Active Vibration Control of Piezoelectric Intelligent Structures

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Abstract—**The study of the applications of piezoelectric materials in the active vibration control of flexible structures has been ongoing for more than a decade. Based on the Linear Quadratic Gauss (LQG) optimal control method, the paper introduces an effective procedure to suppress the vibration of flexible structures with the sensors/actuators are symmetrically collocated on both sides of the same position on the host structure. The model of a piezoelectric intelligent cantilever beam is built by the application of ANSYS software, and through the modal analysis, the first 4 ranks of modal frequencies and vibration shapes are extracted. The D-optimal design principle is adopted which is an optimal method which chosen by the maximum determinant of Fisher Information Matrix as the criteria function. Studies on the modal vibration shapes and dynamic characteristics of the structure, and converts the selected modes into a unitary form by a simple method, the optimal position of the piezoelectric elements can be confirmed by the mode shapes of the structures. Finally, the paper takes a piezoelectric cantilever beam as an example and gives the step response curve of the system at the two circumstance of when piezoelectric elements are bonded onto the optimal position of the host structure and not, the simulation results show the effectiveness of the method in this paper.**

Index Terms—**piezoelectric materials, intelligent structures, LQG optimal control, D-optimal design principle, active vibration control, ANSYS software**

I. INTRODUCTION

Piezoelectric materials are finding increasing applications in active vibration control of flexible structures, recently, a great progress has been achieved in the development of flexible structures associated with piezoelectric materials as sensors and actuators. Piezoelectric ceramics provide cheap, reliable and non-intrusive means of actuation and sensing in flexible structures. A flexible structure used in vibration control can be defined as a structure or structure's component with bonded or embedded sensors and actuators as well as an associated control system, which enable the structure to respond simultaneously to external stimulus exerted on it and then suppresses undesired effects or enhance desired effects [1]. In 1985, Bailey et al. [2] performed an experimental research on active vibration control using surface-bonded piezoelectric polymer actuators on a cantilever beam. Their experiment has greatly inspired the study of the related field. Recently, much research has been developed in the field of intelligent materials and structures. Piezoelectric is a kind of intelligent material, due to the following two characteristics: the first is direct and inverse piezoelectric effects and the second is the ability to be used as the sensor or actuator in active vibration control systems.

A significant proportion of the mainstream research in the vibration control of flexible structures has been concentrated on suppressing vibrations at a pre-specified point which is often where the sensor and actuator located, so, the position of piezoelectric sensors/actuators plays an important role in the design procedure of active vibration control. In some applications, this might be a satisfactory design criterion. However, it could be argued that there are a large number of applications where the vibrations of the entire structure or at least a certain region should be minimized. Many researchers have focused on the development of the optimal position of the piezoelectric elements. Padula and et al [3] explained the importance of sensors/actuators selection, reviewed the optimal techniques and summarized some experimental and numerical results. Han and Lee [4] built the controllable Grammian matrix of the system, and took the maximal eigenvalue as performance function. Genetic algorithms were used to find the effective locations of piezoelectric sensors and actuators on smart composite plates. Sadri and et al [5] presented two criteria for the optimal position of piezoelectric actuators using the controllability of modes. Genetic algorithms were also used to place two piezoelectric actuators on a simply supported isotropic plate. Gao and et al [6] considered a vibration suppression problem with the objective to minimize the total radiated acoustic power or acoustic potential energy. They used genetic algorithms with immune diversity to search for the optimal position for actuators. Zhang and et al [7] developed a performance function based on maximizing the dissipated energy which due to the control action. According to this characteristic, a float-encoded genetic algorithm is presented which is capable of solving this optimization problem. Guo and et al [8] presented a global optimization of sensors position based on the damage

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detection method for structural health monitoring systems. Martin Kögl and et al [9] presented a novel approach to the design of piezoelectric plates and shell actuators using topology optimization. In this approach, the optimization problem consists of distributing the piezoelectric actuators in such a way as to achieve a maximum output displacement in a given direction at a given point of the structure. Cao and et al [10] used the element sensitivities of singular values to identify optimal position for actuators.

In a word, there have been many performance criterion were presented, such as observability and controllability of the control system measures, dissipation energy measures and system stability measures. However, in order to make use of the above mentioned measures, the state space equation of the system should be modeled first by the given position of piezoelectric elements.

The D-optimal design principle is an optimal method presented by Bayard and et al [11] which chosen by the maximum determinant of Fisher Information Matrix as the criteria function. The paper simplified the selected modes into a unitary form by a simple method to determine the optimal position of the piezoelectric elements by the using of this principle. Joshi [12] concluded that, the bonded piezoelectric elements have a significant influence to the plate's natural frequencies and the associated modal shapes. Accordingly, with respect to this influence, the paper used the Linear Quadratic Gauss (LQG) optimal control method, applied filtration technology and designed a Kalman filter to filtrate the process noise and the measurement noise, so as to exactly estimate the state and output of the system, and as well as reached the best control effect.

II. Modeling and Theory

A. Physical Model of the System

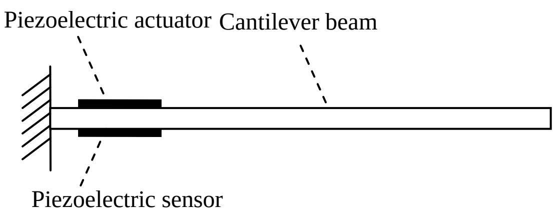

Figure1. Physical model of flexible cantilever beam

Consider the flexible cantilever beam as in Fig.1 shows, the beam is fixed at one end and free at the other, there is a pair of piezoelectric patches which will be used as sensor/actuator, and they are collocated on both sides of the same position of the structure. The research of reference [13] is verified that the symmetrical collocation can avoid observation spillover and control spillover induced by modal truncation, and ensures the controlled system is minimum phase system.

B. Piezoelectric Constitutive Equations

For the flexible structures shape control and active vibration control, piezoelectric material is often used as sensor and actuator. When the piezoelectric element is used as a sensor, the strain ε which the component produces is known, the dielectric displacement D which the component inducts must be solved; when the piezoelectric element is used as actuator, the applied electrical field E is known, and the stress σ which the electrical field E produces must be solved. The following two types of piezoelectric constitutive equations can describe the material performance in vibration control.

$$\begin{cases} \{\sigma\} = -[c]\{E\} + [s]\{\varepsilon\} \\ \{D\} = [e]\{E\} + [c]\{\varepsilon\} \end{cases} \quad (1)$$

Where ε is the strain tensor matrix, s is the elastic stiffer constant matrix when the electrical field E is zero, σ is the stress tensor matrix, c is the piezoelectric stress constant, e is the dielectric constant when the strain ε is zero, E is the applied electrical field vector, D is the electric displacement density vector. Because the terminal conditions are $\varepsilon = 0$ and $E = 0$, the piezoelectric constitutive equations for sensors and actuators applied into active vibration control of flexible structures are simplified as follows:

$$\begin{cases} \sigma_\lambda = -c_{j\lambda} E_j + s^E_{\lambda u} \varepsilon_u \\ D_i = e^\varepsilon_{ij} E_j + c_{iu} \varepsilon_u \end{cases} \quad (2)$$

C. Sensing Equations

The external electrical field will be zero if there is not any vibration, but the piezoelectric sensors will produce electrical charge while the beam has some external stimulus. From (1) and (2), the output voltage of the *i*th piezoelectric sensor can be gained as follows:

$$V_i = k e_{31} b_p r_s \int_{x_{1i}}^{x_{2i}} \frac{\partial^2 w(x,t)}{\partial x^2} dx \quad (3)$$

The voltage is an amplified one by the electric charge amplifier. Where $i = 1, \cdots, r$, r and b_p represent the number and the width of the sensors respectively, r_s is the distance from the middle axis of the sensors to the middle axis of the smart beam, k is the amplified multiple of the electric charge amplifier, x_{1i} and x_{2i} is the coordinate value of the ith sensor. Then, the vibration of the smart beam can be observed effectively based on the equations.

D. Actuating Equations

Consider the dynamic equations of a cantilever beam with clamped-mass boundary conditions and bounded piezoelectric sensors and actuators, the paper takes a cantilever beam bonded with rectangular shaped piezoelectric sensors and actuators as the study object, and the beam's transverse vibration equation can be expressed by a partial differential equation as follow [14]:

$$E_b J_b \frac{\partial^4 w}{\partial x^4} + \rho_b A_b \frac{\partial^2 w}{\partial t^2} = M[\delta'(x - x_2) - \delta'(x - x_1)] \quad (4)$$

Where E_b , J_b , A_b and ρ_b represent respectively the Young's modulus, moment of inertia, cross-section area, and the linear mass density of the beam, δ is the Dirac delta function, M is the bending moment and it can be gained from the input voltage of u :

$$M = K_a u \tag{5}$$

Where $K_a = bd_{31}E_p(t_p + t_b)$, in which b , t_p and E_p represent respectively the width, the thickness and the Young's modulus of the piezoelectric elements, t_b is the thickness of the base structure, d_{31} is the piezoelectric constant.

To use the assumed modes method, the function of $w(x,t)$ can be expanded as follows:

$$w(x,t) = \sum_{i=1}^{n} \phi_i(x)\eta_i(t) \tag{6}$$

Where $\phi_i(x)$ is the modal function, $\eta_i(t)$ is the generalized coordinate. The boundary condition for the cantilever beam is:

$$\begin{cases} x = 0B & _1[w(x,t)] = 0 \\ x = lB & _2[w(x,t)] = 0 \end{cases} \tag{7}$$

Resulting from the substitution of (7) into (4) ~ (6), the modal vibration shapes are assumed to be expressed as:

$$\phi_i(x) = A[\sin a_i x - \sinh a_i x + \frac{\sin a_i l + \sinh a_i l}{\cos a_i l + \cosh a_i l}(\cosh a_i x - \cos a_i x)] \tag{8}$$

Where $a_1 l = 1.875, a\ _2 l = 4.694, a\ _3 l = 7.855$, when $i = 4,5,\cdots,\infty$, $a_i l = \frac{2i-1}{2}\pi$. From the equations (4) ~ (8), substituting the value of the moment on the dynamic equation of the system, we can conclude that the vibration of the host structure would be controlled by means of put some voltage on the piezoelectric actuators.

E. Simplified Model

From the exited research, we can conclude that the finite element method could analyze the arbitrary geometry models and the anisotropic properties of the piezoelectric materials. Considering the piezoelectric effect, special finite elements with a degree of voltage are developed. These elements have become available in some commercial finite element software such as ANSYS already. In this paper, the solid45 3-d solid elements are used to model the host structure and the solid5 3-d solid elements are used to model the piezoelectric elements. All of the material characters of them are shown in Tab. I and Tab. II, through modeling and modal analysis to the piezoelectric flexible structure by ANSYS software, the first 4 ranks of modal frequencies are shown in Tab. III, at the same time, Figs. 2 ~ 5 show the modal vibration shapes of the system with no piezoelectric elements bonded, where Figs. 6 ~ 9 show the modal vibration shapes of the system with a pair of piezoelectric elements bonded to the host structure, compare the Figs. 2 ~ 5 to 6 ~ 9, it can be conclude that, even though the vibration shapes are similar at the two circumstance, the modal frequencies of the system are changed at a certain extent, it explains that the bonded piezoelectric elements have some influence to the natural frequencies of the host structure.

TABLE I.
CHARACTER OF THE ALUMINUM MATERIAL

Aluminum	EX (N/m^2)	Dens(kg/m^3)	Nuxy
SOLID45	7.5842×10^{10}	2743	0.27

TABLE III.
THE FIRST 4 RANKS OF MODAL FREQUENCIES OF THE SYSTEM

Ranks		1	2	3	4
Frequencies	Before bonded	2.7	16.9	47.7	94.32
	After bonded	3.0	18.7	52.6	103.9

TABLE II.
CHARACTER OF THE PIEZOELECTRIC MATERIAL

PZT-5H	Dens (kg/m^3)	Relative piezoelectric constant		Piezoelectric strain constant (c/m^2)			Flexible coefficient (10^{10}N/m^2)					
SOLID5	7700	$\varepsilon_{11}^S / \varepsilon_0$	$\varepsilon_{33}^S / \varepsilon_0$	e33	e31	e15	c11	c12	c13	c33	c44	c66
		1697.53	1468.26	23.3	-6.5	17	12.6	7.95	8.41	11.7	23	23.3

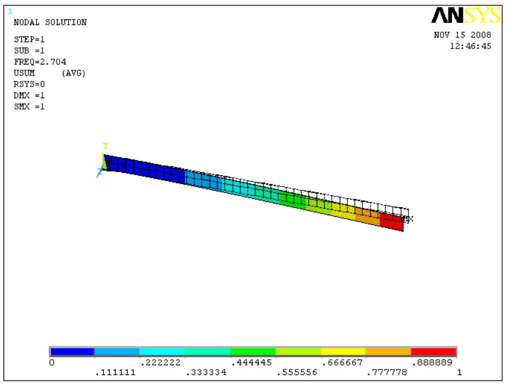

Figure2. The first rank of modal vibration shape with no piezoelectric elements bonded

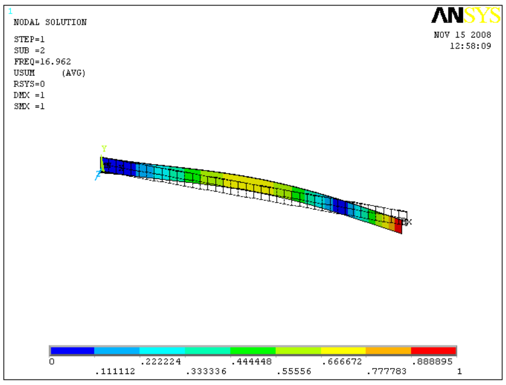

Figure3. The second rank of modal vibration shape with no piezoelectric elements bonded

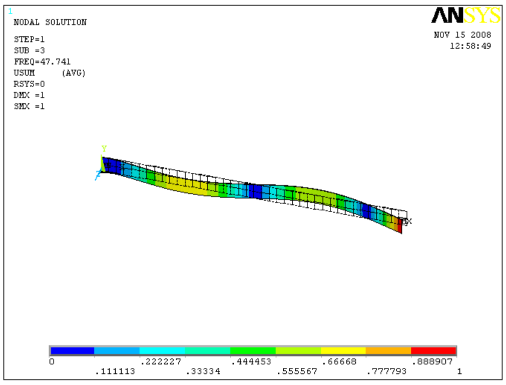

Figure4. The third rank of modal vibration shape with no piezoelectric elements bonded

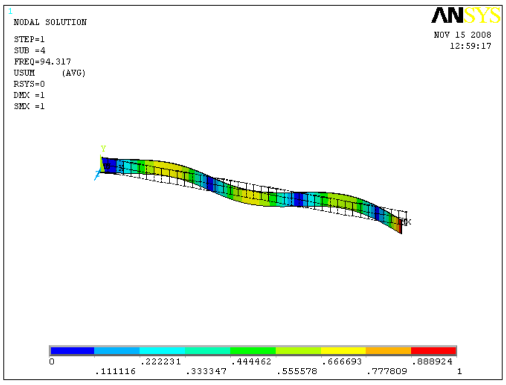

Figure5. The forth rank of modal vibration shape with no piezoelectric elements bonded

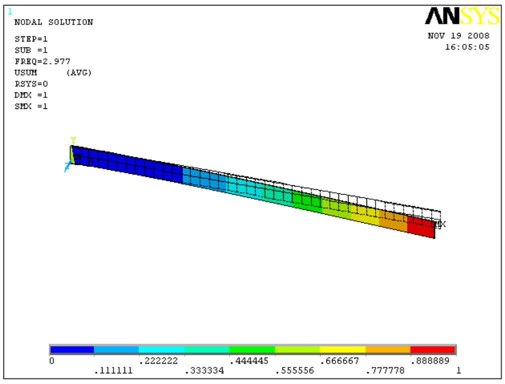

Figure6. The first rank of modal vibration shape with a pair of piezoelectric elements bonded

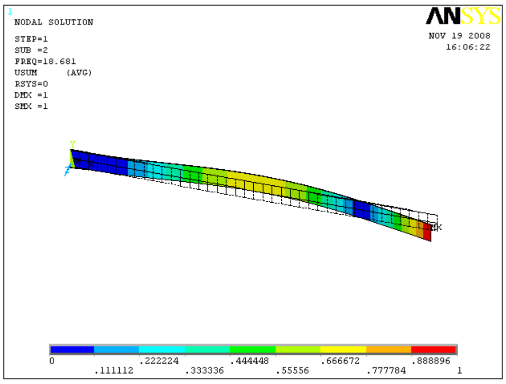

Figure7. The second rank of modal vibration shape with a pair of piezoelectric elements bonded

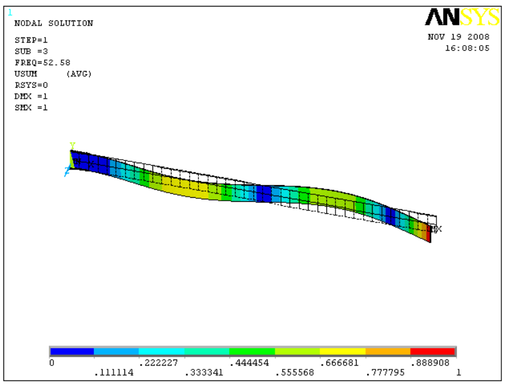

Figure8. The third rank of modal vibration shape with a pair of piezoelectric elements bonded

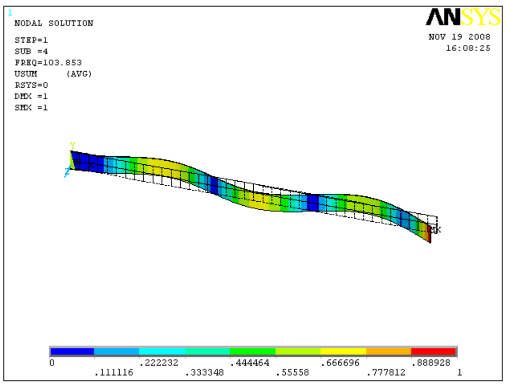

Figure9. The forth rank of modal vibration shape with a pair of piezoelectric elements bonded

III. OPTIMAL POSITIONS OF PIEZOELECTRIC ELEMENTS

A. Performance Criterion of D-optimal Design Principle

Actually, the sensors are used to estimate the state parameters [15]. Based on the theory of mathematical statistics, the determinant of Fisher Information Matrix $\det(F)$ has inverse ratio to the low bound of variance of parameter unbiased estimation. Due to the symmetrical collocation of the piezoelectric elements, if the position of sensors is confirmed, the actuators will be at the same position as the sensors. For a lightly damped structure, the D-optimal design principle can be simplified to the decouple problems of placement of sensors/actuators and input control, the principle can be written as:

$$\max(\det(F)) \tag{9}$$

Subject to $\beta \in B_m$, $B_m = \{\beta : \sum_{k=1}^{M} \beta_k = m\}$, Where β is the position selection matrix, B_m is the set of possible positions. The objective function can also be written as:

$$S(m) = \max_{\beta} \sum_{i=1}^{N} \log\left(\sum_{k=1}^{M} \beta_k \frac{(\gamma_k^T \phi_i)^2}{\varphi_k^2}\right) \tag{10}$$

Where N and M are the number of modes and the possible positions of sensors, m is the number of the sensors, β_k is composed of 0 and 1, if $\beta_k = 1$, it denotes that a sensor is located on the position, by contrast, if $\beta_k = 0$, it denotes that the position has no a sensor, γ_k is a vector coefficient related to the kth position of sensor; ϕ_i is the ith normalized modal shape vector, φ_k is the covariance of sensor signal noise, the value of φ_k relates to the piezoelectric materials, if both the sensors and actuators are use the same materials, φ_k is a constant, and this constant has no influence to the optimal results, so assuming that $\varphi_k = 1$.

The physical sense of (10) can be regard as finding the position of maximum charge or voltage output, therefore, $\gamma_k^T \phi_i$ is equivalent to the output charge of sensors. Supposes that the sensor area is much small compared to the beam, the sensor charge can be written as:

$$q = D \times A = A \times (D_x + D_y) \tag{11}$$

$$D_x = d_{31} E_p \varepsilon_x = d_{31} E_p \frac{\partial^2 w}{\partial x^2} \frac{t_p}{2} \tag{12}$$

$$D_y = d_{31} E_p \varepsilon_y = d_{31} E_p \frac{\partial^2 w}{\partial y^2} \frac{t_p}{2} \tag{13}$$

Where q is the charge output by sensors, D_x is the electric displacement generated by the x axis strain, D_y is the electric displacement generated by the y axis strain, and A is the area of the sensor. To the beam structure, electric displacement is generated by unidirectional strain, d_{31} is the piezoelectric strain constant, ε_x and ε_y are represent the piezoelectric strains generated by the x axis and y axis respectively, E_p and t_p are the Young's modulus and the thickness of the piezoelectric elements, w is the deflection of the structure. Dimensions of the piezoelectric elements and the finite element model are shown in Fig.10.

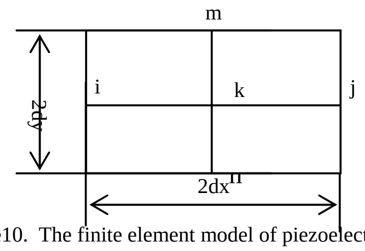

Figure10. The finite element model of piezoelectric elements

Based on the second-order difference scheme, $\frac{\partial^2 w}{\partial x^2}\Big|_k$ and $\frac{\partial^2 w}{\partial y^2}\Big|_k$ can be written as:

$$\frac{\partial^2 w}{\partial x^2}\Big|_k \approx d_x^2 w = \frac{w(i) + w(j) - 2w(k)}{(dx)^2} \tag{14}$$

$$\frac{\partial^2 w}{\partial y^2}\Big|_k \approx d_y^2 w = \frac{w(m) + w(n) - 2w(k)}{(dy)^2} \tag{15}$$

Substitutes (12), (13), (14), (15) into (11) to yield:

$$q = d_{31} E_p A \lambda_k \tag{16}$$

Where

$$\lambda_k = \frac{t_p}{2}\left(\frac{w(i)+w(j)-2w(k)}{(dx)^2} + \frac{w(m)+w(n)-2w(k)}{(dy)^2}\right);$$

the deflection of the structure w can be expressed by the mode shape function ϕ_i:

$$w = \sum_{i=1}^{N} \eta_i \phi_i \tag{17}$$

Where η_i is the *i*th modal generalized coordinate, substitutes (17) into (16) and assumes that $\sum_{i=1}^{N} \gamma_k^T \phi_i = q$, γ_k^T can be written as:

$$\gamma_k^T = \frac{1}{2} \eta_i A d_{31} E_p \lambda_k^i \tag{18}$$

Where

$$\lambda_k^i = \phi_i \frac{t_p}{2}\left(\frac{1}{(dx)^2}[0 \ldots 1_i \quad -2_k \quad 1_j \ldots 0] + \frac{1}{(dy)^2}[0 \ldots 1_m \quad -2_k \quad 1_n \ldots 0]\right),$$

in which ϕ_i is the *i*th normalized mode shape vector, substitutes (18) into (10), the objective function can be simplified as:

$$S(m) = \max_{\beta} \sum_{i=1}^{N} \left(\sum_{k=1}^{M} \eta_i \beta_k \left|\lambda_k^i\right|\right) \tag{19}$$

B. The Simplify and Improvement of D-optimal Design Principle

Equation (19) illustrates that the objective function is composed of all ranks of the modal shapes with the coefficient η_i. With the vibration generating force, the kinematics equation in modal coordinates can be written as:

$$\ddot{\eta}_i(t) + 2\zeta_i\omega_i\dot{\eta}_i(t) + \omega_i^2\eta_i(t) = \frac{1}{M_i} f(t) \quad (20)$$

Where $\ddot{\eta}_i$, $\dot{\eta}_i$ and η_i represent the modal acceleration, velocity and displacement respectively, ω_i and ζ_i represent the natural frequency and damping ratio of the *i*th mode.

For a different force, every modal shape has a different proportion in the vibration of a structure. Assuming that the vibration generating force is taken as a unit impulse, structure vibrates according to the lower mode shape and the other modal shapes can be ignored. Assuming that the vibration generating force is taken as sine force with a frequency of θ, the vibration of the structure has relation to the frequency of the external force rather than uniquely according to one modal shape. For the above-mentioned, in order to develop the performance of the vibration, the bonded elements must control every modal shape. So, the piezoelectric elements should be placed on the maximum of all the modal strains. Unitary modal strain can be written as:

$$\overline{\lambda_k^i} = \lambda_k^i / \max(\phi_i) \quad (21)$$

Integrates (21) and (19), a new objective function can be written as:

$$S^*(m) = \max_{\beta} \sum_{i=1}^{N} (\sum_{k=1}^{M} \beta_k \left|\overline{\lambda_k^i}\right|) \quad (22)$$

Apparently, the performance criterion of the sensors/actuators position can be obtained using mode shapes of the structure.

IV. LINEAR QUADRATIC GAUSS (LQG) OPTIMAL CONTROLS

In this section, it explains the basic ideas behind a new approach to active vibration control referred to as the spatial LQG approach. From (4) ~ (6), because of the perpendicular character of the function $\phi_i(x)$, the system's modal coordinate equation can be gained:

$$\ddot{\eta}_i(t) + \omega_i^2\eta_i(t) = B_i u \quad (23)$$

Where $B_i = K_a[\phi_i'(x_2) - \phi_i'(x_1)]$, assuming that the structure itself has a diagonally damp, then,

$$\ddot{\eta}_i(t) + 2\zeta_i\dot{\eta}_i + \omega_i^2\eta_i(t) = B_i u \quad (24)$$

Where ζ_i is the damping ratio of the *i*th mode.

The output voltage of sensor is:

$$u_s = K_s[w'(x_2) - w'(x_1)] = K_s \sum_{i=1}^{n}[\phi_i'(x_2) - \phi_i'(x_1)]\eta_i(t) = \sum_{i=1}^{n} C_i\eta_i(t) \quad (25)$$

Where $K_s = \frac{bt_b g_{31}}{2C_p}$, $C_i = K_s[\phi_i'(x_2) - \phi_i'(x_1)]$, g_{31} and C_p represent respectively the piezoelectric constant and the capacitance. Introduce the state vector $X = \{\eta_1, \eta_2, \cdots, \eta_n, \ \dot{\eta}_1, \dot{\eta}_2, \cdots, \dot{\eta}_n\}$, (23) and (24) can be written as follows:

$$\dot{X}(t) = AX(t) + BU(t) + \varepsilon_1(t) \quad (26.a)$$

$$X(t_0) = X_0 \quad (26.b)$$

$$Y(t) = CX(t) + \varepsilon_2(t) \quad (26.c)$$

Where $A = \begin{bmatrix} 0_{n\times n} & I_{n\times n} \\ -\Omega^2 & -2\zeta_i\omega_i \end{bmatrix}$, $B = \begin{bmatrix} 0_{n\times 1} \\ B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix}$, $C = \begin{bmatrix} C_1 & C_2 & \cdots & C_n & 0_{1\times n} \end{bmatrix}$, $\Omega = \begin{bmatrix} \omega_1 & & & \\ & \omega_2 & & \\ & & \ddots & \\ & & & \omega_n \end{bmatrix}$, $X(t)$ is the state vector of the system, $U(t)$ is the control force of the structure, B and C represent the position matrix and the output matrix respectively, $Y(t)$ is the output vector, $\varepsilon_1(t)$ and $\varepsilon_2(t)$ are zero-mean Gauss white noise, they can be expressed as:

$$E[\varepsilon_1(t)] = 0E \qquad [\varepsilon_2(t)] = 0 \quad (27.a)$$

$$E[\varepsilon_1(t)\varepsilon_1^{\mathrm{T}}(\tau)] = Q_{\mathrm{e}}\delta(t-\tau) \quad (27.b)$$

$$E[\varepsilon_2(t)\varepsilon_2^{\mathrm{T}}(\tau)] = R_{\mathrm{e}}\delta(t-\tau) \quad (27.c)$$

A spatial LQG controller attempts to minimize the vibration of the entire structure by minimizing a cost function that relates to the spatial behavior of the composite system, and a LQG controller consists of a state feedback control law designed by solving a linear quadratic regulator problem plus a Kalman filter. The quadratic performance criterion function given by:

$$J = \frac{1}{2}\int_{t_0}^{\infty}[X^T(t)QX(t) + U^T(t)RU(t)]dt \quad (28)$$

Firstly, design the optimal feedback control force $U(t)$ by the application of classical LQR control method:

$$U(t) = -KX(t) \quad (29)$$

Where $K = R^{-1}B^{T}P$, P is the stabilizing solution to the Riccati equation:

$$-PA - A^{T}P + PBR^{-1}B^{T}P - Q = 0 \quad (30)$$

When the full state is not available, the state is replaced in the control law by an optimal state estimate generated by a Kalman filter given by:

$$\dot{\hat{X}}(t) = (A - BK - K_{e}C)\hat{X}(t) + K_{e}Y \quad (31.a)$$

$$\hat{X}(t_{0}) = \hat{X}_{0} \quad (31.b)$$

$$\hat{Y}(t) = C\hat{X}(t) \quad (31.c)$$

Where $\hat{X}(t)$ is the state estimate to $X(t)$, K_{e} is the gain of the Kalman filter given by:

$$K_{e} == P_{e}C^{T}R^{-1} \quad (32)$$

In which P_{e} is the stabilizing solution to the Riccati equation:

$$P_{e}A^{T} + AP_{e} - P_{e}C^{T}R_{e}^{-1}CP_{e} + Q_{e} = 0 \quad (33)$$

The LQG optimal controller of the system is shown in Fig.11.

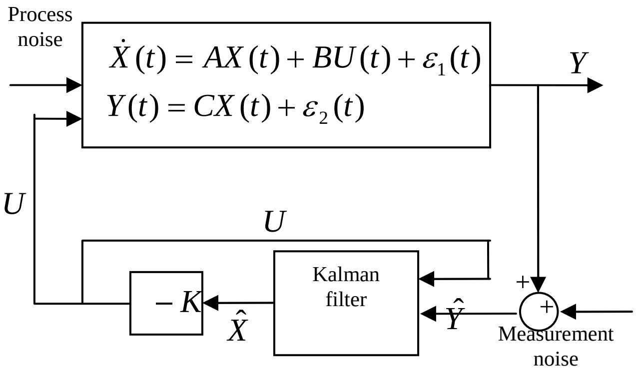

Figure11. LQG optimal controller of the system

V. Numerical Examples

Now, uses the method of the paper introduced, builds a model of piezoelectric cantilever beam by the application of ANSYS software. The geometrical size of the beam is $l_{b} \times w_{b} \times h_{b} = 0.504 \times 0.0254 \times 0.0008(m)$, the parameters of piezoelectric elements are $l_{p} \times w_{p} \times h_{p} = 0.0254 \times 0.0254 \times 0.0002(m)$, $E_{p} = 63GPa$, $d_{31} = 0.12nm/V$, $g_{31} = 16\mu V/N$, $C_{p} = 35pF$, and the other parameters are shown in Tab.I and Tab.II. Based on the calculation results of ANSYS software and the application of D-optimal design principle, the optimal position of the piezoelectric elements is 0.151m away from the cantilevered end. Accordingly, puts the piezoelectric sensors/actuators on this place, a LQG controller is designed for the piezoelectric intelligent cantilever beam using the procedure described in the previous section. Fig.12 shows the first two state variables of x_{1} and x_{2}, and the output variables of y_{1} and y_{2}.

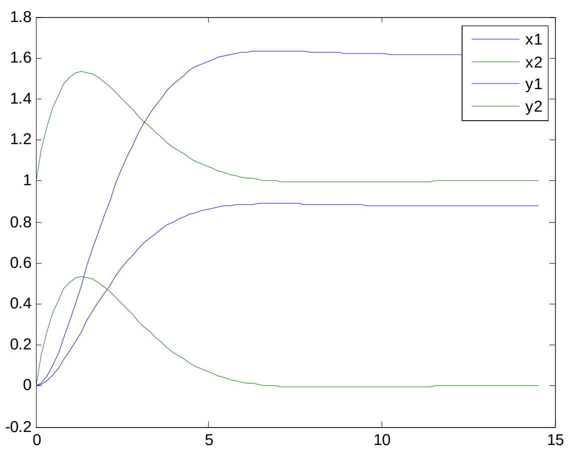

Figure12. State variables and output variables curves

Fig.13 and 14 show both the property of sensor and actuator, from them we can conclude that both the two curves are liner, they demonstrate that the vibration of the host structure could be well measured by the bonded sensor, then, this information is feedback to the controller, which applies a control voltage to the piezoelectric actuator. Accordingly, the vibration of the host structure could be greatly reduced by this way.

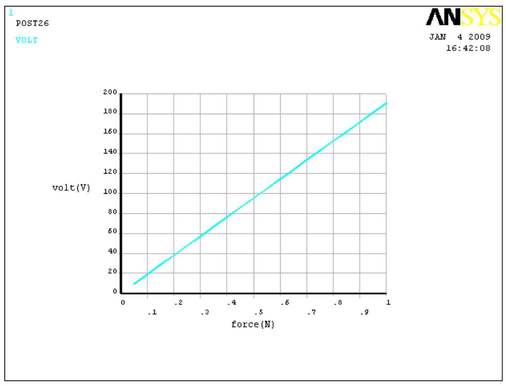

Figure13. Property of sensor

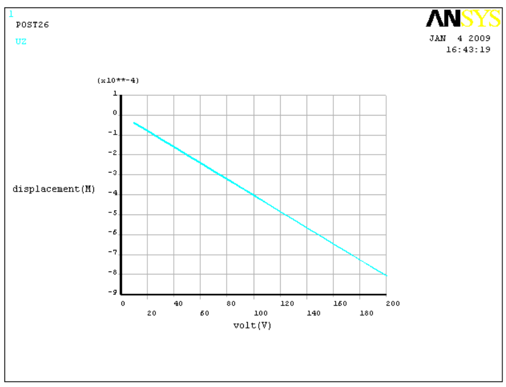

Figure14. Property of Actuator

Finally, Fig.15 shows the step response curve when the piezoelectric elements are located on the optimal position and not. From it we can conclude that not only the vibration amplitude is reduced consumedly, the vibration time is reduced rapidly too.

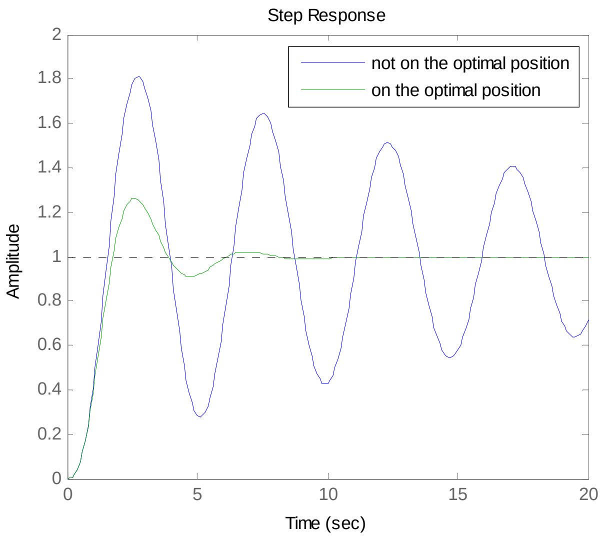

Figure15. Step response curve of the system with optimal position and not

VI. Conclusions

This paper is concerned with the application of piezoelectric materials in active vibration control of flexible structures using LQG control approach, the model of a piezoelectric intelligent cantilever beam is built by the application of ANSYS software, and through the modal analysis, the first 4 ranks of modal frequencies and vibration shapes are extracted. The D-optimal design principle performance function is simplified and gains a method of determine the optimal position of sensors/actuators which only by the use of modes shapes of structures. Considering the actual system concludes some process noise and measurement noise inevitably, a Kalman filter is designed based on the Linear Quadratic Gauss (LQG) optimal control method. Finally, the paper takes a piezoelectric cantilever beam as an example and gives the step response curve of the system at the two circumstance of when piezoelectric elements are bonded onto the optimal position of the host structure and not, the simulation results show the effectiveness of the method in this paper.

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