBO stability of the bandpass digital IF filter section in Figs. 5 and 6 over the CSD multiplier coefficient space.

Let us consider a bilinear-LDI bandpass digital IF filter section consisting of (floating radix-point) CSD multiplier coefficients $\hat{m}_{ik_i} \in CSD(W,w)$, where $CSD(W,w))$ represents the set of all possible CSD numbers having a wordlength of W digits and a maximum number of w nonzero digits. In this way, the CSD multiplier coefficients $\hat{m}_{ik_i}$ can be expressed in the general form

$$\hat{m}_{ik_i} = \sum_{n=1}^{W} (D_{ik_i})_n 2^{R-n} \tag{11}$$

with

$$\sum_{k=1}^{W} |(D_{ik_i})_n| \leq w \tag{12}$$

where $0 < R < W$ represents a floating radix-point. Moreover, in order to be valid, CSD numbers must satisfy the constraints

$$(D_{ik_i})_n \in \{1, -1, 0\} \tag{13}$$

and

$$(D_{ik_i})_n \times (D_{ik_i})_{n+1} = 0 \tag{14}$$

Let the total wordlength W be represented as $W = W_I + W_F$, where W_I represents the wordlength associated with the integer part, and where W_F represents that associated with the fractional part of the respective CSD numbers. In practical situations, the integer wordlength W_I is chosen to accommodate the representation of the maximum integer value of all the infinite-precision multiplier coefficients m_{ik_i}, and the fractional wordlength W_F is chosen based on precision requirements. The resulting CSD multiplier coefficients $\hat{m}_{ik_i} \in CSD(W,w)$ usually turn out to be very sparse, permitting computationally efficient hardware/software implementation of the multiplication operations in terms of limited number of shift and add operations *only*.

It may be plausible to form a digital IF filter chromosome by concatenating the CSD representation of the values of the constituent multiplier coefficients $\hat{m}_{ik_i}$. However, in the course of the underlying DCGA optimization, the crossover and mutation operations may lead to offspring digital IF filter chromosomes which no longer conform to the CSD number system constraints (13) and (14). In [17], this problem was successfully resolved by employing an exhaustive indexed LUT of permissible CSD multiplier coefficient values such that the indices of the multiplier coefficient values form a closed set under the operations of crossover and mutation. In this way, the digital IF filter chromosomes are formed by concatenating the LUT indices of the values of the CSD multiplier coefficients $\hat{m}_{ik_i}$ (instead of concatenating the values of the CSD multiplier coefficients $\hat{m}_{ik_i}$ *themselves*), ensuring that the resulting digital IF filter chromosomes automatically conform to the CSD number system throughout the course of DCGA optimization.

Having adopted the above LUT approach, there still remains the fundamental problem that the offspring digital IF filter chromosomes generated in various stages of the DCGA optimization may not be BIBO stable. This problem can be circumvented by discarding any offspring digital IF filter chromosome that is not BIBO stable immediately after its generation. However, this makes the DCGA optimization more time consuming than necessary, essentially rendering it computationally inefficient.

In the following, a novel approach is presented for the design and DCGA optimization of guaranteed BIBO stable digital IF filters over the CSD multiplier coefficient space (without generating intermediate digital IF filter chromosomes that are not BIBO stable).

By using the bilinear-LDI lattice digital filter realization technique in reverse order, the CSD bandpass digital IF filter section can be back-transformed to an equally resistively terminated symmetrical lattice reactance two-port network as shown in Fig. 7. The series arm reactance

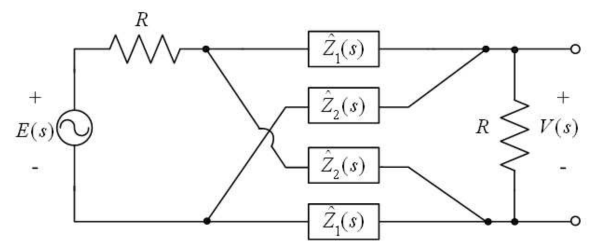

Figure 7. Back-Transformed Analog Prototype Reference Bandpass Filter Section

$\hat{Z}_1(s)$ and the lattice arm reactance $\hat{Z}_2(s)$ are as shown in Figs. 8a and 8b, where $\hat{C}_{il_i}$ and $\hat{L}_{il_i}$ (for $l_1 = 1$ and $l_2 = 1, 2$) are as given in Tables III and IV, and where the multiplier coefficients $\hat{m}_{ik_i}$ represent the (quantized) CSD counterparts of the infinite-precision multiplier coefficients m_{ik_i}.

Based on the properties of the bilinear analog-to-digital frequency transformation, in order for the bandpass digital IF filer section in Figs. 5 and 6 to be BIBO stable, it is both necessary and sufficient for the values of the elements $\hat{C}_{il_i}$ and $\hat{L}_{il_i}$ in Tables III and IV to be positive [21]. In this way, one can guarantee the BIBO stability of the offspring digital IF filter chromosomes generated in various stages of DCGA optimization *indirectly*, simply by ensuring that $\hat{C}_{il_i} > 0$ and $\hat{L}_{il_i} > 0$ (see also [22] for some related discussions).

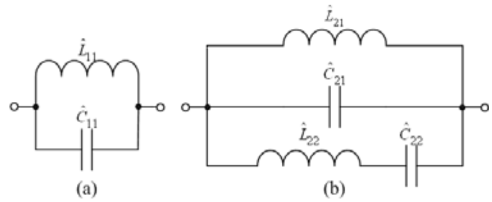

Figure 8. Realization of Back-Transformed Reactances $\hat{Z}_1(s)$ and $\hat{Z}_2(s)$

TABLE III.
ELEMENTS OF BACK-TRANSFORMED SERIES ARM REACTANCE $\hat{Z}_1(s)$

$\hat{C}_{11}$	$-T(\hat{m}_{11}\hat{m}_{12} + 2\hat{m}_{11} - 4)/4\hat{m}_{11}$
$\hat{L}_{11}$	$T/\hat{m}_{12}$

TABLE IV.
ELEMENTS OF BACK-TRANSFORMED LATTICE ARM REACTANCE $\hat{Z}_2(s)$

$\hat{C}_{21}$	$-T\left(\hat{m}_{22} + 2 - \frac{4}{\hat{m}_{21}} - \frac{4\hat{m}_{24}}{\hat{m}_{23}\hat{m}_{24}-4}\right)/4$
$\hat{L}_{21}$	$T/\hat{m}_{22}$
$\hat{C}_{22}$	$-4T/\hat{m}_{23}(\hat{m}_{23}\hat{m}_{24} - 4)$
$\hat{L}_{22}$	$T(\hat{m}_{23}\hat{m}_{24} - 4)^2/16\hat{m}_{24}$

From Table III,

$$\hat{C}_{11} > 0 \Rightarrow -T(\hat{m}_{11}\hat{m}_{12} + 2\hat{m}_{11} - 4)/4\hat{m}_{11} > 0 \quad (15)$$

$$\hat{L}_{11} > 0 \Rightarrow T/\hat{m}_{12} > 0 \quad (16)$$

and from Table IV,

$$\hat{C}_{21} > 0 \Rightarrow \frac{-T}{4}(\hat{m}_{22} + 2 - \frac{4}{\hat{m}_{21}} - \frac{4\hat{m}_{24}}{\hat{m}_{23}\hat{m}_{24} - 4}) > 0 \quad (17)$$

$$\hat{L}_{21} > 0 \Rightarrow T/\hat{m}_{22} > 0 \quad (18)$$

$$\hat{C}_{22} > 0 \Rightarrow \frac{-4T}{\hat{m}_{23}(\hat{m}_{23}\hat{m}_{24} - 4)} > 0 \quad (19)$$

$$\hat{L}_{22} > 0 \Rightarrow \frac{T(\hat{m}_{23}\hat{m}_{24} - 4)^2}{16\hat{m}_{24}} > 0 \quad (20)$$

Therefore, the inequality constraint (16) implies

$$\hat{m}_{12} > 0 \quad (21)$$

and the inequalities (18) and (20) imply

$$\hat{m}_{22} > 0 \quad (22)$$

$$\hat{m}_{24} > 0 \quad (23)$$

In this way, it remains to satisfy the inequality constraint (15), and the inequalities constraints (17) and (19). In order to simplify matters, it is expedient to assume that the remaining multiplier coefficients $\hat{m}_{11}$, and $\hat{m}_{21}$ and

$\hat{m}_{23}$ also have positive values[4]. Although this assumption precludes the DCGA optimization from obtaining other potential digital IF filters that may also satisfy the given design specifications, it has the advantage of reducing the size of the three LUTS, and, consequently, leading to increased convergence speed.

In accordance with the above assumption, the inequality constraint (15) can be simplified as

$$\hat{m}_{11}\hat{m}_{12} + 2\hat{m}_{11} - 4 < 0 \qquad (24)$$

and the inequality constraints (17) and (19) can be simplified as

$$\hat{m}_{23}\hat{m}_{24} - 4 < 0 \qquad (25)$$

$$\hat{m}_{22} + 2 - \frac{4}{\hat{m}_{21}} - \frac{4\hat{m}_{24}}{\hat{m}_{23}\hat{m}_{24} - 4} < 0 \qquad (26)$$

Then, in order to make the CSD bandpass digital IF filter section BIBO stable, it is necessary and sufficient to choose the values of the multiplier coefficients $\hat{m}_{ik_i} \in CSD(W, w)$ such that the inequality constraints (24), (25), and (26) are satisfied. In this paper, the aforementioned constraints are satisfied by employing a novel LUT-based approach as discussed in the next section.

V. NOVEL CSD LUTS FOR GUARANTEED DIGITAL IF FILTER BIBO STABILITY

The proposed approach for DCGA optimization of BIBO stable digital IF filters over the CSD multiplier coefficient space is based on three separate indexed CSD LUTs, where the constituent indices form closed sets under the DCGA operations of crossover and mutation. These LUTs are constructed as follows:

- The first LUT is constructed to include all permissible values for the pairs $(\hat{m}_{11} \in CSD(W, w), \hat{m}_{12} \in CSD(W, w))$ satisfying the inequality constraint (24).
- The second LUT is constructed to include all permissible values for the pairs $(\hat{m}_{23} \in CSD(W, w), \hat{m}_{24} \in CSD(W, w))$ satisfying the inequality constraint (25).
- The third LUT is constructed to satisfy the inequality constraint (26). However, since this constraint concerns the values of four multiplier coefficients (as opposed to the values of two multiplier coefficients for the previous two LUTs), its construction involves different considerations. To this end, let us replace the inequality constraint (26) by a corresponding equality constraint in accordance with

$$\hat{m}_{22} = -\epsilon + \frac{4\hat{m}_{24}}{\hat{m}_{23}\hat{m}_{24} - 4} + \frac{4}{\hat{m}_{21}} - 2 \qquad (27)$$

  where $\epsilon > 0$ is an infinite-precision slack variable. In order to satisfy the equality constraint (27), the third LUT is constructed to include all permissible values for the monads $\hat{m}_{21} \in CSD(W, w)$. Then, the remaining problem is to select the value of $\epsilon > 0$ judiciously so as to enable the determination of the value of the multiplier coefficient $\hat{m}_{22} \in CSD(W, w)$. In this way, in the process of encoding the bandpass digital IF filter section into a corresponding chromosome, one can bypass any *direct* reference to the value of the multiplier coefficient $\hat{m}_{22}$. Instead, one can incorporate the value of the slack variable ϵ as a gene in the construction of the bandpass digital IF filter chromosome. This is best facilitated by replacing the slack variable ϵ by a corresponding finite-precision counterpart $\hat{\epsilon}$, and by representing $\hat{\epsilon}$ by a binary number having a suitable wordlenth[5]. Then, the equality constraint (27) is replaced by the equality constraint

$$\tilde{\hat{m}}_{22} \approx -\hat{\epsilon} + \frac{4\hat{m}_{24}}{\hat{m}_{23}\hat{m}_{24} - 4} + \frac{4}{\hat{m}_{21}} - 2 \qquad (28)$$

  where $\tilde{\hat{m}}_{22} \in CSD(W, w)$ is determined by truncating the infinite-precision multiplier coefficient $\hat{m}_{22}$ to its nearest CSD counterpart in the respective LUT. This guarantees that $\tilde{\hat{m}}_{22}$ conforms to the specified CSD format while satisfying the inequality constraint (26).

Finally, in accordance with the above discussions, the bandpass digital IF filter chromosome is formed by an ordered concatenation of

- the first LUT index referencing the values of the multiplier coefficient pair $(\hat{m}_{11}, \hat{m}_{12})$
- the second LUT index referencing the values of the multiplier coefficient pair $(\hat{m}_{23}, \hat{m}_{24})$,
- the third LUT index referencing the value of the multiplier coefficient monad $\hat{m}_{21}$, and
- the finite-precision value of the slack variable $\hat{\epsilon}$.

If necessary, the above three CSD LUTs are reduced in size in such a manner that the ρ-th LUT (for $\rho = 1, 2, 3$) incorporates 2^{B_ρ} rows. As a results, the indices for the ρ-th LUT can be represented by B_ρ-bit binary numbers, and the resulting index sets become automatically closed under the operations of crossover and mutations in the course of the underlying DCGA optimization.

VI. DESIGN METHODOLOGY FOR THE PROPOSED DIGITAL IF FILTER DCGA OPTIMIZATION

The design methodology for the proposed DCGA optimization of BIBO stable digital IF filters over the CSD multiplier coefficient space can be summarized as follows:

A. Generation of the initial population pool

The starting point in the proposed DCGA optimization is to obtain a corresponding initial infinite-precision digital IF filter by using the bilinear-LDI lattice digital

[4]Then, the resulting bilinear-LDI digital IF filter realization becomes compatible with the original LDI digital filter realization technique in [23].

[5]The value of this worldlength is set empirically, starting with a large value which is reduced later so as not to slow down the DCGA optimization. In the ensuing application examples, it has been found that a wordlength of 16 bits is more than adequate for this purpose.

filter realization technique. Then, the resulting infinite-precision multiplier coefficients m_{ik_i} are quantized to obtain the corresponding finite-precision multiplier coefficients $\hat{m}_{ik_i} \in CSD(W, w)$.

Let the value of the finite-precision slack variable $\hat{\epsilon}$ be represented as a B_4-bit binary number. Then, the initial digital IF filter seed chromosome is formed in an ordered manner by concatenating

- the B_1 block of bits representing the binary index for the multiplier coefficient pair (m_{11}, m_{12}),
- the B_2 block of bits representing the binary index for the multiplier coefficient pair (m_{23}, m_{24}),
- the B_3 block of bits representing the binary index for the multiplier coefficient m_{21}, and
- the B_4 block of bits representing the binary value of the slack variable $\hat{\epsilon}$ (c.f. Eqn. (27)).

In this way, the first three blocks of bits in the chromosome represent the binary indices of the CSD multiplier coefficient values $\hat{m}_{11}$, $\hat{m}_{12}$, $\hat{m}_{21}$, $\hat{m}_{23}$, and $\hat{m}_{24}$ in the three LUTs, whereas the fourth block of bits represents the binary value of the slack variable $\hat{\epsilon}$.

Finally, an initial population pool of N chromosomes is formed by scanning the digital IF filter seed chromosome successively B_ζ block of bits at a time (for $\zeta = 1, 2, 3, 4$), and by randomly flipping bits in the ζ-th block of bits in accordance with the probabilistic relationship $p_F \times 0.5^{B_{\zeta+1} - b_\zeta}$ (with $1 \leq b_\zeta \leq B_\zeta$ from the most significant to the least significant bit), where p_F is a fixed probability factor.

B. Formation of the next generation population pool

The current population pool $P(t)$ of size N is replaced by an enlarged population pool $\hat{P}(t)$ of size $2N$ by using the following genetic operations:

- *Crossover Operations*: Chromosomes in the population pool $P(t)$ are randomly paired as parents in such a manner that each chromosome is chosen only once as a parent [16]. The resulting parent chromosome pairs undergo two-point crossover operations, reproducing two offspring for each parent pair. The resulting offspring are then combined with the initial population pool $P(t)$ to form the enlarged population pool $\hat{P}(t)$.
- *Mutation Operations*: A few chromosomes (determined by a small mutation probability) in the enlarged population pool $\hat{P}(t)$ undergo mutation operation by randomly flipping their bits to enhance diversity.

Duplicate chromosomes are eliminated so as to maintain diversity, and the chromosomes in the enlarged population pool $\hat{P}(t)$ are ranked by evaluating their fitness values (c.f. Eqn. 7). The next generation population pool of $P(t+1)$ is then formed by using the CPSS scheme (c.f. Eqn 10).

C. Fitness evaluation

The fitness of each of the resulting digital IF filter chromosomes in the enlarged population pool $\hat{P}(t)$ is evaluated in accordance with

$$fitness = -20log_{10}[max(\varepsilon_p, \varepsilon_t, \varepsilon_s)] \tag{29}$$

where

$$\varepsilon_p = \underbrace{max}_{\omega \in \Omega_p}[W_p|H(e^{j\omega}) - 1|] \tag{30}$$

with Ω_p representing the passband frequency region(s), where

$$\varepsilon_t = \underbrace{max}_{\omega \in \Omega_t}[W_t|H(e^{j\omega}) - 0.7079|] \tag{31}$$

with Ω_t representing the 3 dB attenuation frequency points, and where

$$\varepsilon_s = \underbrace{max}_{\omega \in \Omega_s}[W_s|H(e^{j\omega}) - 0.001|] \tag{32}$$

with Ω_s representing the 60 dB stopband attenuation region(s). Here, W_p, W_t, and W_s represent the passband, transition band, and stopband weighing factors, respectively.

VII. Application Examples

This section presents the application of the above DCGA optimization technique to the design of a pair of eighteenth-order digital IF filters for two different design specifications.

A. Application example 1

In this subsection, the proposed DCGA optimization is applied to the design of a BIBO stable digital AMPS 455 kHz IF filter, where the design specifications consist of frequency-response constraints given in Table V.

In accordance with the given frequency response design specifications, the values for the elements C_{il_i} and L_{il_i} in the analog prototype reference bandpass filter section in Fig. 1 are obtained as given in Table VI. These element values are obtained as follows:

- By using the bilinear analog-to-digital frequency transformation in Eqn. (2) to transform the (discrete-time) z-domain bandpass frequency response specifications in Table V to corresponding (continuous-time) s-domain bandpass frequency response specifications,
- by transforming the resulting s-domain bandpass frequency response specifications to corresponding s-domain lowpass frequency response specifications (in accordance with the conventional analog lowpass-to-bandpass frequency transformation),
- by realizing the corresponding analog prototype reference lowpass filter section by using the classical techniques (see, e.g., [24]), and
- by transforming the analog prototype reference lowpass filter section to the corresponding bandpass filter section in Fig. 1.

Finally, the values of the multiplier coefficients m_{ik_i} for the infinite-precision seed digital IF filter are obtained

TABLE V.
DIGITAL IF FILTER DESIGN SPECIFICATIONS FOR EXAMPLE 1

Center frequency	455 kHz
Bandwidth	21 kHz
3 dB attenuation frequency regions	
Lower passband	Upper passband
(443 kHz, 446 kHz)	(464 kHz, 467 kHz)
60 dB attenuation frequency regions	
Lower stopband	Upper stopband
(404 kHz, 406 kHz)	(504 kHz, 506 kHz)

TABLE VI.
ANALOG ELEMENT VALUES C_{il_i} AND L_{il_i} FOR EXAMPLE 1

C_{11}	2.4045×10^{-6}
L_{11}	3.1389×10^{-8}
C_{21}	4.2878×10^{-7}
L_{21}	1.7602×10^{-7}
C_{22}	1.7312×10^{-8}
L_{22}	4.3597×10^{-6}

in terms of the values of C_{il_i} and L_{il_i} (c.f. Table I and II) as shown in Table VII.

The magnitude frequency-response associated with the initial finite-precision seed digital IF filter is as shown in Fig. 9, and enlarged in Fig. 10 around the transition band regions. It is observed that the 3 dB attenuation frequency point is violated by 2 kHz, and that the 60 dB attenuation frequency point is violated by 27 kHz.

TABLE VII.
INFINITE-PRECISION MULTIPLIER COEFFICIENT VALUES FOR EXAMPLE 1

m_{ik_i}	Decimal Value
m_{11}	0.1081
m_{12}	17.5044
m_{21}	0.0806
m_{22}	23.7373
m_{23}	31.7382
m_{24}	0.0630

Based on the infinite-precision values in Table VII for the multiplier coefficients m_{ik_i}, the CSD LUTs for the proposed DCGA optimization are constructed by choosing the remaining design parameters as shown in Table VIII.

TABLE VIII.
PARAMETER VALUES FOR CONSTRUCTING THE CSD LUTS FOR EXAMPLE 1

W_I	6 bits
W_F	6 bits
w	5
$\hat{m}_{21}$ table size	$2^{11} \times 12$
$\hat{m}_{11}$,$\hat{m}_{12}$ table size	$2^{16} \times 24$
$\hat{m}_{23}$,$\hat{m}_{24}$ table size	$2^{17} \times 24$

By employing the proposed DCGA optimization technique, the values of the quantized multiplier coefficients $\hat{m}_{ik_i}$ are obtained as given in Table IX (where the overbared digit $\bar{1}$ is used to represent -1).

TABLE IX.
FINITE-PRECISION MULTIPLIER COEFFICIENT VALUES AFTER DCGA OPTIMIZATION FOR EXAMPLE 1

$\hat{m}_{ik_i}$	CSD Representation	Decimal Value
$\hat{m}_{11}$	000000.000101	0.0781
$\hat{m}_{12}$	10$\bar{1}$010.$\bar{1}$00010	25.5313
$\hat{m}_{21}$	000000.000101	0.0781
$\hat{m}_{22}$	10$\bar{1}$000.$\bar{1}$01000	23.6250
$\hat{m}_{23}$	00100$\bar{1}$.00000$\bar{1}$	6.9844
$\hat{m}_{24}$	000000.00010$\bar{1}$	0.0469

The magnitude frequency-response of the resulting digital IF filter is as shown in Fig. 11, and enlarged in Fig. 12 around the transition band region.

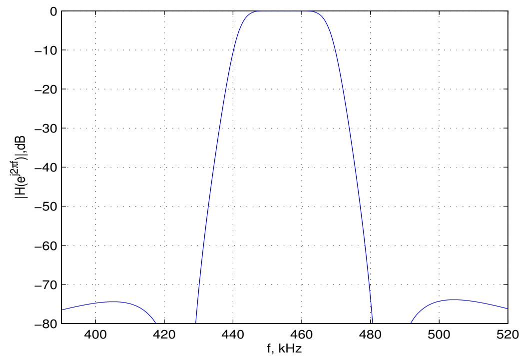

Figure 9. Magnitude Frequency-Response of Infinite-Precision Digital IF Filter in Example 1

By inspection of Figs. 11 and 12, the 60 dB attenuation frequency points are at 404.6 kHz and 505.7 kHz, and the 3 dB attenuation frequency points are at 444.3 kHz and 465.5 kHz, both satisfying the desired design specifications.

The convergence speed for the above DCGA optimization for various values of the shape coefficient parameter $0.6 \leq c \leq 0.95$ for a fixed α-value of 0.4 is as shown in Fig. 13, with a best convergence at 373 generations when $c = 0.8$. In Fig. 14, the convergence speed is shown for various values of the exponent parameter $0.1 \leq \alpha \leq 0.7$ for a fixed c-value of 0.8, leading to a best convergence at 328 generations when $\alpha = 0.3$.

For the sake of comparison, Fig. 15 shows the convergence speed of a corresponding conventional GA applied to the same optimization problem. The conventional GA optimization takes 2688 generations to converge, whereas it takes only 328 generations for DCGA to converge (c.f. Fig. 16).

B. Application example 2

In this subsection, the proposed DCGA optimization is applied to the design of a BIBO stable digital IF filter with

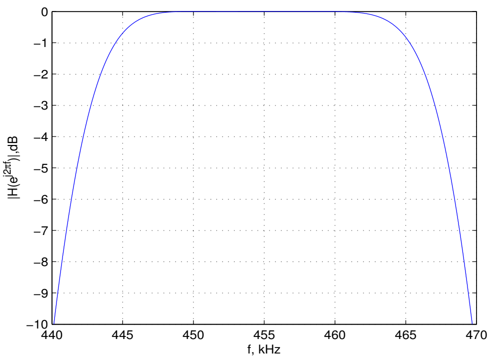

Figure 10. Transition Band Magnitude Frequency-Response of Infinite-Precision Digital IF Filter in Example 1

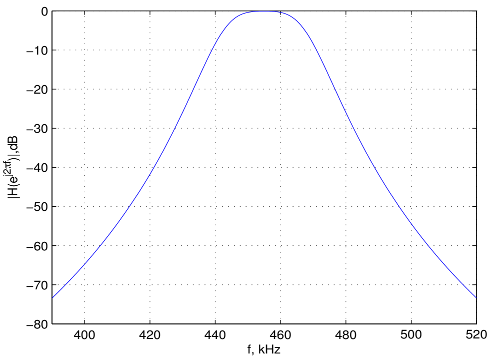

Figure 11. Magnitude Frequency-Response of Optimized Digital IF Filter for Example 1

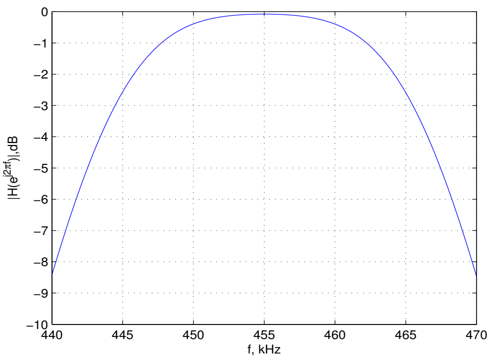

Figure 12. Transition Band Magnitude Frequency-Response of Optimized Digital IF Filter for Example 1

its center frequency located at 910 kHz. The remaining design specifications consist of frequency-response constraints given in Table X.

In much the same way as for the previous example, the values of C_{il_i} and L_{il_i} are obtained as given in Table XI. Similarly, the values of the multiplier coefficients m_{ik_i} for the infinite-precision seed digital IF filter are obtained as shown in Table XII in terms of the values of C_{il_i} and L_{il_i} (c.f. Table I and II).

The magnitude frequency-response associated with the initial finite-precision seed digital IF filter is as shown in

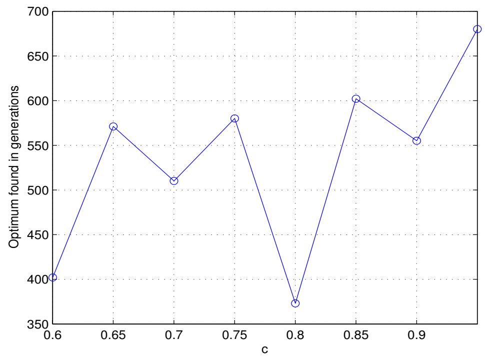

Figure 13. Convergence Speed for Various c-Values in Example 1

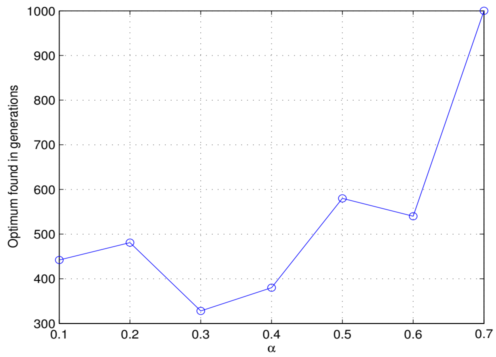

Figure 14. Convergence Speed for Various α-Values in Example 1

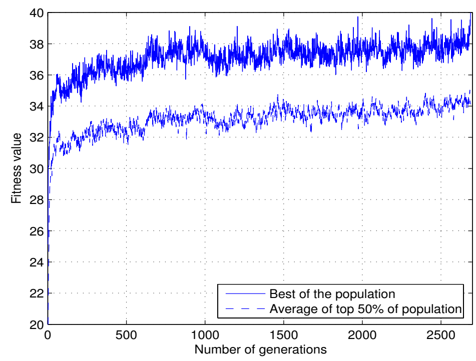

Figure 15. Best and Average Fitness Values for Conventional GA Optimization for Example 1

TABLE X.
DIGITAL IF FILTER DESIGN SPECIFICATIONS FOR APPLICATION EXAMPLE 2

Center frequency	910 kHz
Bandwidth	21 kHz
3 dB attenuation frequency regions	
Lower passband	Upper passband
(898 kHz, 901 kHz)	(919 kHz, 922 kHz)
60 dB attenuation frequency regions	
Lower stopband	Upper stopband
(859 kHz, 861 kHz)	(959 kHz, 961 kHz)

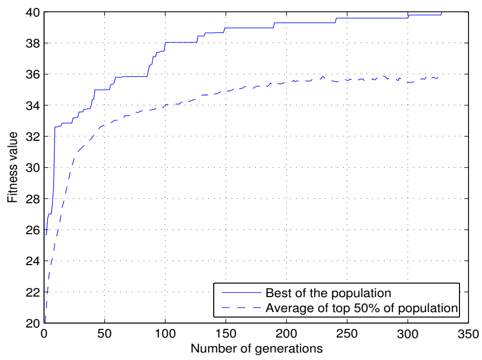

Figure 16. Best and Average Fitness Values for DCGA optimization for Example 1

TABLE XI.
ANALOG ELEMENT VALUES C_{il_i} AND L_{il_i} FOR EXAMPLE 2

C_{11}	2.3590×10^{-6}
L_{11}	7.9984×10^{-9}
C_{21}	5.1362×10^{-7}
L_{21}	3.6763×10^{-8}
C_{22}	4.4185×10^{-9}
L_{22}	4.2703×10^{-6}

Fig. 17, and enlarged in Fig. 18 around the transition band region. It is observed that the 3 dB attenuation frequency point is violated by 2.5 kHz, and that the 60 dB attenuation frequency point is violated by 26 kHz.

TABLE XII.
INFINITE-PRECISION MULTIPLIER COEFFICIENT VALUES FOR EXAMPLE 2

m_{ik_i}	Decimal Value
m_{11}	0.0566
m_{12}	34.3476
m_{21}	0.0397
m_{22}	49.3043
m_{23}	62.1765
m_{24}	0.0322

Based on the infinite-precision values for the multiplier coefficients m_{ik_i} (c.f. in Table XII), the CSD LUTs for the proposed DCGA optimization are constructed by choosing the remaining design parameters as shown in Table XIII.

TABLE XIII.
PARAMETER VALUES FOR LUTS IN EXAMPLE 2

W_I	7 bits
W_F	6 bits
w	5
$\hat{m}_{21}$ table size	$2^{12} \times 12$
$\hat{m}_{11}$,$\hat{m}_{12}$ table size	$2^{17} \times 24$
$\hat{m}_{23}$,$\hat{m}_{24}$ table size	$2^{18} \times 24$

By employing the proposed DCGA optimization technique, the values of the quantized multiplier coefficients $\hat{m}_{ik_i}$ are obtained as given in Table XIV. The magnitude frequency-response of the resulting digital IF filter is as shown in Fig. 19, and enlarged in Fig. 20 around the transition band region. By inspection of Figs. 19 and 20, the 60 dB attenuation frequency points are at 867 kHz and 956 kHz, and the 3 dB attenuation frequency points are at 899.4 kHz and 920.5 kHz, both satisfying the desired design specifications.

TABLE XIV.
FINITE-PRECISION MULTIPLIER COEFFICIENT VALUES AFTER DCGA FOR EXAMPLE 2

$\hat{m}_{ik_i}$	CSD Representation	Decimal Value
$\hat{m}_{11}$	0000000.000010	0.03125
$\hat{m}_{12}$	$10000\bar{1}00.0\bar{1}010$	61.9060
$\hat{m}_{21}$	0000000.000010	0.03125
$\hat{m}_{22}$	1000000.000100	64.0630
$\hat{m}_{23}$	$010\bar{1}001.001001$	25.1410
$\hat{m}_{24}$	0000000.000001	0.01563

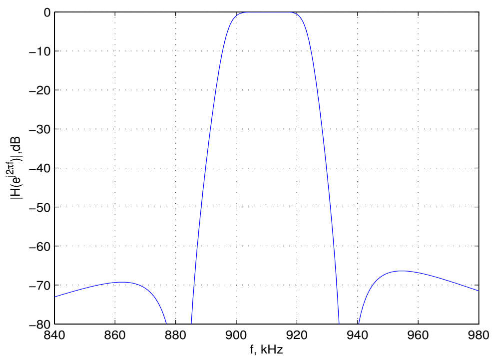

Figure 17. Magnitude Frequency-Response of Infinite-Precision Digital IF Filter in Example 2

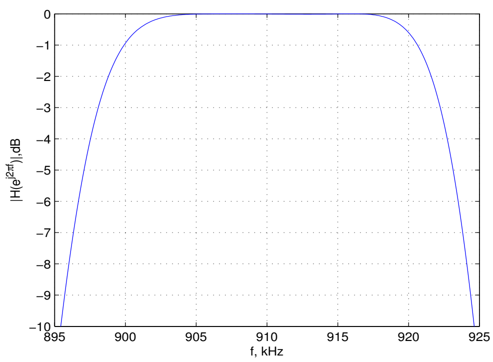

Figure 18. Transition Band Magnitude Frequency-Response of Infinite-Precision Digital IF Filter in Example 2

The convergence speed for the above DCGA optimization for various values of the shape coefficient parameter $0.6 \leq c \leq 0.95$ for a fixed α-value of 0.4 is as shown in Fig. 21, with a best convergence within 505 generations when $c = 0.75$. In Fig. 22, the convergence speed

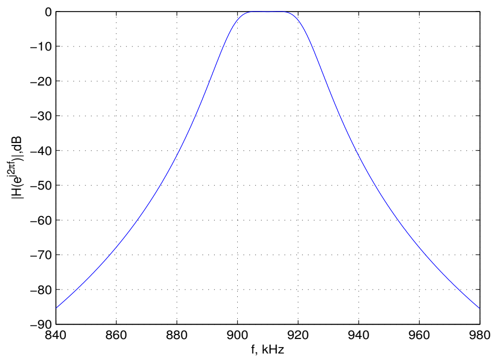

Figure 19. Magnitude Frequency-Response of Optimized Digital IF Filter in Example 2

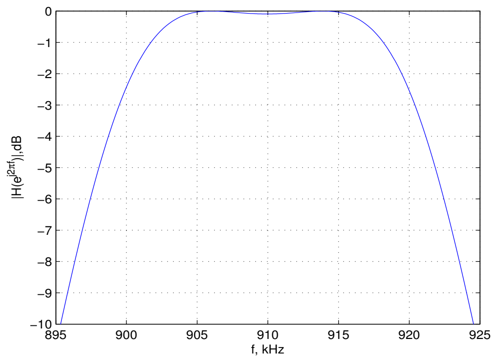

Figure 20. Transition Band Magnitude Frequency-Response of Optimized Digital IF Filter in Example 2

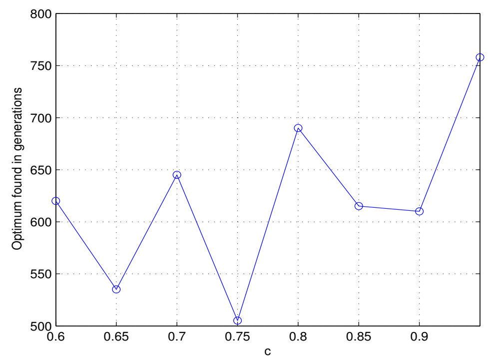

Figure 21. Convergence Speeds for Various c-Values in Example 2

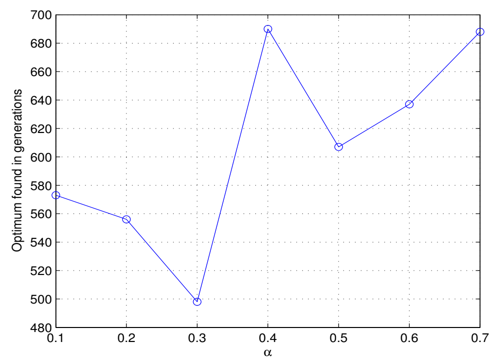

Figure 22. Convergence Speeds for Various α-Values in Example 2

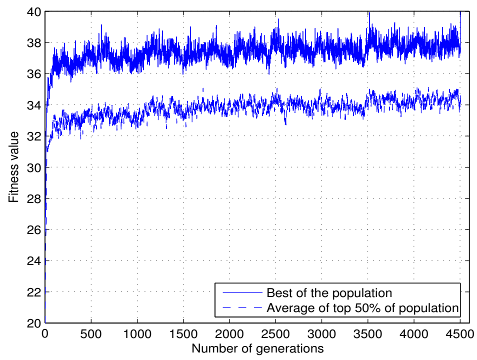

Figure 23. Best and Average Fitness Values for Conventional GA Optimization for Application Example 2

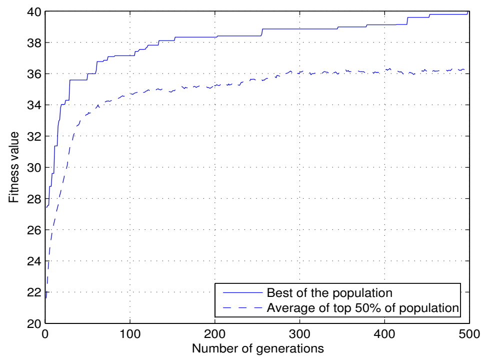

Figure 24. Best and Average Fitness Values for DCGA optimization for Application Example 2

is shown for various values of the exponent parameter $0.1 \leq \alpha \leq 0.7$ for a fixed c-value of 0.8, leading to a best convergence within 498 generations when $\alpha = 0.3$.

Once again, for the sake of comparison, Fig. 23 shows the convergence speed of a corresponding conventional GA for the same optimization problem. The conventional GA optimization takes 4505 generations to converge, whereas it takes only 498 generations for DCGA to converge (c.f. Fig. 24).

VIII. CONCLUSION

This paper has presented a novel technique for the optimization of digital IF filters over the (finite-precision) canonical signed-digit (CSD) multiplier coefficient space based on a diversity controlled (DC) genetic algorithm (GA). This optimization technique exploits the bilinear-LDI lattice digital filter design approach for the realization of the required infinite-precision seed digital IF filter chromosome. A novel look-up table (LUT) approach has been proposed to ensure that the finite-precision CSD digital IF filter chromosomes generated in the course of DCGA optimization are guaranteed to be BIBO stable. The resulting DCGA exhibits high convergence speeds, achieved by permitting external control over the population diversity and parent selection pressure throughout the course of optimization. The proposed DCGA optimization has been illustrated through its application to the design of a pair of digital IF filters satisfying practical design specifications. It has been shown that the DCGA optimization results in around an order of magnitude improvement in the convergence speed as compared to a corresponding conventional GA optimization.

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