A New MLS Chaotic Ssystem and its Backstepping Sliding Mode Synchronization Control

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***Abstract*—A new four-dimensional system is proposed in this paper, which is named modified Lorenz-Stenflo (MLS) system. The system is constructed by replacing one of quadratic nonlinear terms of Lorenz-Stenflo system with a piecewise linear (PWL) function. The simple structure makes the MLS system much easier to be implemented with electronic circuits. Based on adaptive control and sliding mode control methods, an adaptive sliding mode controller is derived to synchronize two MLS chaotic systems. Meanwhile the uncertain parameters in response system are identified. The error systems are deduced by backstepping method. Numerical simulations are used to verify the effectiveness of the proposed controller.**

***Index Terms*—modified Lorenz-Stenflo (MLS) chaotic system, synchronization control, sliding mode control, backstepping control**

I. INTRODUCTION

The well-known Lorenz chaotic attractor [1] was discovered in a three-dimensional autonomous system in 1963. Another famous chaotic attractor is Rössler [2], which is found in 1976. Then in 1996, Lennart Stenflo constructed the Lorenz-Stenflo (LS) system [3] based on Lorenz system. In 1999 the Chen system [4] was constructed via a state feedback controller to the Lorenz system. Later, the Lorenz system family and the generalized Lorenz canonical form were presented [5-8].

During the study on the chaotic control and synchronization, scientists find it is hard to build simple electronic circuits for these nonlinear systems since they have many quadratic nonlinear terms. Much work has been done to simplify these quadratic nonlinear terms without losing their chaotic behaviors. One of the effective methods is to replace these nonlinear terms with the piecewise linear (PWL) functions [9]. In 2002, Lü found a new PWL chaotic system [10]. In 2004 Zheng proposed a novel method for simultaneously creating two symmetrical chaotic attractors in a three-dimensional linear autonomous system [11]. Then modified Chen's system, modified Lü system and unified PWL chaotic family were generated in [12-14].

In this paper, a new four-dimensional chaotic system, named the modified Lorenz-Stenflo (MLS) system, is constructed by replacing one of quadratic nonlinear terms of LS system with a PWL function. The PWL function is mainly composed of standard sign function. The new chaotic system is slightly different from LS system in structures, but the shape of the new chaotic attractors is much larger than that of LS chaotic attractors. And the simple structure makes the MLS system much easier to be implemented with electronic circuits.

In recent years, chaotic control and chaotic synchronization have attracted increasing attention due to their potential application in secure communication, etc. Many theories have been developed to achieve chaotic synchronization, such as feedback control [15], coupled control [16]. In fact, from the perspective of hardware implementation, the resistance and capacitors in transmitter circuit will always differ slightly from those in receiver circuit. Hence, the two chaotic circuits will not be identical due to the mismatch in the system's parameters. So synchronization control of two chaotic systems with uncertain parameters is a critical issue.

Besides adaptive control method [17,18], sliding mode control is a nonlinear control scheme widely used for controlling uncertain nonlinear systems [19,20]. In this paper, an adaptive sliding mode controller is derived to synchronize two MLS chaotic systems. The error system between two MLS chaotic systems is deduced using backstepping control theory [21].

This paper is organized as follows. In section 2 the MLS chaotic system is proposed and the chaotic behaviors are analyzed in detail. In section 3 the backstepping sliding mode controller is proposed to synchronize two MLS systems. In section 4 simulations show that this method can achieve synchronization effectively, meanwhile the uncertain parameters in response system are correctly identified. Section 5 is the conclusions.

doi:10.4304/jcp.5.3.456-463

II. THE MLS CHAOTIC SYSTEM

In 1996, Lennart Stenflo studied the equations which govern atmospheric waves. By using a low-frequency and short-wavelength approximation, he managed to derive a set of simplified equations. He then used the same strategy to construct the Lorenz-Stenflo (LS) system [3,22]. The autonomous differential equations are described by:

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + dx_4 \\ \dot{x}_2 = x_1(c - x_3) - x_2 \\ \dot{x}_3 = x_1 x_2 - bx_3 \\ \dot{x}_4 = -x_1 - ax_4 \end{cases} . \tag{1}$$

where $a = 1, b = 0.7, c = 26, d = 1.5$. The Lorenz system and this LS system share the same basic foundation. The LS system has three stationary points:

$S_0 = (0,0,0,0),$

$S_\pm = \left((\pm)\sqrt{bz_s(1+d/a^2)}, \pm\sqrt{bz_s(1+d/a^2)}, z_s, \mp\sqrt{bz_s(d+a^2)}\right)$

where $z_s = c - 1 - d/a^2$.

The LS system is similar to the famous Lorenz equation, but differ from it in the new control parameter d and the new state variable x_4 . Some dynamical behaviors of the LS equation, including the familiar period-doubling route to chaos, are reported in [3,21].

By replacing a quadratic-nonlinear term $x_1 x_2$ in the third equation with a PWL function, a new four-dimensional chaotic system is generated and named modified Lorenz-Stenflo (MLS) system. The system has the following form:

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + dx_4 \\ \dot{x}_2 = x_1(c - x_3) - x_2 \\ \dot{x}_3 = sign(x_2) \cdot x_1 - bx_3 \\ \dot{x}_4 = -x_1 - ax_4 \end{cases} . \tag{2}$$

where $sign(\cdot)$ denotes the standard sign function, and $a, b, c, d \in R$ are constant parameters. When $a = 1$, $b = 0.7, c = 26, d = 1.5$, the chaotic attractors are shown in Fig.1(a) and 1(b).

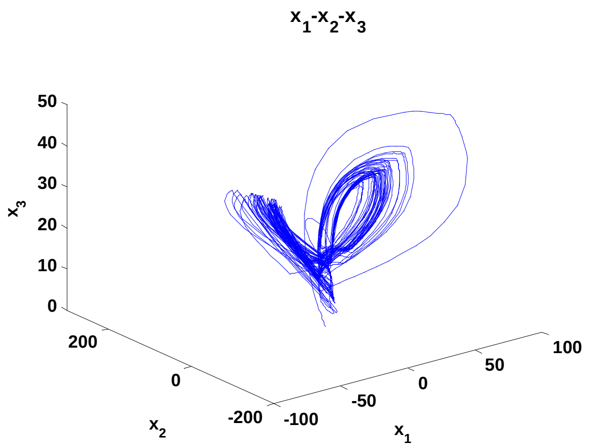

Figure 1(a). The attractors of MLS system in three-dimensional view (variable x_1-x_2-x_3).

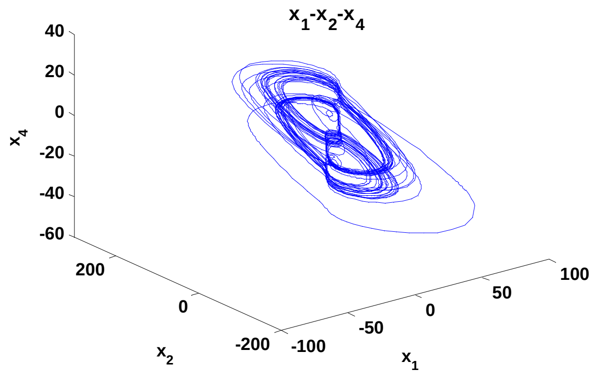

Figure 1(b). The attractors of MLS system in three-dimensional view (variable x_1-x_2-x_4).

From the MLS system (2), we obtain:

$$\nabla V = \frac{\partial \dot{x}_1}{\partial x_1} + \frac{\partial \dot{x}_2}{\partial x_2} + \frac{\partial \dot{x}_3}{\partial x_3} + \frac{\partial \dot{x}_4}{\partial x_4} = -(2a + b + 1)$$

When $(2a + b + 1) > 0$, ∇V is negative. Obviously system (2) is a dissipative system, and an exponential contraction of system (2) is:

$$dV / dt = e^{-(2a+b+1)}$$

Then, we discuss the equilibria of the MLS system. Let:

$$\begin{cases} a(y - x) + dw = 0 \\ x(c - z) - y = 0 \\ sign(y) \cdot x - bz = 0 \\ -x - aw = 0 \end{cases}$$

The system (2) has three equilibrium points, which are respectively described as follows:

$$S_0 = (0,0,0,0),$$

$$S_\pm = \left(\pm\frac{bp}{a}, \pm\frac{bp}{a^2}\left(\frac{d}{a} + a\right), \frac{p}{a}, \mp\frac{bp}{a^2}\right)$$

where $p = ac - a - \dfrac{d}{a}$.

The MLS system is invariant under the transformation of $(x_1, x_2, x_3, x_4) \to (-x_1, -x_2, x_3, -x_4)$. Therefore, if (x_1, x_2, x_3, x_4) is a solution of system (2), $(-x_1, -x_2, x_3, -x_4)$ is also a solution of system (2) and all these solutions exist in pairs.

We consider the condition of $a = 1, b = 0.7, c = 26,$ $d = 1.5$. For equilibrium $S_0 = (0,0,0,0)$, system (2) is linear and the Jacobian matrix is defined as:

$$J_0 = \begin{bmatrix} -a & a & 0 & d \\ c - z & -1 & -x & 0 \\ sign(x_2) & 0 & -b & 0 \\ -1 & 0 & 0 & -a \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & 1.5 \\ 26 & -1 & 0 & 0 \\ 0 & 0 & -0.7 & 0 \\ -1 & 0 & 0 & -1 \end{bmatrix}$$

The eigenvalues of matrix J_0 are:

$$\lambda_1 = -0.7, \lambda_2 = -1, \lambda_3 = 3.9497, \lambda_4 = -5.9497 \tag{3}$$

In formula (3) $\lambda_3 = 3.9497 > 0$, which implies that the equilibrium S_0 is unstable.

System (2) is linearized at equilibrium points $S_\pm$. When $a=1, b=0.7, c=26, d=1.5$, $S_\pm = (\pm 16.45, \pm 41.125, 23.5, \mp 16.45)$. The Jacobian matrices of $S_\pm$ are:

$$J_\pm = \begin{bmatrix} -a & a & 0 & d \\ c-x_3 & -1 & -x_1 & 0 \\ sign(x_2) & 0 & -b & 0 \\ -1 & 0 & 0 & -a \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & 1.5 \\ 2.5 & -1 & \mp 16.45 & 0 \\ \pm 1 & 0 & -0.7 & 0 \\ -1 & 0 & 0 & -1 \end{bmatrix}$$

Matrices J_+ and J_- have the same eigenvalues as follows:

$$\lambda_1 = -3.5878, \lambda_2 = -1, \lambda_{3,4} = 0.4439 \pm 2.0948i \tag{4}$$

Results show that λ_1 and λ_2 are negative, λ_3 and λ_4 form a complex conjugate pair and their real parts are positive. So equilibriums $S_\pm$ are saddle-focus points, and these equilibriums are unstable.

As is well known, the Lyapunov exponents can measure the exponential rates of divergence or convergence of nearby trajectories in phase space. When $a=1, b=0.7, c=26, d=1.5$, the largest positive Lyapunov exponents of the MLS system is calculated as $\lambda_{L\max} = 13.1$.

The spectrum of MLS system (2) is also shown in Fig.3.

As is well known, the Lyapunov exponents can measure the exponential rates of divergence or convergence of nearby trajectories in phase space. When $a=1, b=0.7, c=26, d=1.5$, the largest positive

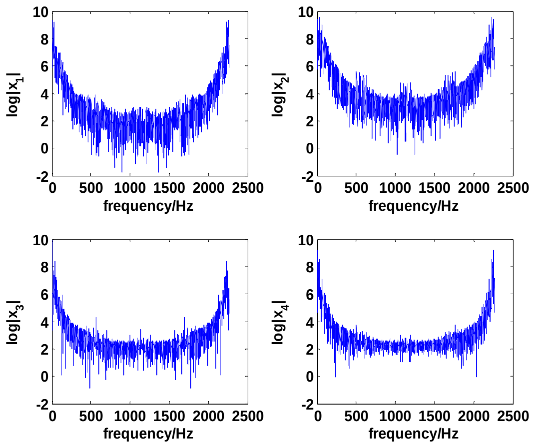

Figure 2. Spectrum of four variables in MLS system.

Lyapunov exponents of the MLS system is calculated as $\lambda_{L\max} = 13.1$. The spectrum of MLS system (2) is also shown in Fig.2.

In MLS system (2), the shape of chaotic attractor is changed with the variation of parameters a, b, c, d. Several simulations have been carried out. Findings are summarized as follows:

1) Let $b=0.7, c=26, d=1.5$.

a) When $a=10$, the eigenvalues of $J_\pm$ for $S_\pm$ are: $\lambda_1 = -11.512, \lambda_2 = -10.696, \lambda_{3,4} = 0.254 \pm 3.76i$. The attractors in three-dimensional view are shown in Fig. 3(a) and 3(b).

b) When $a=22.02$, the eigenvalues of $J_\pm$ for $S_\pm$ are: $\lambda_{1,2} = \pm 4.0252i$, $\lambda_{3,4} = -22.87 \pm 0.787i$. The attractors in three-dimensional view are shown in Fig. 3(c) and 3(d).

2) Let $a=1, c=26, d=1.5$.

a) When $b=5$, the eigenvalues of $J_\pm$ for $S_\pm$ are: $\lambda_1 = -1, \lambda_2 = -7.6874, \lambda_{3,4} = 0.344 \pm 3.894i$.

The attractors in three-dimensional view are shown in Fig. 4(a) and 4(b).

b) When $b=9.75$, the eigenvalues of $J_\pm$ for $S_\pm$ are: $\lambda_1 = -1, \lambda_2 = -11.75, \lambda_{3,4} = \pm 4.416i$. The attractors in three-dimensional view are shown in Fig. 4(c) and 4(d).

c) When $b=15$, the eigenvalues of $J_\pm$ for $S_\pm$ are: $\lambda_1 = -1, \lambda_2 = -16.478, \lambda_{3,4} = -0.261 \pm 4.618i$. The attractors in three-dimensional view are shown in Fig. 4(e) and 4(f).

3) Let $a=1, b=0.7, d=1.5$.

a) When $c=7.9$, the eigenvalues of $J_\pm$ for $S_\pm$ are: $\lambda_1 = -1, \lambda_2 = -2.7, \lambda_{3,4} = \pm 1.183i$. The attractors in three-dimensional view are shown in Fig. 5(a) and 5(b).

b) When $c=18$, the eigenvalues of $J_\pm$ for $S_\pm$ are: $\lambda_1 = -1, \lambda_2 = -3.281, \lambda_{3,4} = 0.291 \pm 1.795i$. The attractors in three-dimensional view are shown in Fig. 5(c) and 5(d).

4) Let $a=1, b=0.7, c=26$.

a) When $d=5$, the eigenvalues of $J_\pm$ for $S_\pm$ are: $\lambda_1 = -1, \lambda_2 = -3.4631, \lambda_{3,4} = 0.382 \pm 1.974i$. The attractors in three-dimensional view are shown in Fig. 6(a) and 6(b).

b) When $d=19.6$, the eigenvalues of $J_\pm$ for $S_\pm$ are: $\lambda_1 = -1, \lambda_2 = -2.7, \lambda_{3,4} = \pm 1.1832i$. The attractors in three-dimensional view are shown in Fig. 6(c) and 6(d).

c) When $d=22$, the eigenvalues of $J_\pm$ for $S_\pm$ are: $\lambda_1 = -1, \lambda_2 = -2.477, \lambda_{3,4} = -0.112 \pm 0.914i$. The attractors in three-dimensional view are shown in Fig. 6(e) and 6(f).

III. BACKSTEPPING SYNCHRONIZATION CONTROL OF MLS SYSTEM

Based on the backstepping control method, we consider using adaptive sliding mode control technique to obtain synchronization. This controller is robust to the parameter uncertainty and guarantees the synchronization of the drive-response MLS chaotic systems. Considering system (2) as drive system, the response system is designed as:

$$\begin{cases} \dot{y}_1 = a(y_2 - y_1) + dy_4 + u_1 \\ \dot{y}_2 = y_1(c' - y_3) - y_2 + u_2 \\ \dot{y}_3 = sign(y_2) \cdot y_1 - b' y_3 + u_3 \\ \dot{y}_4 = -y_1 - ay_4 + u_4 \end{cases} \tag{5}$$

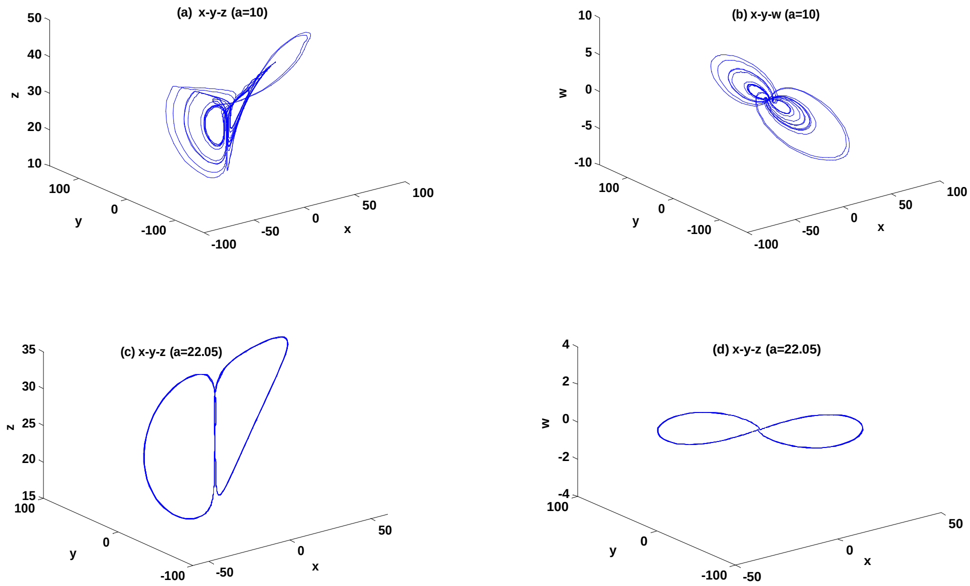

Fig.3. The attractors of MLS system in three-dimensional view when $b = 0.7, c = 26, d = 1.5$ (a)(b) $a = 10$, (c)(d) $a = 22.05$

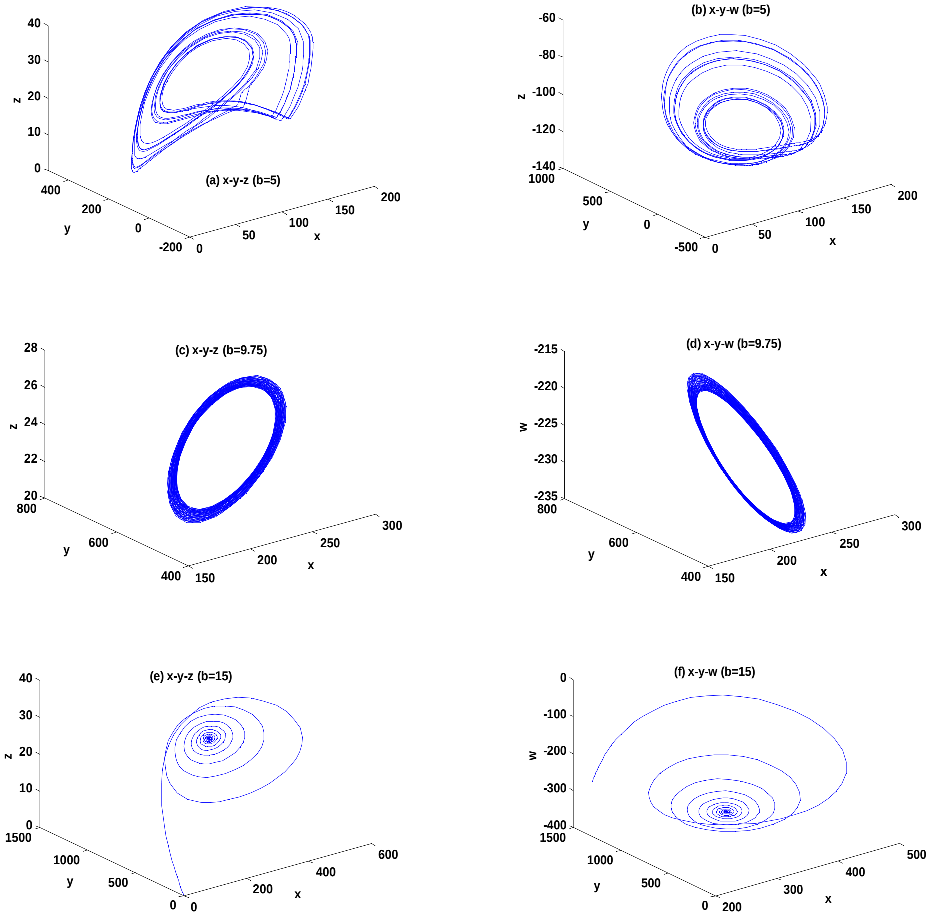

Fig.4. The attractors of MLS system in three-dimensional view when $a = 1, c = 26, d = 1.5$ (a)(b) $b = 5$, (c)(d) $b = 9.75$, (e)(f) $b = 15$

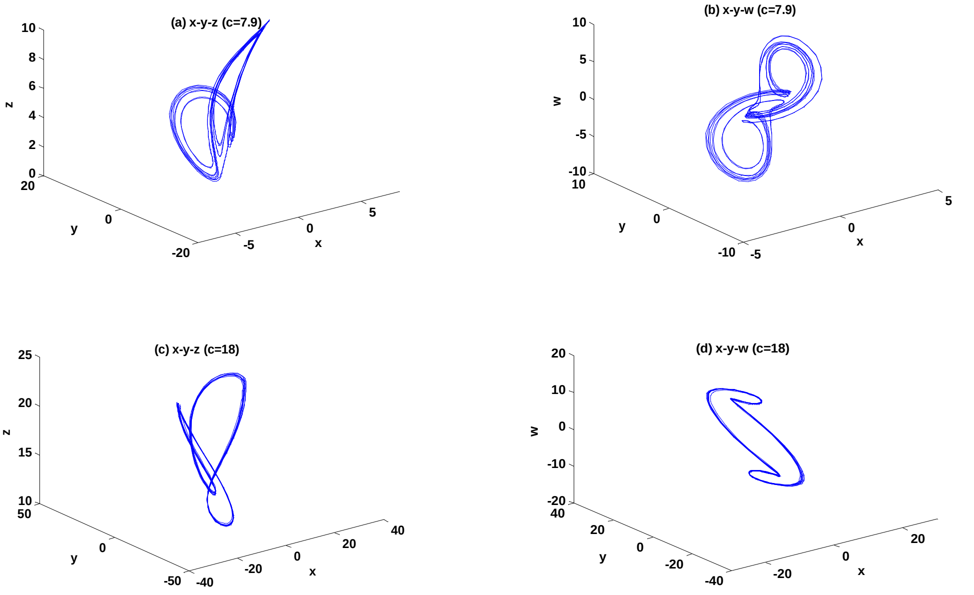

Fig.5. The attractors of MLS system in three-dimensional view when $a=1, b=0.7, d=1.5$ (a)(b) $c=7.9$, (c)(d) $c=18$

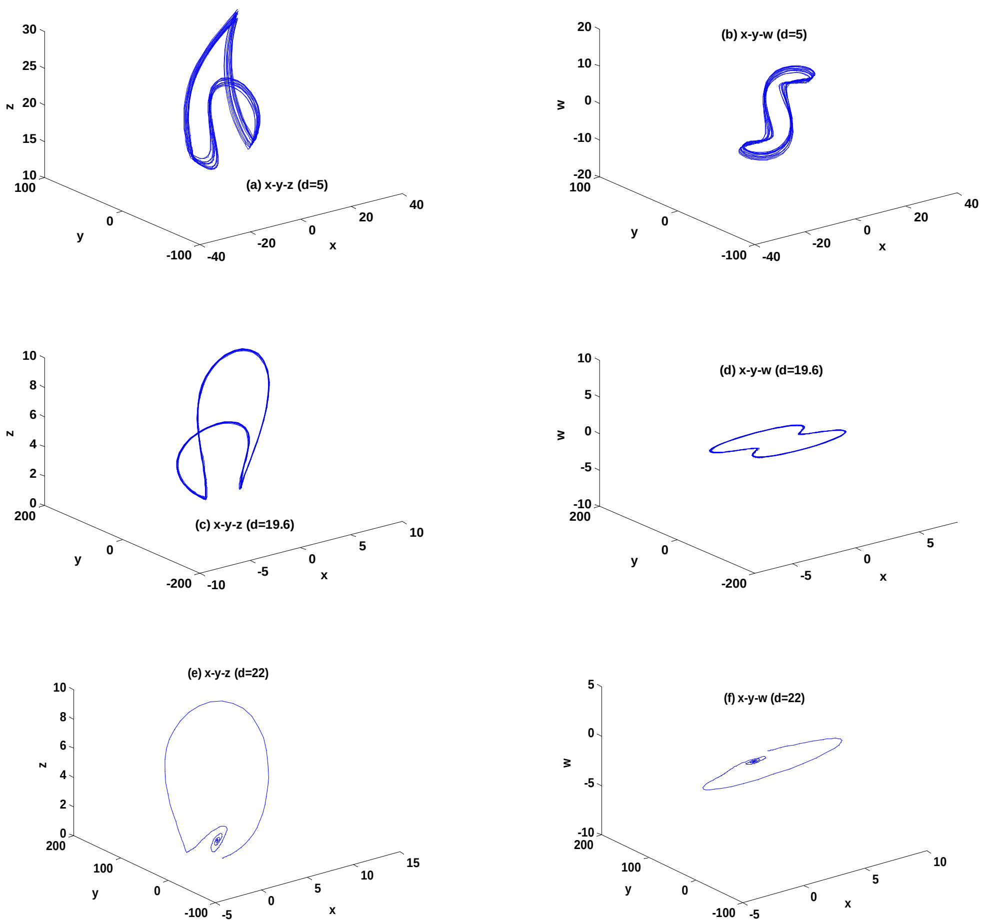

Fig.6. The attractors of MLS system in three-dimensional view when $a=1, b=0.7, c=26$ (a)(b) $d=5$, (c)(d) $d=19.6$, (e)(f) $d=22$

We set: $e_i = y_i - x_i\ (i = 1 \sim 4), e_b = b' - b, e_c = c' - c$, the error system is constructed as:

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \\ \dot{e}_4 \end{bmatrix} = \begin{bmatrix} a(e_2 - e_1) + de_4 + u_1 \\ ce_1 - e_2 - y_1 y_3 + x_1 x_3 + e_c y_1 + u_2 \\ -be_3 + sign(y_2)\cdot y_1 - sign(x_2)\cdot x_1 - e_b y_3 + u_3 \\ -e_1 - ae_4 + u_4 \end{bmatrix} \quad (6)$$

From error system (6), we find there is not any uncertain parameters in e_1 and e_4. In order to simplify the structures of controllers, we set $u_4 = 0$.

Theorem 1: If the sliding mode controllers satisfy:

$$\begin{cases} u_1 = -de_4 \\ u_2 = (x_3 - c)e_1 - m_1 sign(s_1) \\ u_3 = sign(x_2)\cdot x_1 - sign(y_2)\cdot y_1 - m_2 sign(s_2) \\ \dot{e}_b = e_3 y_3 \\ \dot{e}_c = -e_2 y_1 \end{cases} \quad (7)$$

the error system (6) will be gradually stable and the uncertain parameters are identified. In formula (7), $m_1, m_2 > 0$ and the switching surfaces are $s_1 = e_2, s_2 = e_3$.

Proof: Backstepping control method is applied to prove theorem 1.

1) Set: $w_1 = e_4$, so $\dot{w}_1 = -e_1 - ae_4 + u_4$. Given Lyapunov function $V_1 = w_1^2/2$, we get:

$$\dot{V}_1 = w_1\dot{w}_1 = -ae_4^2 - e_1e_4 \quad (8)$$

$\alpha_1(w_1)$ is considered as virtual control to e_1. When $\alpha_1(w_1) = e_1 = 0$, $\dot{V}_1 = -ae_4^2 \le 0$, so e_4 is gradually stable.

2) Set: $w_2 = e_1 - \alpha_1(w_1)$, which is the error of virtual control $\alpha_1(w_1)$ and e_1. So $\dot{w}_2 = a(e_2 - e_1) + de_4 + u_1$. There is not any uncertain parameters in $\dot{w}_2$, so we set $V_2 = V_1 + w_2^2/2$, then:

$$\begin{aligned} \dot{V}_2 &= V_1 + e_1[a(e_2 - e_1) + de_4 + u_1] \\ &= -ae_4^2 - ae_1^2 + e_1[ae_2 + de_4 + u_1] \end{aligned} \quad (9)$$

If $u_1 = -de_4$, $\dot{V}_2 = -ae_4^2 - ae_1^2 + ae_1e_2$. Then $\alpha_2(w_1, w_2)$ is considered as virtual control to e_2. When $\alpha_2(w_1, w_2) = e_2 = 0$, $\dot{V}_2 = -ae_4^2 - ae_1^2 \le 0$, so e_1, e_4 are gradually stable.

3) Set: $w_3 = e_2 - \alpha_2(w_1, w_2)$, which is the error of virtual control $\alpha_2(w_1, w_2)$ and e_2. We get $\dot{w}_3 = ce_1 - e_2 - y_1e_3 - e_1x_3 + e_cy_1 + u_2$. Let $V_3 = V_2 + (w_3^2 + e_c^2)/2$, then:

$$\begin{aligned} \dot{V}_3 = &-ae_4^2 - ae_1^2 - e_2^2 + \\ &e_2[(c - x_3)e_1 - y_1e_3 + e_cy_1 + u_2] + e_c\dot{e}_c \end{aligned} \quad (10)$$

If $u_2 = (x_3 - c)e_1 - m_1 sign(s_1), \dot{e}_c = -e_2y_1$ and the switching surface is $s_1 = e_2$, we will obtain:

$$\dot{V}_3 = -ae_4^2 - ae_1^2 - ae_2^2 - m_1e_2 sign(e_2) - y_1e_3$$

Now $\alpha_3(w_1, w_2, w_3)$ is considered as virtual control to e_3. When $\alpha_2(e_4, w_2) = e_3 = 0$, we get $y_1e_3 = 0$, so $\dot{V}_2 = -ae_4^2 - ae_1^2 - e_2^2 - m_2e_2 sign(e_2) \le -ae_4^2 - ae_1^2 - e_2^2 - m_1|e_2| \le 0 \le 0$. That is to say, e_1, e_2, e_4 are gradually stable.

4) Set: $w_4 = e_3 - \alpha_3(w_1, w_2, w_3)$, which is the error of virtual control $\alpha_3(w_1, w_2, w_3)$ and e_3. We get $\dot{w}_4 = -be_3 + sign(y_2)\cdot y_1 - sign(x_2)\cdot x_1 - e_by_3 + u_3$. Let $V_4 = V_3 + (w_4^2 + e_b^2)/2$, then:

$$\begin{aligned} \dot{V}_4 = V_3 - be_3^2 + e_3[&sign(y_2)\cdot y_1 - sign(x_2)\cdot x_1 \\ &- e_by_3 + u_3] + e_b\dot{e}_b \end{aligned} \quad (11)$$

If $u_3 = sign(x_2)\cdot x_1 - sign(y_2)\cdot y_1 - m_2 sign(s_2), \dot{e}_b = e_3y_3$ and the switching surface is $s_2 = e_3$, we obtain:

$$\begin{aligned} \dot{V}_4 &= -ae_4^2 - ae_1^2 - e_2^2 - be_3^2 - m_1e_2 sign(e_2) \\ &\quad -m_2e_3 sign(e_3) \\ &\le -ae_4^2 - ae_1^2 - e_2^2 - be_3^2 - m_1|e_2| - m_2|e_3| \\ &\le 0 \end{aligned}$$

From the formulas (8)~(11), we can conclude the error system (6) is gradually stable under the control of formula (7). That is to say, system (2) and system (5) achieve complete synchronization. □

IV. SIMULATIONS

Considering MLS system (2) as the drive system and system (5) as the response system, we use formula (7) as the synchronization controller in simulations. The state initial values of two MLS systems are set as (2,4,6,8) and (9,7,5,3). In drive systems (2), the parameters are $a = 1, b = 0.7, c = 26, d = 1.5$. The response system (5) has two uncertain parameters and their initial values are set as $b' = 1, c' = 20$. The function of Runge-Kutta algorithm in MATLAB is applied with the unit step of 0.01.

Firstly, the two MLS chaotic systems achieve complete synchronization when the proportion parameters are set as $m_1 = 1, m_2 = 1$. Fig.7 shows that error systems of two MLS systems turn to zero gradually in about five seconds. Meanwhile the uncertain parameters b' and c' are identified to be 0.7 and 26 in Fig.8, which are same with those of drive system (2).

Secondly, in order to increase the synchronization speed, we set $m_2 = 6$. Other parameters and simulation speed are unchanged. Obviously, from Fig.9, the synchronization speed is faster than that of the previous simulation. The error e_2 turns to zero and parameter c' is identified to 26 only in two seconds.

Then we set $m_1 = m_2 = 0$, which means the switching surfaces of sliding mode control are omitted from the controllers. The simulation results are shown in Fig.10 and Fig.11. Contrasting to Fig.8~11, two MLS systems are hardly to synchronize each other and parameter c' can not be stabled to 26 without the sliding mode control. The contrasts indicate the importance and effectiveness of sliding mode control.

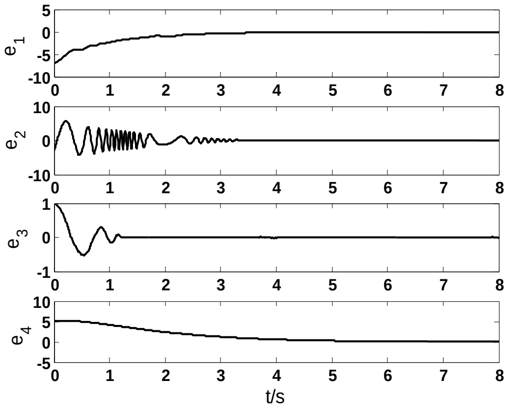

Figure 7. The plots of error systems when $m_1=m_2=1$.

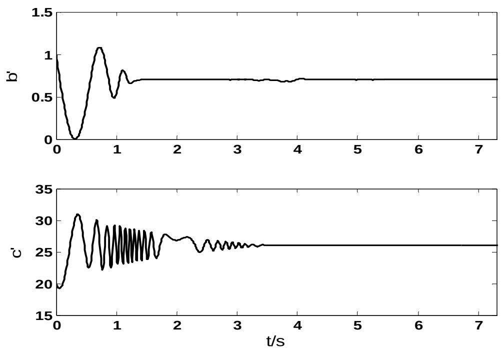

Figure 8. The plots of uncertain parameters when $m_1=m_2=1$.

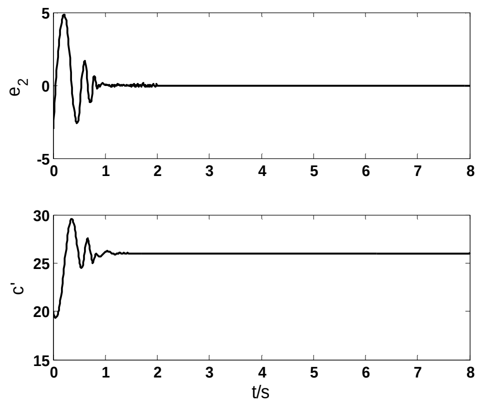

Figure 9. The plots of error systems and uncertain parameters when $m_2=6$

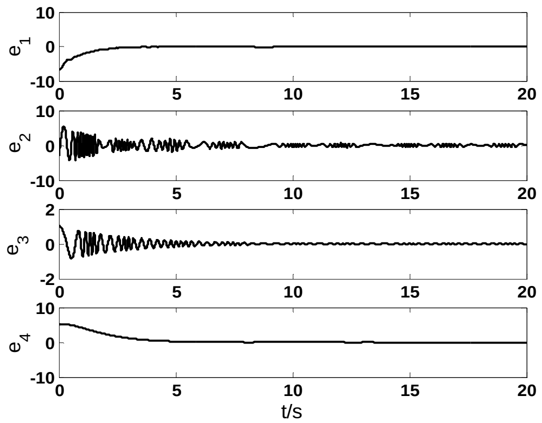

Figure 10. The plots of error without sliding mode control.

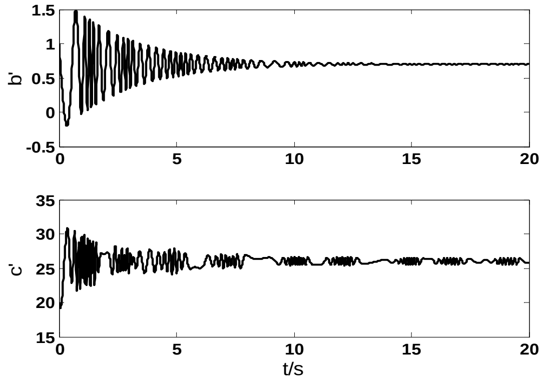

Figure 11. The plots of uncertain parameters without sliding mode control

V. CONCLUSIONS

A new modified Lorenz-Stenflo (MLS) system is proposed in this paper. Dynamical analysis shows the MLS chaotic system has the different chaotic behaviors from LS system and Lorenz system. Compared with the original LS system, the MLS system can be more easily implemented in electronic circuits and more practicable applied in secure communication.

Based on adaptive control and sliding mode control methods, an adaptive sliding mode controller is derived to synchronize two MLS chaotic systems with uncertain parameters. The error system between two MLS chaotic systems is deduced using backstepping control theory. Numerical simulations are used to verify the effectiveness of the obtained controller.

ACKNOWLEDGMENT

It is a project supported by China Postdoctor Science Foundation of No.20080430170 and Jiangsu Postdoctor Science Foundation of No. 0801046B.

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