A New Judging and Revising Method for Ordinal Consistency of Fuzzy Judgment Matrix

Xixiang Zhang[1]
[1]Mathematics and Information Engineering School of Jiaxing University, Jiaxing, China
Email:Zhangmiddle@126.com
Guangxue Yue[1,2,3,4] , Xiaojing Liu[1] and Fei Yu[5]
[2]College of Computer and Communication, Hunan University, Changsha, China
[3] Department of Computer Science and Technology, Huaihua University, Huaihua, China
[4]Electronic Commerce Market Application Technology, Guangdong University of Business Studies, Guangzhou, China
[5] Jiangsu Provincial Key Laboratory of Computer Information Processing Technology, Suzhou, China
ChinaEmail: {ygx, liuxiaojing99999}@163.com

Abstract—**In uncertainty decision making, experts can use interval-fuzzy number, triangular fuzzy number, trapezoidal fuzzy number, linguistic 2-tuple or linguistic indices to express their preferences. Using kernel function, the fuzzy numbers can be converted into fuzzy number range from 0 to 1. Thus, different fuzzy judgment matrix can be expressed in a unified style. A judging method for ordinal consistency of fuzzy judgment matrix was proposed according to the transitivity of binary relation. And two concepts of non-transitive route number(NTRN) and non-transitive route contribution number(NTCN) were put forward. Through the non-transitive route number and non-transitive route contribution number guidance, a revising method for fuzzy judgment matrix without ordinal consistency was put forward, in which the irrational element can be identified. The revising method can help the decision-maker revise his/her judgment matrix effectively. Finally, the non-financial performance evaluation attributes were selected and experts were asked to give the judgment matrix, fuzzy matrix without ordinal consistency was used to demonstrate the idea of the new judging and revising method for ordinal consistency of the fuzzy judgment matrix.**

Index Terms—**fuzzy judgment matrix, ordinal consistency, revising method, transitivity**

I. Introduction

Due to the uncertainty, complexity of the problem and the limited knowledge of decision-makers, sometimes it will be difficult for decision-makers to use fixed values to express their preferences on alternatives. Fuzzy linguistic terms such as "very bad, bad, no difference, good, very good" will be easier for decision-makers to express their preferences [1]. Interval-value number, triangular fuzzy number, trapezoidal fuzzy number, linguistic indices and linguistic 2-tuple can be used to present and process fuzzy linguistic terms [2-5]. Using kernel functions, Interval-value number, triangular fuzzy number, trapezoidal fuzzy number, linguistic index and 2-tuple linguistic presentation model can be converted into an literal with its value between 0 and 1[2,3,6]. A matrix whose elements meet the condition $0 \le a_{ij} \le 1$ for $\forall i, j = 1,2,\cdots,n$ is called a fuzzy judgment matrix. Therefore, research on consistency of fuzzy matrix is necessary and instructive.

Consistency is a research topic on Analytic Hierarchy Process (AHP for short), a procedure for evaluating alternatives introduced by Saaty in 1977. Since Saaty put forward the concept of consistency and judged the decision-maker's judgment matrix by its consistency ratio, there were many studies about the cardinal consistency of judgment matrix. Lamata and Pelaez(2002) defined the consistency index CI of a matrix using the average of the consistency index of the matrix triplets[7]. Li and Ma (2006) developed a model that can assist in making a consistent decision and used Gower plots to detect major inconsistencies graphically [8]. Alonso and Lamata (2006) introduced a statistical criterion for accepting/rejecting the pairwise reciprocal comparison matrices in the AHP[9].

Ma(1994) studied the prerequisites of ordinal consistency and concluded that the ordinal consistency was the basic condition to judge the thinking continuousness of a decision-maker, and the sorting weight induced by a judgment matrix without ordinal consistency was irrational[10]. Luo(2004) studied the revising method of judgment matrix and considered that ordinal consistency was the prerequisite of cardinal consistency[11]. Zhu, Wang and Liu(2007) also demonstrated that consistent analysis should be based on ordinal consistency[12]. Comparing researches on cardinal consistency of a judgment matrix, less research on ordinal consistency, however, were studied according to references analysis.

Ma(1994) proposed a repetition judging method for ordinal consistency according to the concepts of it[10]. Basile and Dapuzzo (2002) studied transitivity of the decision-maker's preference relation and used the complete strict simple order to judge the ordinal consistency of judgment matrix[13] . Fan and Jiang(2004)

Manuscript received May 1, 2009; revised June 25, 2009; accepted July 20, 2009.

doi:10.4304/jcp.5.4.573-580

also used complete strict simple order to judge the ordinal consistency of linguistic label matrix[14]. Li and Ma (2006) used Gower plots to judge the ordinal consistency graphically and put forward a model to minimize the ordinal inconsistency[15]. Dai, Li and Xue(2006) introduced the concept of increasing ordered shadow matrix and used its properties to judge the ordinal consistency of a judgment matrix[15,16].

Ordinal consistency can tell the continuous thinking of a decision-maker and is the prerequisite of cardinal consistency[10,11]. To further study properties of ordinal consistency, the paper analyzed the transitivity of a binary relation and used the transitivity of a complete preference matrix to judge the ordinal consistency of a fuzzy judgment matrix. Based on the transitive route, the concepts of non-transitive route number and non-transitive route contribution number were firstly defined, which can be used to identify the most irrational element and tell the revising direction of it. Finally, a method to revise the judgment matrix without ordinal consistency was put forward, which will guide the decision-maker to revise his/her inconsistent judgment matrix and improve the revising efficiency. Then an example was introduced to illustrate it. The example showed that the proposed judging and revising method for ordinal consistency of a fuzzy judgment matrix was simple and effective. Decision-making support system can let the experts to revise their judgment matrices interactively with the help of the method.

Ⅱ. PREMIER OF FUZZY JUDGMENT MATRIX

In uncertainty environment, decision-maker will like use fuzzy linguistic terms such as “good”, ”very good” to express his/her preference. Fuzzy linguistic terms can be represented by interval-valued fuzzy number, triangular fuzzy number, linguistic label or linguistic 2-tuple representation model. Yager introduced the concepts of kernel function to measure the different represented fuzzy numbers[2].

Definition 1[2] Suppose $\widetilde{A}=(x,\mu_{\widetilde{A}}(x))$ be a fuzzy number. The kernel function of the fuzzy number $\widetilde{A}$ can be expressed as follows.

$$K(\widetilde{A})=\frac{\int_0^1 x\mu_{\widetilde{A}}(y)dx}{\int_0^1 \mu_{\widetilde{A}}(y)dx} \tag{1}$$

Using the above formula, Hou and Wu gave the kernel functions for interval-valued fuzzy number, triangular fuzzy number, trapezoidal fuzzy number and studied the additional consistency of type Ⅰ fuzzy judgment matrix[3].

If $\widetilde{A}=(a,b),a,b\in[0,1]$ be an interval-valued number, then the kernel function of $\widetilde{A}$ is [3]

$$K(\widetilde{A})=\frac{a+b}{2} \tag{2}$$

If $\widetilde{A}$ be a triangular fuzzy number, $\widetilde{A}=(a,b,c),a,b,c\in[0,1]$, then the kernel function of $\widetilde{A}$ is

$$K(\widetilde{A})=\frac{a+4b+c}{6} \tag{3}$$

If $\widetilde{A}$ be a trapezoidal fuzzy number, $\widetilde{A}=(a,b,c,d),a,b,c,d\in[0,1]$, then the kernel function of $\widetilde{A}$ is

$$K(\widetilde{A})=\frac{a+b+c+d}{3}+\frac{ab-cd}{3(c-b+d-a)} \tag{4}$$

If $\widetilde{A}=(s_i),i=0,1,2,\cdots g/2,(g/2)+1,\cdots,g$, g is a linguistic index and is an even number, s_i represents a fuzzy linguistic term, the kernel function (or the membership function) of it is

$$K(\widetilde{A})=\frac{i}{g} \tag{5}$$

Herrera and Martinez analyzed the representation method for linguistic fuzzy preference and proposed a new linguistic fuzzy preference presentation method (the linguistic 2-tuple representation model)[18]. The linguistic 2-tuple representation model takes a basis of the symbolic model and symbolic translation to represent the linguistic information using a pair of values called linguistic 2-tuple (written as (s_i,α), s_i is a linguistic term and α is a numeric value between –0.5 and 0.5).

Suppose S={s_0, s_1, ⋯, s_g} be a set of labels assessed in a linguistic term set with odd elements, which has the following properties: ①ordered: when the index i⩾j, there must exist s_i⩾s_j; ②a negation operator: Neg(s_i)= s_{g-i}; ③ there exists a min and max operator: s_i⩾ s_j means max(s_i, s_j)=s_i and min(s_i, s_j)=s_j[17].

Definition 2[18] Let S={s_0, s_1, ⋯, s_g} be a linguistic term set and $\beta\in[0,g]$ be a value representing the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:

$$\nabla:[0,g]\rightarrow S\times[-0.5,0.5]$$

$$\nabla(\beta)=(s_i,\alpha),with\begin{cases} s_i, i=round(\beta) \\ \alpha=\beta-i,\alpha\in[-0.5,0.5]\end{cases} \tag{6}$$

Where round(.) is the usual round operation, s_i had the closest index label to β.

Let S={s_0, s_1, ⋯, s_g} be a linguistic term set and (s_i,α) be a 2-tuple. There is always a ∇^{-1} function, such that, from a 2-tuple it returns its equivalent numerical value $\beta\in[0,g]$, which is[18]:

$$\nabla^{-1}: S \times [-0.5, 0.5] \to [0, g]$$
$$\nabla^{-1}(s_i, \alpha) = i + \alpha = \beta \tag{7}$$

If $\widetilde{A}$ is a 2-tuple linguistic fuzzy representation model $\widetilde{A} = (s_i, \alpha_i)$, the kernel function of it is

$$K(\widetilde{A}) = \frac{\nabla(s_i, \alpha_i)}{g} \tag{8}$$

In Ref.[3] Xu(2006) defined the concept of expected value function of fuzzy number and gave the formula to convert fuzzy number (such as interval-valued fuzzy number, triangular fuzzy number, trapezoidal fuzzy number) into fixed real value. Although the expected value Xu defined was not between 0 and 1, it can be modified to meet the requirement.

From the literal above, the value of the kernel functions is located in the range [0, 1]. Definition 3, 4 were put forward by Zhang and Qiu in Reference[19]

Definition 3 Suppose $A = (a_{ij})_{n \times n}$ be a judgment matrix given by a decision-maker, if $0 \le a_{ij} \le 1$ for $\forall i, j = 1,2,\cdots,n$, then $A = (a_{ij})_{n \times n}$ is called a fuzzy judgment matrix, where a_{ij} is the preference degree of the X_i to X_j.

$$a_{ij} = \begin{cases} 0, X_i \ is\ strictly\ infer\ to\ X_j \\ (0,0.5) \quad , X_i\ is\ infer\ to\ X_j \\ 0.5 \quad , X_i\ is\ identical\ to\ X_j \\ (0.5,1), X_i\ is\ superior\ to\ X_j \\ 1, X_i\ is\ strictly\ superiro\ to\ X_j \end{cases} \tag{9}$$

Definition 4 Suppose $A = (a_{ij})_{n \times n}$ be a fuzzy judgment matrix. $A = (a_{ij})_{n \times n}$ is called fuzzy complementary judgment matrix if it follows.

$$a_{ij} + a_{ji} = 1, \forall i, j = 1,2,\cdots,n \tag{10}$$

Definition 5 Suppose $A = (a_{ij})_{n \times n}$ be the fuzzy complementary fuzzy matrix. If the following condition is true, $A = (a_{ij})_{n \times n}$ is ordinal consistent.

$$a_{ik} > 0.5, a_{kj} > 0.5 \Rightarrow a_{ij} > 0.5 \tag{11}$$

or

$$a_{ik} < 0.5, a_{kj} < 0.5 \Rightarrow a_{ij} < 0.5 \tag{12}$$

Fuzzy judgment matrix shows the preferences of pairwise comparison on alternatives $X = \{X_1, X_2, \cdots, X_n\}$, and whose entries are preference ratios: $a_{ij} > 0.5$ means the decision-maker prefer X_i to X_j and $a_{ij} < 0.5$ shows the reverse preference, $a_{ij} = 0.5$ means that X_i and X_j have the same preference. If the decision-maker thinks that X_i is preferred to X_k and X_k is preferred to X_j, he usually considers that X_i is preferred to X_j, which shows his/her continuous decision-making. Ordinal consistency is the measurement tool to judge if the decision-maker gave the judgment with continuous thinking. Ordinal consistency is the prerequisite of a fuzzy judgment matrix, if a fuzzy judgment matrix is not ordinal consistent, the fuzzy judgment matrix is not acceptable because the decision-maker's thinking is not continuous and there exists contradiction [9-11]. Therefore, ordinal consistency of the fuzzy judgment matrix should be ensured before calculating the weight of each alternative.

III. THE TRANSITIVITY OF A BINARY RELATION

The relationships between elements of sets are represented using the structure called relation. The most direct way to express the relationships between elements of two sets is ordered pairs made of two related elements. In Ref.[20] , Bernard, Bushy and Cutler demonstrated the idea of binary relation.

Definition 6 If A and B are two finite nonempty sets, the product set $A \times B$ is the set of all ordered pairs (a,b) with a∈A and b∈B, that is

$$A \times B = \{(a,b) \mid a \in A \wedge b \in B\} \tag{13}$$

Suppose A and B are two finite nonempty sets, a binary relation from A to B is a subset of $A \times B$.

In simple words, a binary relation from A to B is a set R composed of ordered pairs where the first element of each pair comes from A and the second element comes from B. If A and B are the same set, then we say the relation R is a relation on the set A.

Definition 7 Suppose A, B and C are three nonempty sets, R is the binary relation from A to B, and S is the relation from B to C. The composite of R and S is the relation consisting of ordered pairs (a,c), where a∈A, c ∈C , and for which there exists an element b∈B such that (a,b)∈R and (b,c) ∈S. The composite of R and S is denoted as

$$R \circ S = \{(a,c) \mid a \in A \wedge c \in C \wedge$$
$$\exists b(b \in B \wedge (a,b) \in R \wedge (b,c) \in S)\} \tag{14}$$

Obviously, if R is a relation on the set A, the composite of R and R is $R \circ R$, and can be denoted as R^2.

$$R^2 = \{(a,c) \mid a \in A \wedge c \in A \wedge$$
$$\exists b(b \in A \wedge (a,b) \in R \wedge (b,c) \in R)\} \tag{15}$$

Suppose R is the relation on set A and $a,b,c \in A$. We say that R is transitive if whenever $(a,b) \in R$ and $(b,c) \in R$, then $(a,c) \in R$. And we say that R is not transitive if there exists a, b and c in A such that $(a,b) \in R$ and $(b,c) \in R$, but $(a,c) \notin R$.

Theorem 1 A relation R on the set A is transitive if and only if $R^2 \subseteq R$.

Relation matrix is also another way to express a binary relation.

Definition 8 Suppose R is a relation on the set A. The relation matrix of R is $M_R = (m_{ij})_{n\times n}$, where

$$m_{ij} = \begin{cases} 1, if (a_i, a_j) \in R \\ 0, if (a_i, a_j) \notin R \end{cases}, \forall a_i, a_j \in A \quad (16)$$

Suppose R and S are two relations on the set A. If every element in matrix M_R is smaller than the respond element in matrix M_S, then we can say that $M_R \le M_S$.

$$r_{ij} \le s_{ij}, \forall i, j = 1,2,\cdots,n \Rightarrow M_R \le M_S$$

Theorem 2 Suppose R is a relation on the set A, and the relation matrix of R is $M_R = (m_{ij})_{n\times n}$. The relation matrix of the composite of R and R is $M_{R^2} = M_R \cdot M_R = (m_{ij}^2)_{n\times n}$,

$$m_{ij}^2 = \bigvee_{k=1}^{n} (m_{ik} \wedge m_{kj}), \forall i, j = 1,2,\cdots,n \quad (17)$$

Proof: if there exists k such that m_{ik}=1 and m_{kj}=1, then $m_{ij}^2 = 1$. And m_{ik} =1 and m_{kj} =1 means that $(a_i, a_k) \in R$ and $(a_k, a_j) \in R$. According to the definition of composite of the relation R and R, we know that $(a_i, a_j) \in R^2$. Therefore, $M_{R^2} = M_R \cdot M_R = (m_{ij}^2)_{n\times n}$ represent the composite relation R^2.

Theorem 3 Suppose R is a relation on the set A, and the relation matrix of R is $M_R = (m_{ij})_{n\times n}$. R is transitive if and only if $M_R^2 \le M_R$.

$A = (a_{ij})_{n\times n}$ is fuzzy complementary judgment matrix, if $a_{ik} > 0.5, a_{kj} > 0.5 \Rightarrow a_{ij} > 0.5$ for $\forall i, j = 1,2,\cdots,n$, then $A = (a_{ij})_{n\times n}$ is ordinal consistent. For R is a relation on the set A, and $M_R = (m_{ij})_{n\times n}$ is the relation matrix of A, if $m_{ik} = 1, m_{kj} = 1 \Rightarrow m_{ij} = 1$, then R is transitive.

The ordinal consistency of a fuzzy complementary judgment matrix and the transitivity of a relation have the same conditions. The transitivity of a relation can be easily judged. The ordinal consistency of a fuzzy complementary judgment matrix can be judged based on the transitivity of the respond relation.

Ⅳ. JUDGING ORDINAL CONSISTENCY OF A FUZZY MATRIX

When the decision-makers give their judgment matrices through pairwise comparing on alternatives, they can use fuzzy numbers (such as interval-value fuzzy number, triangular fuzzy number and so on) to express their preferences. Using kernel functions mentioned above, the judgment matrix can be converted into a fuzzy judgment matrix. Element a_{ij} in the fuzzy judgment matrix shows the degree of the decision-maker's preference over alternatives X_i and X_j : $a_{ij} > 0.5$ means he prefers X_i to X_j and $a_{ij} < 0.5$ means the decision-maker preferred X_j to X_i. Suppose R is the superior relation on the alternatives set X, if $a_{ij} > 0.5$, we call that X_i is superior to X_j and the respond element in the relation matrix M_R is designated to 1, otherwise, the respond element in M_R is designated to 0. That is to say, the superior relation matrix is $M_R = (r_{ij})_{n\times n}$, where

$$r_{ij} = \begin{cases} 1, a_{ij} > 0.5 \\ 0, otherwise \end{cases} \quad (18)$$

Generally, a decision-maker has continuous thinking, which means if $a_{ik} > 0.5$ and $a_{kj} > 0.5$, then $a_{ij} > 0.5$. If $a_{ik} > 0.5$ and $a_{kj} > 0.5$, then $a_{ij} > 0.5$, the related elements in its superior relation matrix $M_R = (r_{ij})_{n\times n}$ will be 1. If $r_{ik} = 1$ and $r_{kj} = 1$, then $r_{ij} = 1$, the matrix $M_R = (r_{ij})_{n\times n}$ is transitive. Thus, we can say that the fuzzy judgment matrix is ordinal consistent if its superior relation matrix $M_R = (r_{ij})_{n\times n}$ is transitive.

Theorem 4 Suppose a decision-maker gives his/her fuzzy judgment matrix A $=(a_{ij})_{n\times n}$ through pairwise comparison , and its superior relation matrix is $M_R = (r_{ij})_{n\times n}$. The judgment matrix A is ordinal consistency if and only if $M_R^2 \le M_R$.

Proof: Suppose A is ordinal consistent judgment matrix, and m_{ij}^2 is an element of its superior relation matrix. If $m_{ij}^2 = 0$, then $m_{ij}^2 \le m_{ij}$ is true. If $m_{ij}^2 = 1$, there must exist k such that $m_{ik} = 1$ and $m_{kj} = 1$. Because $M_R = (r_{ij})_{n\times n}$ is the superior relation matrix of A, the respond elements a_{ik} and a_{kj} in judgment matrix A must be greater than 0.5. Because the fuzzy judgment matrix A is a matrix with ordinal consistency, if $a_{ik} > 0.5$ and $a_{kj} > 0.5$, then $a_{ij} > 0.5$. Therefore, the respond element m_{ij} in superior relation matrix is also 1. Thus $m_{ij}^2 \le m_{ij}$ is also true.

Suppose $M_R^2 \le M_R$, then $m_{ij}^2 \le m_{ij}$ for $\forall i, j = 1,2,\cdots,n$. If $a_{ik} > 0.5$ and $a_{kj} > 0.5$, then $m_{ik} = 1$ and $m_{kj} = 1$. According to theorem 2 we get $m_{ij}^2 = 1$. Because $m_{ij}^2 \le m_{ij}$, then $m_{ij} = 1$. $M_R = (r_{ij})_{n\times n}$ is the superior relation matrix of fuzzy judgment matrix A. Therefore, and the element a_{ij} in judgment matrix must be greater than 0.5($a_{ij} > 0.5$). If $a_{ik} > 0.5$ and $a_{kj} > 0.5$, we can get $a_{ij} > 0.5$, so the fuzzy judgment matrix A is ordinal consistent.

In decision-making, an expert may give his preference on alternatives using fuzzy judgment matrix. To judge the ordinal consistency of it, we can use the transitivity of its superior relation matrix. The ordinal consistency of fuzzy matrix can be determined with the following three steps.

Step 1 Give the superior relation matrix M_R of the fuzzy judgment matrix A using formula $r_{ij} = \begin{cases} 1, a_{ij} > 0.5 \\ 0, otherwise \end{cases}$;

Step 2 Find composite relation matrix $M_R^2 = M_R \cdot M_R$ according to theorem 2;

Step3 If condition $M_R^2 \le M_R$ is true, the fuzzy judgment matrix A is ordinal consistent; otherwise, the judgment matrix is not ordinal consistent, it need be revised.

Ⅴ. REVISING A JUDGMENT MATRIX WITHOUT ORDINAL CONSISTENCY

Ordinal consistency is a prerequisite for a fuzzy judgment matrix. Before the cardinal consistency of a fuzzy judgment is tested, its ordinal consistency should be judged. If a fuzzy judgment matrix is not ordinal consistent, the decision-maker should be guided to revise it.

If $M_R^2 \le M_R$, we can say that M_R is transitive. If the superior relation matrix M_R of a fuzzy judgment matrix is transitive, the judgment matrix is ordinal consistent. If $M_R^2 \le M_R$ is not true, the judgment matrix need to be revised.

If there exists $m_{ik} = 1, m_{kj} = 1$ and $m_{ij} = 0$, the superior relation matrix M is not transitive. There is a superior path from X_i to X_k, and X_k to X_j, but X_i is not superior to X_j. If there exist two or more different non-transitive paths, the element should be revised greater. Here we introduced two concepts to help decision-maker identify the irrational element and revise it.

Definition 9 Suppose a decision-maker give his/her preference using fuzzy numbers and get the judgment matrix $A = (a_{ij})_{n\times n}$, $M_R = (m_{ij})_{n\times n}$ is the superior relation matrix of A. If there exists $m_{ik} = 1, m_{kj} = 1$ and $m_{ij} = 0$, the path is not transitive, the number of such routes is called non-transitive route number , denoted as $ntrn_{ij}$

$$ntrn_{ij} = |\{k \mid m_{ik} = 1 \wedge m_{kj} = 1 \wedge m_{ij} = 0\}| \quad (19)$$

The greater the non-transitive route number of an element in the superior relation matrix, the more possibility the element in the respond judgment matrix should be revised greater than 0.5.

If there is just a non-transitive route of the element, that is $ntrn_{ij}$=1.The element may be revised greater and the non-transitive route becomes transitive. Or an edge in the non-transitive route is removed, the route is broken and the non-transitive route disappears.

Definition 10 Suppose a decision-maker give his/her preference using fuzzy numbers and get the judgment matrix $A = (a_{ij})_{n\times n}$, $M_R = (m_{ij})_{n\times n}$ is the superior relation matrix of A. The edge perhaps contributes to different the non-transitive routes, the number of the non-transitive routes that it contributes is called the non-transitive route contribution number, denoted as $ntcn_{ik}$.

$$ntcn_{ij} = |\{k \mid m_{ik} = 1 \wedge m_{kj} = 1 \wedge m_{ij} = 0\}| \quad (20)$$

If $ntcn_{ik} \ge 2$, there are more than 2 non-transitive routes between X_i and X_j, Fig.1 shows the situation. X_i is superior to X_{k1} and X_{k1} is superior to X_j, X_i is superior to X_{k2} and X_{k2} is superior to X_j, ⋯. Adding an edge from X_i to X_j, which means X_i is superior to X_j, the non-transitive routes between X_i and X_j become transitive.

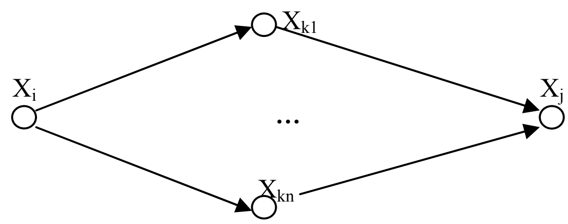

Figure 1 two more non-transitive routes between X_i and X_j

If $ntrn_{ij} = 1$, there may exists two reasons for the non-transitive route. One reason is the element a_{ij} in the fuzzy judgment matrix A is under-evaluated. The other reason is one element in the non-transitive route is highly evaluated. If the second reason leads to it, one edge in the non-transitive route should be removed, and the respond element in the judgment matrix should be modified smaller than 0.5. To measure the possibility of the reason that leads to the non-transitive route, we add 1 to non-transitive route contribution number of the edge because it has contribute to another non-transitive route. Thus $ntcn_{ik} = ntcn_{ik} + 1$, and $ntcn_{kj} = ntcn_{kj} + 1$.

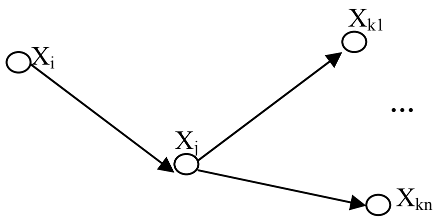

Figure2 edge(X_i,X_j) contributed to different non-transitive routes

If $ntcn_{ij}$ is greater than or equal to 2(Fig.2 shows one edge contribute to two more non-transitive routes), two more non-transitive routes exist because of the edge, and the edge should be erased to break the non-transitive routes. So the respond element in the judgment matrix should be revised smaller.

Based on the concept of non-transitive route number and non-transitive route contribution number, the revising method of a fuzzy judgment matrix without ordinal consistency can be demonstrated as following pseudocode in Fig.3.

> Input the MR
> Compute the route with length 2,donated as $RN^2 = M_R * M_R$;
> Compute $M_R{}^2$;
> NTRN=RN^2, $ntrn_{ij} = 0$ if $m_{ij} \le m_{ij}^2$;
> if there exists $ntrn_{ij} \ge 2$ then
> Select the largest $ntrn_{ij}$ and advise the decision-maker modify the respond element larger;
> Else
> If $ntrn_{ij} = 1$, add 1 to the non-transitive route contribution number of the edges in the non-transitive route, $ntcn_{ik} = ntcn_{ik} + 1$ and $ntcn_{kj} = ntcn_{kj} + 1$;
> Select the largest $ntcn_{ij}$ and let the decision-maker change the respond element in the judgment matrix smaller;
> Endif

Figure 3 the revising pseudocode

If $ntrn_{ij} \ge 2$, an edge should be added between X_i and X_j , and the related non-transitive routes become transitive. Therefore, the respond element in the judgment matrix should be revised greater than 0.5. If $ntrn_{ij} = 1$, the non-transitive route contribution of the related two edges should be add 1. If an edge contributes to two more non-transitive routes, move the edge from the relation graph and the respond non-transitive routes disappear, thus, the respond element in judgment matrix should be revised less than 0.5.

The process of judging and revising a fuzzy judgment matrix based on the transitivity of its superior relation matrix can be demonstrated as follows:

Step 1 Get the superior relation matrix of it. The superior relation matrix is simpler to judge its transitivity than the fuzzy judgment matrix.

Step 2 Judge if the fuzzy judgment matrix is ordinal consistent. If $M_R^2 \le M_R$, the procedure is terminated; otherwise, go step 3.

Step 3 Calculate the non-transitive routes $ntrn_{ij}$. If $ntrn_{ij} \ge 2$, to step 5; If $ntrn_{ij} = 1$, go step 4.

Step 4 Calculate the non-transitive route contribution $ntcn_{ij}$ of every element in the non-transitive route: $ntcn_{ij} = ntcn_{ij} + 1$.

Step 5 And guide the decision-maker to select the most irrational element to be revised, which is the element with the biggest $ntrn_{ij}$ or the element with the biggest $ntcn_{ij}$. If the most irrational element is the element with biggest $ntrn_{ij}$, modify the element a_{ij} in the fuzzy judgment matrix bigger than 0.5; otherwise, modify the element a_{ij} in the fuzzy judgment matrix smaller than 0.5. And we get the revised fuzzy judgment matrix $A^{(1)} = (a_{ij}^{(1)})_{n\times n}$, $a_{ij} \Rightarrow a_{ij}^{(1)}$.

Step 6 Get the superior relation matrix of the revised fuzzy judgment matrix, and go Step 2.

VI. ILLUSTRATED EXAMPLE

Through questionnaire investigation, we got that non-financial performance evaluation attributes of small-medium entrepreneur, in which market possessing rate, customer satisfactory, customer sustained rate, commodity supply efficiency and commodity quality are the indices reflecting the customer management ability of a company. Experts from bank, insurance company, small-medium entrepreneur were asked to pairwise the attributes using linguistic terms, the linguistic term set is S={ s0=I Incomparable，s1=SW Significantly Worse，s2=WO Worse，s3=SI Somewhat Inferior，s4=EQ Equivalent，s5=SB Somewhat Better，s6=SU Superior，s7=SS Significantly Superior，s8=CS Certainly Superior }.

The fuzzy linguistic terms were expressed by the following fuzzy numbers. The meaning of the fuzzy number was demonstrated in table 1.

Table 1 Meaning of the fuzzy numbers

value	meaning
0.1	X_i is incomparable to X_j
0.2	X_i is significantly worse than X_j
0.3	X_i is worse than X_j
0.4	X_i is somewhat inferior than X_j
0.5	X_i is equivalent to X_j

0.6	X_i is somewhat better than X_j
0.7	X_i is superior to X_j
0.8	X_i is significantly superior to X_j
0.9	X_i is certainly superior to X_j

The 9 scale fuzzy linguistic terms can show people's capabilities better. The fuzzy number can explain as follows: comparing two attributes, the percentage of each attribute can get.

Experts used fuzzy linguistic terms to represent his preference over the attributes that affect the financial performance of small-medium entrepreneur. We used the above fuzzy numbers to represent the fuzzy linguistic terms and got the fuzzy complementary matrices. One of the matrices was as follows.

$$A=\begin{bmatrix}0.5 & 0.7 & 0.4 & 0.5 & 0.7\\ 0.4 & 0.5 & 0.7 & 0.5 & 0.8\\ 0.6 & 0.3 & 0.5 & 0.5 & 0.9\\ 0.5 & 0.5 & 0.5 & 0.5 & 0.7\\ 0.3 & 0.2 & 0.1 & 0.3 & 0.5\end{bmatrix}$$

The judgment matrix was not ordinal consistent. The above judging and revising method was used to revise the judgment matrix. Steps were as follows.

Firstly, we got its superior relation matrix M_R

$$M_R=\begin{bmatrix}0 & 1 & 0 & 0 & 1\\ 0 & 0 & 1 & 0 & 1\\ 1 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

Secondly, calculate the composite of the superior relation matrix and judge its transitivity.

$$M_R^{\ 2}=\begin{bmatrix}0 & 0 & 1 & 0 & 1\\ 1 & 0 & 0 & 0 & 1\\ 0 & 1 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

Obviously, $M_R^2 \le M_R$ is not true because $m_{13}^2=1, m_{21}^2=1, m_{32}^2=1$ while the respond elements in judgment matrix are 0. So, the judgment matrix A is not ordinal consistent.

Thirdly, calculate the non-transitive routes $ntrn_{ij}$ and non-transitive route contribution $ntcn_{ij}$.

$$NTRN=\begin{bmatrix}0 & 0 & 1 & 0 & 0\\ 1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$NTCN=\begin{bmatrix}0 & 2 & 0 & 0 & 0\\ 0 & 0 & 2 & 0 & 0\\ 2 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

From $NTCN$, we can see $ntcn_{12}, ntcn_{23}=2, ntcn_{31}=2$. We can advise the decision-maker to revise the respond element a_{31} , a_{12} or a_{23}. The decision-maker decided to select a_{12} to revise according to the NTRN and NTCN.

Fourthly, the revised judgment matrix and its superior relation matrix show as follows.

$$A^{(1)}=\begin{bmatrix}0.5 & 0.7 & 0.6 & 0.5 & 0.7\\ 0.4 & 0.5 & 0.7 & 0.5 & 0.8\\ 0.4 & 0.3 & 0.5 & 0.5 & 0.9\\ 0.5 & 0.5 & 0.5 & 0.5 & 0.7\\ 0.3 & 0.2 & 0.1 & 0.3 & 0.5\end{bmatrix}$$

$$Q^{(1)}=\begin{bmatrix}0 & 1 & 1 & 0 & 1\\ 0 & 0 & 1 & 0 & 1\\ 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

Fifthly, judge the ordinal consistency of the revised judgment matrix. We got $M_{R(1)}^2=\begin{bmatrix}0 & 0 & 1 & 0 & 1\\ 0 & 0 & 1 & 0 & 1\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}$. So $M_{R(1)}^2 \le M_{R(1)}$ is True and the revised judgment matrix is ordinal consistent and the revising procedure can be terminated.

Ⅶ. CONCLUSION

In some condition, decision-makers would like express their preferences on alternatives using fuzzy languages. Fuzzy linguistic can be represented by interval-valued number, triangular fuzzy number, trapezoidal fuzzy number, linguistic label or 2-tuple linguistic representation model. The pairwise comparison judgment matrix using fuzzy numbers can be converted into a fuzzy complementary fuzzy judgment matrix with the kernel function. Through studying the transitive properties of the relation matrix, we found that the ordinal consistency of a fuzzy complementary matrix is equal to the transitivity of its superior relation matrix. Therefore, we concluded that a judgment matrix was ordinal consistent if and only if its superior relation matrix meet the condition $M_R^2 \le M_R$. If the judgment matrix is not ordinal consistent, it need to be revised. To guide the decision-maker to modify the judgment matrix effectively, the concepts of non-transitive route number and non-transitive route contribution number were put forward. A revising process was proposed to guide the decision-maker to select the most irrational element to be revised based on the concepts.

The method to determine the ordinal consistency of a fuzzy complementary matrix is simple and can be used to guide the decision-maker to select the most irrational element to be revised. The proposed judging and revising method for ordinal consistency of fuzzy complementary matrix was applied in non-financial performance assessment attributes and the result showed that it was appropriate.

The new judging and revising method provides an idea to guide a decision-maker to revise his/her judgment matrix, which can improve the revising efficiency and lead to interactivity between decision-making supported system and the decision-maker.

Consistency of the judgment matrix is a key problem in AHP, which includes cardinal consistency and ordinal consistency. The revising method integrate cardinal consistency into ordinal consistency should be studied further.

ACKNOWLEDGMENT

This work is supported by Hunan Provincial Natural Science Foundation of China under grant No. 07JJ6140, 07JJ6109, 05FJ3018, Zhejiang Provincial Natural Science Foundation of China under grant No. Y1080901, and Jiaxing Technology Division of China under Grant 2008AY2015.

REFERENCES

[1] D.Ben-Arieh, Z.Chen, "Linguisitic group decision-making: opinion aggregation and measures of consensus," Fuzzy Optim Decis Making, Vol. 5,ppt.371-386, Octobor 2006.

[2] Yager R R, "A procedure for ordering fuzzy subsets of the unit interval".Inform Sci,Vol.24, ppt. 143-161, 1981.

[3] F.J.Hou, Q.Z.Wu, " Research on consistency for mixed complementary judgment matrix," Journal of Mathematics Practice and Recoginizition, Vol.35,ppt. 46-55 , April 2005(in Chinese).

[4] F.Herrera, L.Nartinez, " A 2-tuple fuzzy linguistic representation model for computing with words," IEEE Transaction on Fuzzy Systems,Vol.8, No.6, ppt. 746-752, November 2000.

[5] F.Herrera , L.Martinez, "The 2-tuple linguistic computational model—advantages of its linguistic description, accuracy and consistency,"International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, Vol.9, ppt. 33-48, September 2003.

[6] Z.S.Xu, "Group decision-making under different incomplete judgment matrices", Control and Decison,Vol.20, No.1, ppt.28-33, Janary 2006 (In Chinese).

[7] M. T. Lamata, J. I. Pelaez, " A method for improving the consistency of judgments," International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems,Vol. 10, ppt. 677-686, November 2003.

[8] H.L.Li, L.C.Ma, "Adjusting ordianl and cardianl Inconsistencies in decisin preferences based on GOWER PLOTS," Asia-Pacific Journal of Operational Research,Vol. 23, ppt.329-346,May 2003.

[9] J.A Aloson, M.T Lamata, "Consistency in the analytic hierarchy process: a new approach," International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems,Vol. 14, ppt.445−459,July 2006.

[10] W.Y.Ma, "A practical method for judging the ordinal consistency of judgment matrix," System Engineering Theory and Pracitice,Vol.11,ppt.103-105,November 1996 (in Chinese).

[11] Z.Q.Luo," A new revising method for judgment matrix without conxistency in AHP," System Engineering Theory and Pracitice,Vol.6, ppt.83-92 ,June 2004(in Chinese).

[12] J.J.Zhu, M.G.Wang, S.X.Liu, "Research on serveral problems for revising judgment matrix in AHP," System Engineering Theory and Pracitice, Vol.1,ppt.90-94, Jul y2007(in Chinese).

[13] L.Basilel, L.Dapuzzol, " Weak consistency and quasi-linear means imply the actual ranking," International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems , Vol.10,ppt. 227-239,May 2002.

[14] Z.P.Fan, Y.PJiang, " The judgement method for satisfatory consistency of linguisticjudgment matrix," Control and Decison,Vol.19, ppt.903-906,October 2004 (in Chinese).

[15] J.H.Dai, J.Li, H.X.Xue, "Research on the sorting method for a fuzzy judgment matrix under satisfatory consistency ," Operational and Management, Vol.15, ppt.18-24, June 2006 (in Chinese).

[16] J.H.Dai, J.Li, W.M.Shen, H.X.Xue, "The judgment for satisfactory consistency of a fuzzy judgment matrix," System Engineering and Electronic Technology,Vol.28, ppt.1174-1178, October 2006 (in Chinese).

[17] C.Wren,A.Azarbayejani,T.Darrell,etal. Pfinder: Real-time tracking of the human body. IEEE Transon Pattern Analysis and Machine Intelligence,Vol.19, ppt.780-785, Jul y1997.

[18] F.Herrera, L.Nartinez,"A 2-tuple fuzzy linguistic representation model for computing with words,"IEEE Transaction on Fuzzy Systems,Vol.8, ppt. 746-752, June 2000.

[19] W.X.Zhang , G.F.Qiu, "The uncertainty decision-making based on rough set," Beijing: Qinghua Univeristy Press,2006.1 (in Chinese).

[20] K.Bernard, B.R.C.Bushy, R.S.Cutler. Discrete mathematical structures(Fifth Edition)[M]. Beijing, Higher Education Press,2008 (Reprint).