# Vibration Characteristics and Effectiveness of Floating Slab Track System

Jun Yuan
School of Civil Engineering, Xi'an University of Architecture and Technology, Xi'an, P.R. China
Email: yuanjun710055@hotmail.com

Yongchao Zhu
Savills Valuation and Professional Services Limited, Beijing, P.R. China
Email: yczhu@savills-bj.com

Minzhe Wu
School of Civil Engineering, Xi'an University of Architecture and Technology, Xi'an, P.R. China
Email: xauat710055@ hotmail.com

***Abstract*—Ground-borne vibration excited by metro traffic is becoming a more and more important problem for the rapidly developing transport system in urban areas and increasing public concern about environmental problems. In order to attenuate the vibration influence down to an acceptable level, floating slab track have been applied worldwide on metro line. An analytical dynamic model is employed in this paper to assess vibration characteristics and effectiveness of floating slab system. The dispersion equations of floating slab track system are solved by means of Fourier transformation method. The contour integration method is used to convert the vibration responses of the track system. The result demonstrates that the dispersive characteristics of the track system act as a continuous layered wave conductor with an infinite length slab. It will generate same-phase oscillation and infinite wavelengths in track system when excitation frequency is close to the cut-off frequencies. An increase of slab mass is most effective to expand the vibration isolation range, whereas it has no obvious effect to enhance the vibration isolation efficiency. A relatively high slab searing damping and low slab bearing stiffness generally decrease force transmission to improve the vibration isolation effectiveness and decrease resonant response of the track system.**

***Index Terms*—floating slab track, metro trains, vibration response, wave conductor, vibration isolation effectiveness**

## I. INTRODUCTION

The vibration excited by metro traffic is a growing environmental concern in urban areas. It can cause annoyance and discomfort to inhabitants, malfunction to sensitive equipment, and damage to historic structures. In order to attenuate the vibration influence down to an acceptable level, many countermeasures have been developed [1-6]. Floating slab track system is one of the most effective measures for mitigating ground-borne vibration induced by metro trains. There are two main models used to account for the tracks: Euler-Bernoulli beam and Timoshenko beam [7-12]. Euler Bernoulli's theory is sufficiently accurate for wave lengths approximately $\lambda \geq 10\gamma$ or frequencies $f \geq c/10\gamma$ with the velocity of travelling waves in case of beams with circular cross-section of radius $\gamma$ [13, 14]. The Timoshenko model is especially for non-slender beam and for high-frequency responses which shear or rotary effects are not negligible. Thus, layered Euler-Bernoulli beam is used to estimate the vibration isolation performance of the floating slab track in this paper.

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Corresponding author. E-mail: yuanjun710055@hotmail.com.

## II. DOUBLE-BEAM MODEL OF FLOATING SLAB TRACK SYSTEM

A double-beam model for the floating slab track on rigid foundation is shown in Fig. 1. Both of the rails and the track are assumed to act as infinitely long Euler-Bernoulli beam. Railpads and the slab bearings are represented by continuous layers with uniform stiffness ($k_1$ and $k_2$) and damping constant ($c_1$ and $c_2$).

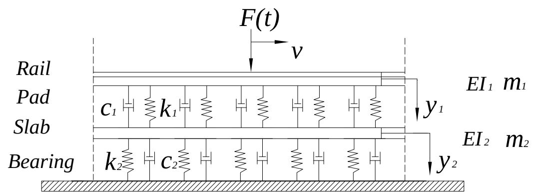


Figure 1. Double-beam model for floating slab track.

The motion equations of rail and slab can be written as [15-17]

$$EI_1\frac{\partial^4 y_1}{\partial x^4}+m_1\frac{\partial^2 y_1}{\partial t^2}+k_1(y_1-y_2)+c_1(\frac{\partial y_1}{\partial t}-\frac{\partial y_2}{\partial t})=F(x,t) \quad (1)$$

$$\begin{aligned}&EI_2\frac{\partial^4 y_2}{\partial x^4}+m_2\frac{\partial^2 y_2}{\partial t^2}+k_2 y_2\\&\quad -k_1(y_1-y_2)+c_2\frac{\partial y_2}{\partial t}-c_1(\frac{\partial y_1}{\partial t}-\frac{\partial y_2}{\partial t})=0\end{aligned} \quad . \quad (2)$$


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Where $F(x,t)=e^{i\varpi t}\delta(x-vt)$ is the unit moving harmonic load subjected on the rails and $\delta$ is the Dirac delta function. Transforming equations (1) and (2) by the double Fourier transformation form the space-time domain $(x,t)$ to the wavenumber-frequency domain $(\xi,\omega)$, the equations can be written by matrix form as

$$\begin{bmatrix} EI_1\xi^4-m_1\omega^2 \\ +k_1+c_1i\omega & -k_1-c_1i\omega \\ -k_1-c_1i\omega & EI_2\xi^4-m_2\omega^2+k_1 \\ & +k_2+i\omega(c_1+c_2) \end{bmatrix}\times\begin{bmatrix}\tilde{y}_1\\ \tilde{y}_2\end{bmatrix}=\begin{bmatrix}2\pi\delta(\omega+\xi v-\varpi)\\ 0\end{bmatrix}. \quad (3)$$

The rails and slab displacement solutions of equation (3) are

$$\tilde{y}_1(\xi,\omega)=\frac{2\pi\delta(\omega+\xi v-\varpi)B}{A}, \quad (4)$$

$$\tilde{y}_2(\xi,\omega)=\frac{2\pi\delta(\omega+\xi v-\varpi)C}{A}. \quad (5)$$

in which,

$$A=\begin{vmatrix} EI_1\xi^4-m_1\omega^2+k_1+c_1i\omega & -k_1-c_1i\omega \\ -k_1-c_1i\omega & EI_2\xi^4-m_2\omega^2+k_1+k_2+i\omega(c_1+c_2) \end{vmatrix}, \quad (6)$$

$$B=EI_2\xi^4-m_2\omega^2+k_1+k_2+i\omega(c_1+c_2), \quad (7)$$

$$C=k_1+c_1i\omega. \quad (8)$$

Transforming (5) from the wavenumber-frequency $(\xi,\omega)$ to space-time domain $(x,t)$, the results can be expressed as

$$y_2(x,t)=\frac{e^{i\varpi t}}{2\pi}\int_{-\infty}^{\infty}\frac{k_1+c_1i(\varpi-\xi v)}{H}d\xi, \quad (9)$$

where

$$H=\begin{vmatrix} EI_1\xi^4+k_1-m_1(\varpi-\xi v)^2+c_1i(\varpi-\xi v) & -k_1-c_1i(\varpi-\xi v) \\ -k_1-c_1i(\varpi-\xi v) & EI_2\xi^4-m_2(\varpi-\xi v)^2+i(\varpi-\xi v)(c_1+c_2)+k_1+k_2 \end{vmatrix}. \quad (10)$$

The force transmitted to the ground for unit moving harmonic load is given by

$$F_T(x,t)=k_2y_2(x,t)+c_2\frac{\partial y_2(x,t)}{\partial t}. \quad (11)$$

## III. RESULTS AND DISCUSSIONS

The numerical results for dispersion curves and transmitted force with various parameters are shown in figures as follows and discussed detailedly. The basic parameters of the model are given in Table 1 which is from the values of Singapore mass rapid transit (SMRT) system [18].

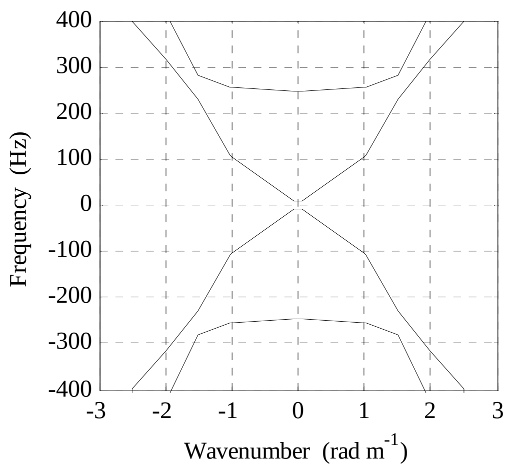
TABLE I.
BASIC PARAMETERS USED FOR THE FLOATING SLAB TRACK

| | Mass(kg/m) | Stiffness(N/m) | Damping constant (Ns/m) |
|---|---|---|---|
| Rails | $m_1$=1.2068E2 | $k_1$=2.8571E8 | $c_1$=8.571E4 |
| Slab | $m_2$=3.280E3 | $k_2$=1.044E7 | $c_2$=5.0E4 |

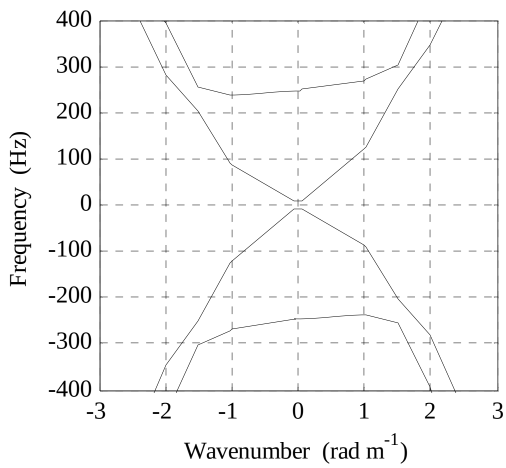
The non-trivial solution of the equation (3) can be written as

$$\begin{vmatrix} EI_1\xi^4-m_1\omega^2+k_1+c_1i\omega & -k_1-c_1i\omega \\ -k_1-c_1i\omega & EI_2\xi^4-m_2\omega^2+k_1+k_2+i\omega(c_1+c_2) \end{vmatrix}=0. \quad (12)$$

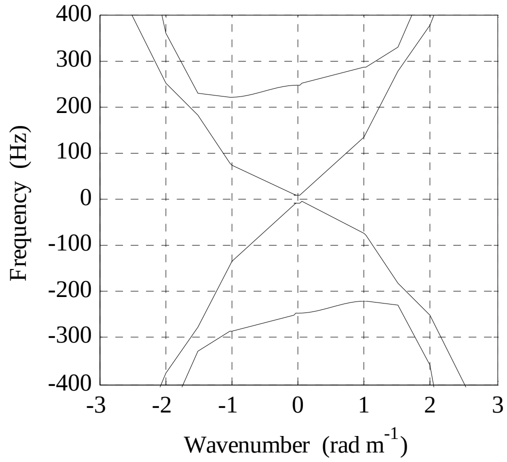
Substituting track parameters given in Table I, the dispersion curves are obtained as follows.



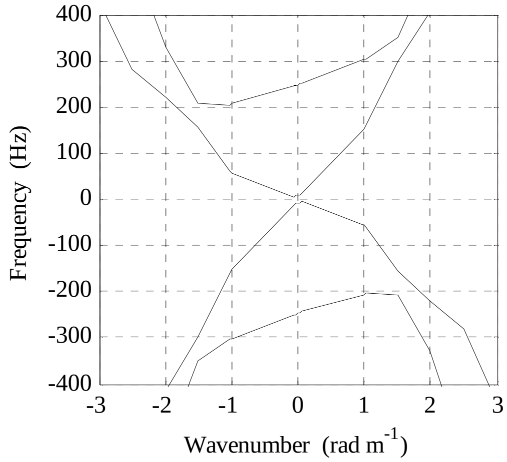
(a) v=0 m/s



(b) v=100 m/s

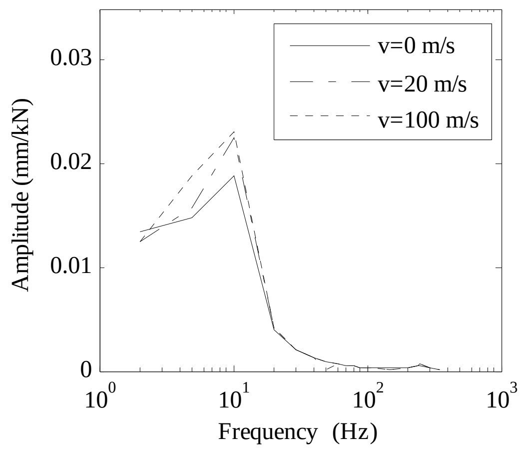



(c) v=200 m/s

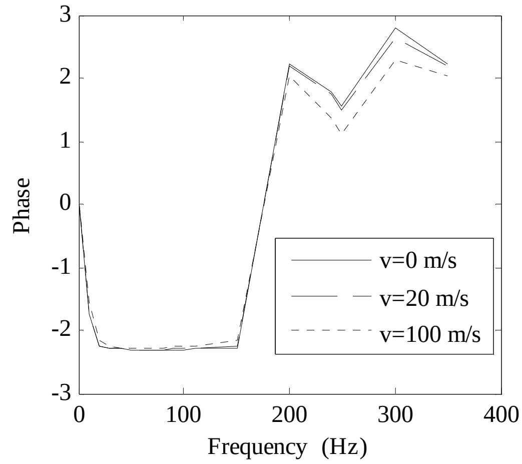



(d) v=300 m/s

Figure 2. Dispersion curves of floating slab track.




(a) Displacement




(b) Phase

Figure 3. Dynamic response curves of the slab.

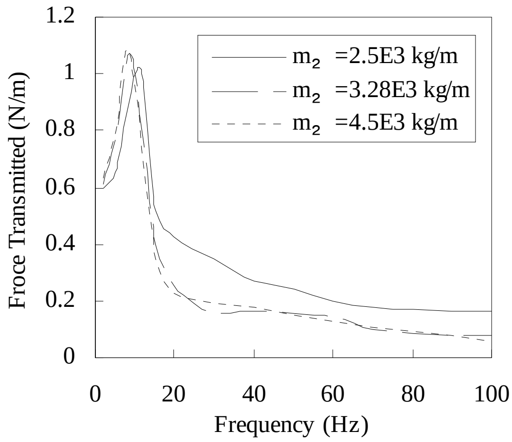
Fig. 2 shows the dispersion curves of the continuous model under the moving load with different velocity. Fig. 2(a) shows that the first cut-off frequency occurs at 8.8182 Hz, approximately equal to natural frequency of floating slab. The second cut-off frequency occurs at 249.3613 Hz, approximately equal to natural frequency of rails. Vibration energy excited by the metro traffic propagates and dissipates along the axial direction of the track system, because it has an infinite length slab act as a continuous wave conductor. It generates same-phase oscillation and infinite wavelengths that tracks vibrate at the cut-off frequencies. Considering the velocity effect of moving load on frequency dispersion, Fig. 2(b), (c) and (d) show the dispersion curves for v=100m/s, 200m/s and 300m/s, respectively. The peak of dispersion curve heads toward the wavenumber axis with the load velocity increase. And the track system will tend to instability when the load velocity reaches to critical load velocity (calculated by the tangent angle of dispersion curves), endangering the metro traffic operation and discomforting passengers.

Fig. 3 shows the curve of dynamic response of the slab against frequency at v=0 m/s (solid line), 20 m/s (long dashed line) and 100 m/s (dashed line). The first peaks of the three curves occur around 9 Hz, which is coincided with the result of dispersion analysis. Frequencies approach the resonant frequency, the transmitted force curve dramatic increased. It drops sharply after the resonant frequency. At frequencies above 35Hz, the slope of the transmitted force curve is less steep. Compared to the cases of v=20 m/s and v=100 m/s, the curve of non-moving oscillating load (with v=0 m/s) is higher.

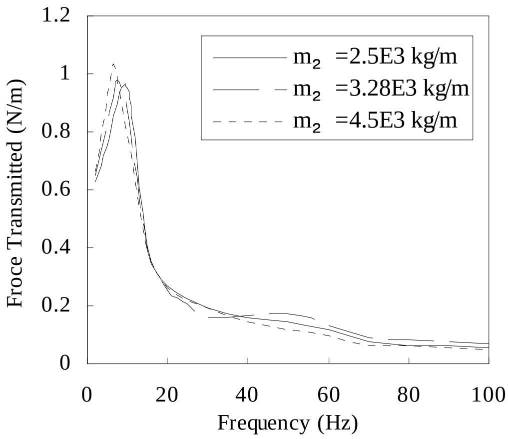
### A. *Effects of the Slab Mass*

In order to assess effects of the slab mass on vibration isolation range and effectiveness, three parameters of mass (2.5E3 kg/m, 3.28E3 kg/m and 4.5E3 kg/m) are calculated in the model corresponded to right-weight slab, medium-weight slab and heavy-weight slab.

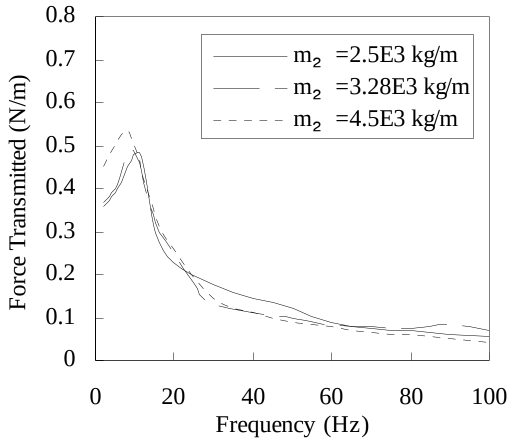
Fig. 4 show the effect curves of slab mass under unit moving harmonic load. Compared with the curves of different mass, the peaks of force curve are shift to right with the mass increasing and to lift with mass decreasing, respectively. This can be seen that large slab mass can get a low resonant frequency for a wide vibration isolation range. Though the force curve of slab with large mass is fall rapidly than others, the effect is not obvious. There




(a) v=0 m/s

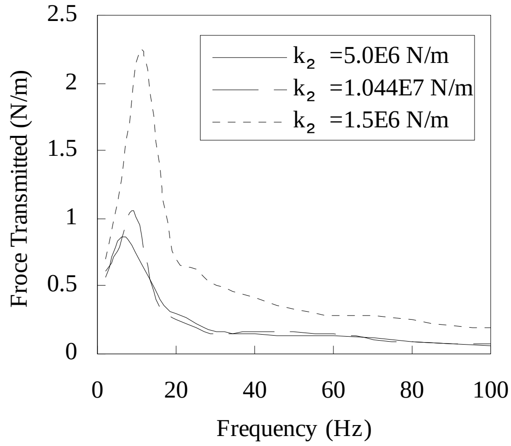



(b) v=20m/s

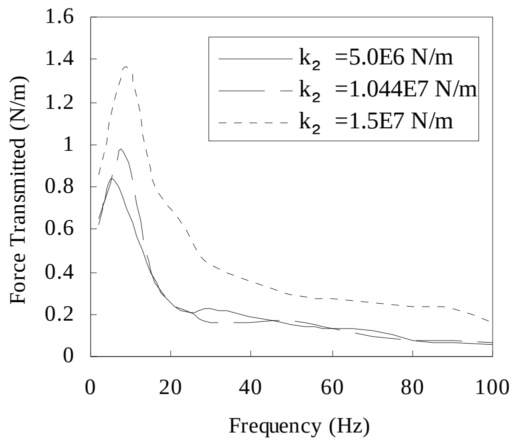



(c) v=100 m/s

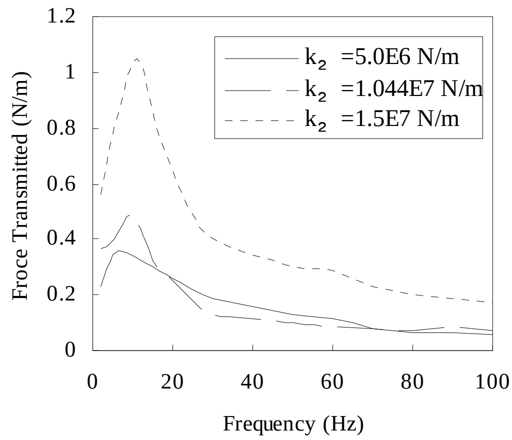
Figure 4. Effects of slab mass on forces transmission.




(a) v=0 m/s




(b) v=20m/s




(c) v=100 m/s

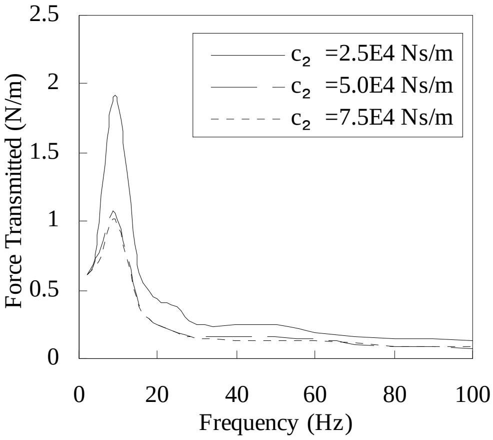
Figure 5. Effects of slab bearing stiffness on forces transmission.

are three ways to increase the slab mass per unit length: add the thickness, width and density of the slab. All of the measurements will cost much and obtain little effectiveness to reduce transmitted force.

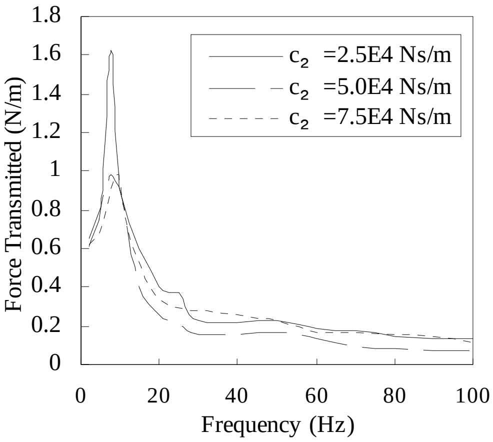
### *B. Effects of the Slab Bearing Stiffness*

Fig. 5 shows under the unit oscillating load, the effect of the stiffness with low and high values (5.0E6 N/m, 1.044E7 N/m and 1.5E7 N/m) on the force transmitted. The values of force curve with lower stiffness are below the higher stiffness at the identical excitation frequency. The peaks of the curves are shift to right with the increase of the stiffness, that is, the resonant frequency is rise. Compared to the influence of the variant slab mass, the changes of transmitted forces are pronounced with the variety of the slab bearing stiffness. Decreasing the stiffness of the elastic bearing must maintain a minimum level of rigidity to ensure rail stability under load.

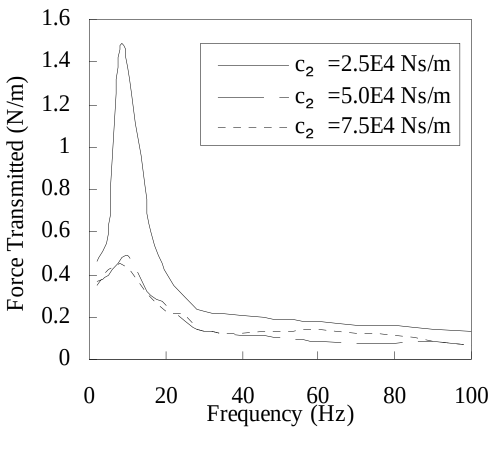
### *C. Effects of the Slab Bearing Damping*




(a) v=0 m/s




(b) v=20m/s




(c) v=100 m/s

Figure 6. Effects of slab bearing damping on forces transmission.

Fig. 6 show the effect of an increasing damping factor (2.5E4 Ns/m, 5.0E4 Ns/m and 7.5E4 Ns/m) on the force transmission under unit moving harmonic load. The force transmitted through the floating slab track with low damping constant is larger than of high damping factor. The results are show that damping is significant effective in reducing the force transmission for improve vibration isolation effectiveness. Excessive deformation and displacement on the floating slab track is endangering the metro traffic operation and discomforting passengers. The floating slab track should be applied without scarifying stability criteria. Therefore, the parameters need to be checked according to traffic operation safety limits: vertical acceleration, deflection and longitudinal displacement [19, 20]. Since the maximum design speed of metro train is restricted to 33.33 m/s in China, the vertical acceleration is proved satisfactory in general. It is compulsory to check the longitudinal displacement and deflection. When dynamic load has been filtered greatly by floating slab track, only static load and residual dynamic load can propagate through damping element into the foundation. Then static displacement of floating slab track is limited to 3 mm. The parameters in this paper are examined by the three traffic operation safety limits, and the results are within the acceptable range.

## IV. CONCLUSIONS

A dynamic model based on double Euler-Bernoulli beam theory for floating slab track system has been applied to study enhancing the vibration isolation effectiveness. It is shown by result that dispersive characteristics of the track system act as a continuous layered wave conductor with an infinite length slab. It will tend to instability when the load velocity reaches to critical load velocity, endangering the metro traffic operation and discomforting passengers. An increase of slab mass is most effective to expand the vibration isolation range, whereas it has no obvious effect to enhance the vibration isolation efficiency. A relatively high slab searing damping and low slab bearing stiffness generally decrease the force transmission to improve the vibration isolation effectiveness and decrease the resonant response of the track system. The bearing stiffness should be in the appropriate range under the security and stability conditions of traffic operation. The damping should be in the appropriate range for a desirable vibration isolation effectiveness and reasonable cost.

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**Jun Yuan** was born in Meishan, Sichuan Province, P.R. China on February 22, 1983. Currently he is a Ph.D. candidate student in the College of Civil Engineering, Xi'an University of Architecture and Technology, Xi'an, P.R. China. He received his M.E. and B.E. degree from Xi'an University of Architecture and Technology in 2008 and 2004, respectively. His research interests include: structural engineering, modeling and simulation, finite elements, and urban rapid rail transit engineering.

He has published several papers on structural engineering. He recently participated in theoretical analysis and numerical computation of the research project supported by Xi'an Science and Technology Bureau of China and the First Railway Survey and Design Institute of China Railway.

Mr. Yuan is a member of China Civil Engineering Society (CCES).

**Yongchao Zhu** was born in Xi'an, Shaanxi Province, P.R. China on June 5, 1982. He has obtained Msc in Project & Programme Management from the University of Warwick, UK and Bachelor Degree in Civil Engineering from Xi'an University of Architecture and Technology, Xi'an, P.R. China.

He is currently employed by Savills Valuation and Professional Services Limited as senior valuer and his major responsibilities are act as consultant to provide professional opinions in varies sectors related with structural engineering, construction project, urban rapid rail transit engineering.

**Minzhe Wu** is a professor in Civil Engineering at the Xi'an University of Architecture and Technology. He received the B.E. degree from Tongji University, Shanghai, P.R. China, and the M.E. degrees in Xi'an University of Architecture and Technology, Xi'an, P.R. China. His research interests include: structural engineering, ancient architecture protection, and urban rapid rail transit engineering.

He has authored a book published by the Shaanxi Science and Technology Press entitled *Finite Element Method for Engineering Applications*, jointly edited an industry code for ministry of metallurgical industry of China. He also has authored a number of papers. For over two decades, his work has been supported by Shanghai Municipal Construction Commission, Education Bureau of Shanxi Province, and Xi'an Science and Technology Bureau of China. He has obtained National College Award for Computer Assisted Instruction, Science and Technology Progress Award from Shanghai Municipal Government.

Prof. Wu has been active in many professional organizations. He was the director of Shaanxi Society of Mechanics, vice-director of Committee of Seismic Control for Structures of China Metallurgical Construction Association, member of the Advisory Committee on the Mechanics Teaching of Ministry of Education of China.