H_∞ Tracking Performance Design for Fuzzy-Model-Based Descriptor Systems Subject to Parameter Uncertainties

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***Abstract*— This paper presents the stability and H_∞ tracking performance design of a fuzzy-model-based descriptor system to attenuate the effects caused by unmodeled dynamics, disturbances, and approximation errors. First, the nonlinear descriptor system is represented by an equivalent Takagi-Sugeno type fuzzy model. Then, a fuzzy observer-based state feedback control scheme is developed to restrain the external disturbances such that the H_∞ tracking performance is achieved. Furthermore, based on Lyapunov stability theorem, the proposed fuzzy control system can guarantee the stability of the whole closed-loop system and obtain good tracking performance as well. Stability conditions are expressed in terms of linear matrix inequalities (LMIs). Finally, a numerical example of a nonlinear descriptor system control problem is given to show the validity and confirm the performance of the proposed scheme.**

***Index Terms*— Descriptor systems, Takagi-Sugeno fuzzy model, state feedback control, H_∞ tracking performance, linear matrix inequalities (LMIs).**

I. INTRODUCTION

Most moving robots, electrical circuits and many other systems which have to be modeled by additional algebraic constraints can be modeled as descriptor systems with nonlinearities and uncertainties. In addition, descriptor systems can describe a larger class of systems than conventional linear state-space models, and is also much tighter than a state-space expression for representing independent parametric perturbations [1]. Descriptor systems are also referred to as singular systems, implicit systems, generalized state-space systems, differential-algebraic systems, or semistate systems. Control of descriptor systems has been extensively studied in the past years due to the fact that descriptor systems better describe physical systems than regular ones in many applications [2]. An important characteristic of descriptor systems is the possible impulse behavior, which is harmful to the physical system and is undesired in system control. Thus, they post additional difficulties to the stabilization control design and, especially, the tracking control design. In order to overcome these kinds of difficulties in the design of a controller, various schemes have been developed in the last decades, among which a successful approach is observer-based fuzzy logic control.

With the developments of fuzzy observer technique, some fuzzy-model-based fuzzy control system design methods have appeared in the fuzzy control field. A typical model is Takagi-Sugeno fuzzy model which represent local linear input-output relations of a nonlinear system and is proved that the fuzzy model is a universal approximator. Recently, a wider class of fuzzy systems described by the descriptor form is considered in [4], where the model is in the extended T-S fuzzy model. In [4], a fuzzy model in the descriptor form is introduced, and stability and stabilization problems for the system are addressed. It is shown that the method therein could lead to simpler conditions than other methods for some systems. However, the results developed in [4] are not valid if the original system considered therein is in the pure descriptor form (i.e., the derivative matrix is not of full rank). This is because the conditions presented in [4] imply that all $\mathbf{E}_k$'s are invertible. The tasks of stabilization and tracking are two typical control problems. It is known that a descriptor model describes a practical system better than a standard dynamic model [5]– [8]. In [9], the authors proposed the parallel distributed compensation (PDC) for fuzzy control design of T-S fuzzy system. Under some conditions, PDC can stabilize the closed-loop fuzzy system asymptotically. Linear matrix inequalities (LMI) methods to find the common positive matrix always plays the key role work in PDC design to achieve tracking control performance [10]. The H_∞ control problem, while being formulated as an LMI optimization problem, can be efficiently solved by MATLAB's LMI Control Toolbox. In general, tracking problems are more difficult than stabilization problems. In [11], a state feedback decentralized fuzzy controller with constant control parameters is proposed to tackle the H_∞ model reference tracking control design problem for nonlinear interconnected systems. The problem of state feedback decentralized H_∞ fuzzy tracking control design is characterized in terms of solving an eigenvalue problem (EVP).

In this paper, a fuzzy logic controller with H_∞ tracking performance for uncertain nonlinear descriptor systems is proposed to attenuate the effects caused by unmodeled dynamics, disturbances, and approximation errors. First, the nonlinear descriptor system is represented by an equivalent Takagi-Sugeno type fuzzy model. Then, a state feedback fuzzy control scheme is developed to restrain the external disturbances such that the model reference H_∞ tracking performance is achieved. Furthermore, based on Lyapunov stability theorem, the proposed fuzzy control system can guarantee the stability of the whole closed-loop system and obtain good tracking performance as well. Stability conditions are expressed in terms of linear matrix inequalities (LMIs). Finally, a numerical example of a nonlinear descriptor system control problem is given to show the validity and confirm the performance of the proposed scheme.

The rest of the paper is organized as follows. In Section II, the problem formulation and the fuzzy description is presented. The fuzzy observer and controller for the uncertain nonlinear descriptor systems is proposed in section III and the analysis of the system stability is given in Section IV. Simulation results are shown in section V and conclusion is given in section VI.

Notations: Throughout the paper, $\mathbb{R}^n$ denotes the n-dimensional real Euclidean space, $\mathbf{I}$ is the identity matrix with proper dimension, the superscripts $\top$ and -1 stand for the matrix transpose and inverse, respectively, and $W > 0$ ($W < 0$) means that W is symmetric and positive (negative) definite.

II. Problem formulation

Consider the Takagi and Sugeno fuzzy model for a descriptor system described by the following fuzzy IF-THEN rules;

Plant Rule i:

If ξ_1 is M_{i1} and ξ_1 is M_{i1} and $\cdots$ and ξ_g is M_{ig} then

$$\begin{cases}\mathbf{E}\dot{\mathbf{x}}(t) = (\mathbf{A}_i + \Delta\mathbf{A}_i)\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t) + \mathbf{w}(t)\\ \mathbf{y}(t) = \mathbf{C}_i\mathbf{x}(t) + \mathbf{v}(t)\end{cases} \tag{1}$$

for $i = 1, 2, \ldots, r$, where r is the number of If-Then rules, ξ_1, ξ_2, $\cdots$, ξ_g are the premise variables, M_{ij}, $j = 1, 2, \ldots, g$, are linguistic values of linguistic variables $\boldsymbol{\xi}$ in the universes of discourse $U \subset \mathbb{R}^n$ for which $\boldsymbol{\xi} = [\xi_1\ \xi_2 \cdots \xi_g]^\top$, $\mathbf{x}(t) \in \mathbb{R}^{n\times 1}$ is the descriptor variable, $\mathbf{u}(t) \in \mathbb{R}^{m\times 1}$ is the control input, $\mathbf{w}(t) \in \mathbb{R}^{n\times 1}$ is the bounded external disturbance, $\mathbf{y}(t)$ is the output vector, $\mathbf{v}(t)$ is the measurement noise, and $\mathbf{A}_i \in \mathbb{R}^{n\times n}$, $\mathbf{B}_i \in \mathbb{R}^{n\times m}$, $\mathbf{E} \in \mathbb{R}^{n\times n}$ with $rank(\mathbf{E}) = n_1 \le n$ are system matrices, and $\Delta\mathbf{A}_i \in \mathbb{R}^{n\times n}$ is the uncertainty and is bounded, i.e., $\|\Delta\mathbf{A}_i\| < \delta_i$ for some positive constant δ_i. It is assumed that $\mathbf{E} = diag\{\mathbf{I}_{n_1}, \mathbf{0}\}$.

It is well known that a fuzzy descriptor system contains not only finite dynamical modes but also infinite modes including infinite nondynamical and dynamical modes. The infinite dynamical modes can generate undesired impulse behavior especially when uncertainties and disturbances exist. If $\det(s\mathbf{E} - \mathbf{A}_i) \neq 0$ for some complex number s, then the pair $(\mathbf{E}, \mathbf{A}_i)$ is called regular. To guarantee the existence and uniqueness of solution for a design system, it is always assumed that the pair $(\mathbf{E}, \mathbf{A}_i)$ is regular. If the number of finite poles is less than $rank(\mathbf{E})$ (i.e., $\bar{n} = \deg\det(s\mathbf{E} - \mathbf{A}_i) < rank(\mathbf{E})$), then the system has impulsive modes; if it has exactly $rank(\mathbf{E})$ finite poles (i.e., $\bar{n} = rank(\mathbf{E})$), then the pair $(\mathbf{E}, \mathbf{A}_i)$ is called impulse-free; if all finite eigenvalues of $(\mathbf{E}, \mathbf{A}_i)$ are stable, we say that $(\mathbf{E}, \mathbf{A}_i)$ is stable; if it is regular and has neither impulsive modes nor unstable finite modes, then the pair $(\mathbf{E}, \mathbf{A}_i)$ is admissible [3]. The impulse immunity avoids impulsive behavior at initial time for inconsistent initial conditions. It is clear that, for nontrivial case $E \neq 0$, impulse immunity implies regularity.

Lemma 1 [3] : The fuzzy descriptor system described by $\mathbf{E}\dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t)$ is regular, impulse-free and stable if and only if there exist a matrix $\mathbf{P}$ such that

$$\begin{aligned}\mathbf{P}^\top\mathbf{E} = \mathbf{E}^\top\mathbf{P} \ge 0\\ \mathbf{P}^\top\mathbf{A}_i + \mathbf{A}_i^\top\mathbf{P} < 0\end{aligned} \tag{2}$$

■

In fact, if (2) is true, then the Lyapunov function is chosen as $\mathbf{V}(t) = \mathbf{x}^\top(t)\mathbf{E}^\top\mathbf{P}\mathbf{x}(t)$. Conversely, if $\mathbf{E}\dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t)$ is regular, impulse-free and stable, then a matrix $\mathbf{P}$ can be chosen such that (2) holds.

By using a center-average defuzzifier, product inference and singleton fuzzifier, the fuzzy system is inferred as follows:

$$\begin{cases}\mathbf{E}\dot{\mathbf{x}}(t) = (\mathbf{A} + \Delta\mathbf{A})\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{w}(t)\\ \mathbf{y}(t) = \mathbf{C}_i\mathbf{x}(t) + \mathbf{v}(t)\end{cases} \tag{3}$$

where

$$\begin{cases}\mathbf{A} = \sum_{i=1}^r \lambda_i(\boldsymbol{\xi})\mathbf{A}_i,\ \Delta\mathbf{A} = \sum_{i=1}^r \lambda_i(\boldsymbol{\xi})\Delta\mathbf{A}_i,\\ \mathbf{B} = \sum_{i=1}^r \lambda_i(\boldsymbol{\xi})\mathbf{B}_i,\ \mathbf{C} = \sum_{i=1}^r \lambda_i(\boldsymbol{\xi})\mathbf{C}_i\end{cases} \tag{4}$$

for which

$$\lambda_i(\boldsymbol{\xi}) = \frac{\beta_i(\boldsymbol{\xi})}{\sum_{i=1}^r \beta_i(\boldsymbol{\xi})},\ \beta_i(\boldsymbol{\xi}) = \prod_{j=1}^{g} M_{ij}(\xi_j) \tag{5}$$

for which $\beta_i(\boldsymbol{\xi})$ is the grade of membership of the system with respect to rule i. We assume that $\beta_i(\boldsymbol{\xi}(t)) \ge 0$, $i = 1, 2, \cdots, r$, and $\sum_{i=1}^r \beta_i(\boldsymbol{\xi}(t)) > 0$, $\forall t$. Hence $\lambda_i(\boldsymbol{\xi}(t))$ satisfies $\lambda_i(\boldsymbol{\xi}(t)) \ge 0$, $i = 1, 2, \cdots, r$, and $\sum_{i=1}^r \lambda_i(\boldsymbol{\xi}) = 1$, $\forall t$.

Consider a reference model as follows:

$$\mathbf{E}\dot{\mathbf{x}}_m(t) = \mathbf{A}_m\mathbf{x}_m(t) + \mathbf{r}(t) \tag{6}$$

where $\mathbf{x}_m(t)$ is the reference state, $\mathbf{A}_m$ the specific asymptotically stable matrix, and $\mathbf{r}(t)$ the bounded reference input. Let $\tilde{\mathbf{w}}(t) = [\mathbf{v}^\top(t)\ \mathbf{w}^\top(t)\ \mathbf{r}^\top(t)]^\top$. It is known that the effect of $\tilde{\mathbf{w}}(t)$ will deteriorate the control performance of the fuzzy control system and even lead to instability of the descriptor system. Therefore, how to eliminate the effect of $\tilde{\mathbf{w}}(t)$ to guarantee tracking control performance is also an important design purpose of fuzzy control systems. Since H_∞ control is the most important control design to efficiently eliminate the effect of $\tilde{\mathbf{w}}(t)$ on the control system, it will be employed to deal with the robust tracking performance control in (1). Let us consider the H_∞ tracking performance related to tracking error $\mathbf{x}(t) - \mathbf{x}_m(t)$ as follows

$$\frac{\int_0^{t_f} \big(\mathbf{x}(t) - \mathbf{x}_m(t)\big)^\top \mathbf{Q}\big(\mathbf{x}(t) - \mathbf{x}_m(t)\big)dt}{\int_0^{t_f} \tilde{\mathbf{w}}(t)\tilde{\mathbf{w}}^\top(t)dt} \le \rho^2 \tag{7}$$

where t_f is the terminal time of control, $\mathbf{Q}$ is the positive definite weighting matrix, and ρ^2 is the prescribed attenuation level.

III. Fuzzy observer and controller design

Assuming that all the pair $(\mathbf{A}_i, \mathbf{C}_i)$ in fuzzy set (1) are selected to be observable, suppose the following fuzzy Luenberger-like full-order linear observer is proposed to deal with the state estimation of the control system:

Observer Rule i:

If $\xi_1(t)$ is M_{i1} and $\cdots$ and $\xi_g(t)$ is M_{ig} then

$$\begin{cases}\mathbf{E}\dot{\hat{\mathbf{x}}}(t) = \mathbf{A}_i\hat{\mathbf{x}}(t) + \mathbf{B}_i\mathbf{u}(t) + \mathbf{L}_i(\mathbf{y}(t) - \hat{\mathbf{y}}(t))\\ \hat{\mathbf{y}}(t) = \mathbf{C}_i\hat{\mathbf{x}}(t)\end{cases} \tag{8}$$

for $i = 1, 2, \ldots, r$, where $\mathbf{L}_i$ is the observer gain of the i-th observer rule to be chosen later. Hence, the overall fuzzy observer can be represented as follows:

$$\begin{cases} \mathbf{E}\dot{\hat{\mathbf{x}}}(t) = \mathbf{A}\hat{\mathbf{x}}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{L}(\mathbf{y}(t) - \hat{\mathbf{y}}(t)) \\ \hat{\mathbf{y}}(t) = \mathbf{C}(t)\hat{\mathbf{x}}(t) \end{cases} \tag{9}$$

where $\mathbf{L} = \sum_{i=1}^{r} \lambda_i(\boldsymbol{\xi})\mathbf{L}_i$ and $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ are defined in (4). Define the estimation error state as

$$\mathbf{e}(t) = \mathbf{x}(t) - \hat{\mathbf{x}}(t) \tag{10}$$

By differentiating (10), we get

$$\begin{aligned} \mathbf{E}\dot{\mathbf{e}}(t) &= \mathbf{E}\dot{\mathbf{x}}(t) - \mathbf{E}\dot{\hat{\mathbf{x}}}(t) \\ &= \sum_{i=1}^{r} \lambda_i(\boldsymbol{\xi})\Big((\mathbf{A}_i + \Delta\mathbf{A}_i)\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t) + \mathbf{w}(t)\Big) \\ &\quad - \sum_{i=1}^{r} \lambda_i(\boldsymbol{\xi})\Big(\mathbf{A}_i\hat{\mathbf{x}}(t) + \mathbf{B}_i\mathbf{u}(t) + L_i(\mathbf{y}(t) - \hat{\mathbf{y}}(t))\Big) \\ &= \sum_{i=1}^{r} \lambda_i(\boldsymbol{\xi})\Big((\mathbf{A}_i + \Delta\mathbf{A}_i)\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t) + \mathbf{w}(t) - \mathbf{A}_i\hat{\mathbf{x}}(t) \\ &\quad + \mathbf{B}_i\mathbf{u}(t) + \sum_{j=1}^{r} \lambda_j(\boldsymbol{\xi})\mathbf{L}_i\mathbf{C}_j(\mathbf{x}(t) - \hat{\mathbf{x}}(t)) + \mathbf{L}_i\mathbf{v}(t)\Big) \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\boldsymbol{\xi})\lambda_j(\boldsymbol{\xi})\Big((\mathbf{A}_i - \mathbf{L}_i\mathbf{C}_j)\mathbf{e}(t) + \Delta\mathbf{A}_i\mathbf{x}(t) \\ &\quad - \mathbf{L}_i\mathbf{v}(t)\Big) + \mathbf{w}(t) \end{aligned} \tag{11}$$

The parallel distributed compensation (PDC) offers a procedure to design a fuzzy controller from the T-S fuzzy model. The designed fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts and has local linear controllers in the consequent parts. The PDC fuzzy controller with r fuzzy rules is constructed from the fuzzy system (5) to deal above estimation errors (11) as follows:

Control Rule j:

If $\xi_1(t)$ is M_{i1} and $\cdots$ and $\xi_g(t)$ is M_{ig},

then $\mathbf{u}(t) = \mathbf{K}_j\Big(\hat{\mathbf{x}}(t) - \mathbf{x}_m(t)\Big)$

for $j = 1, 2, \ldots, r$, where $\mathbf{K}_j$ is the state feedback gain for jth control rule. Hence, the overall fuzzy controller is given by

$$\mathbf{u}(t) = \mathbf{K}(\hat{\mathbf{x}}(t) - \mathbf{x}_m(t)) \tag{12}$$

where $\mathbf{K} = \sum_{j=1}^{r} \lambda_j(\boldsymbol{\xi})\mathbf{K}_j$. Let $\tilde{\mathbf{x}}(t) = [\mathbf{e}^\top(t)\ \mathbf{x}^\top(t)\ \mathbf{x}_m^\top(t)]^\top$. After some algebraic manipulations, the augmented system can be expressed as:

$$\begin{aligned} \mathbf{E}\dot{\tilde{\mathbf{x}}}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\boldsymbol{\xi})\lambda_j(\boldsymbol{\xi})\Big((\tilde{\mathbf{A}}_{ij} + \Delta\tilde{\mathbf{A}}_i)\tilde{\mathbf{x}}(t) \\ &\quad + \tilde{\boldsymbol{\Xi}}_i\tilde{\mathbf{w}}(t)\Big) \end{aligned} \tag{13}$$

where

$$\tilde{\mathbf{A}}_{ij} = \begin{bmatrix} \mathbf{A}_i - \mathbf{L}_i\mathbf{C}_i & 0 & 0 \\ -\mathbf{B}_i\mathbf{K}_j & \mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j & -\mathbf{B}_i\mathbf{K}_j \\ 0 & 0 & \mathbf{A}_m \end{bmatrix} \tag{14}$$

$$\Delta\tilde{\mathbf{A}}_i = \begin{bmatrix} 0 & \Delta\mathbf{A}_i & 0 \\ 0 & \Delta\mathbf{A}_i & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \tilde{\boldsymbol{\Xi}}_i = \begin{bmatrix} -\mathbf{L}_i & \mathbf{I} & 0 \\ 0 & \mathbf{I} & 0 \\ 0 & 0 & \mathbf{I} \end{bmatrix} \tag{15}$$

Then, the uncertainty matrix $\Delta\tilde{\mathbf{A}}_i$ is bounded as $\|\Delta\tilde{\mathbf{A}}_i\| < \delta_i$. Hence, the H_∞ tracking performance in (7) can be rewritten as follows:

$$\begin{aligned} &\int_0^{t_f} (\mathbf{x}(t) - \mathbf{x}_m(t))^\top \mathbf{Q}(\mathbf{x}(t) - \mathbf{x}_m(t))dt \\ &= \int_0^{t_f} [\mathbf{e}(t)^\top\ \mathbf{x}(t)^\top\ \mathbf{x}_m(t)^\top] \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q} & -\mathbf{Q} \\ \mathbf{0} & -\mathbf{Q} & \mathbf{Q} \end{bmatrix} \begin{bmatrix} \mathbf{e}(t) \\ \mathbf{x}(t) \\ \mathbf{x}_m(t) \end{bmatrix} dt \\ &= \int_0^{t_f} \tilde{\mathbf{x}}^\top(t)\tilde{\mathbf{Q}}\tilde{\mathbf{x}}(t)dt \\ &\le \tilde{\mathbf{x}}^\top(0)\tilde{\mathbf{E}}\tilde{\mathbf{P}}\tilde{\mathbf{x}}(0) + \rho^2 \int_0^{t_f} \tilde{\mathbf{w}}^\top(t)\tilde{\mathbf{w}}(t)dt \end{aligned} \tag{16}$$

where $\tilde{\mathbf{Q}}$ is a symmetric positive semi-definite weighting matrix and is given by

$$\tilde{\mathbf{Q}} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q} & -\mathbf{Q} \\ \mathbf{0} & -\mathbf{Q} & \mathbf{Q} \end{bmatrix}$$

The purpose of this paper is to determine a fuzzy controller in (12) for the augmented system in (13) with the guaranteed H_∞ tracking performance in (1) for all $\tilde{\mathbf{w}}^\top(t)$. Then the attenuation level ρ^2 can also be minimized so that the H_∞ tracking performance in (16) is reduced as mall as possible, and, furthermore, the closed-loop system

$$\mathbf{E}\dot{\tilde{\mathbf{x}}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\boldsymbol{\xi})\lambda_j(\boldsymbol{\xi})\big(\tilde{\mathbf{A}}_{ij} + \Delta\tilde{\mathbf{A}}_i\big)\tilde{\mathbf{x}}(t) \tag{17}$$

is quadratically stable.

Remark 1: The premise variables, $\boldsymbol{\xi}$, can be measurable state variables, outputs or combination of measurable state variables. For Takagi-Sugeno fuzzy model, using state variables as premise variables are common, but not always [12]. The limitation of this approach is that some state variables must be measurable to construct the fuzzy observer and fuzzy controller. This is a common limitation for control system design of Takagi-Sugeno fuzzy approach. If the premise variables of the fuzzy observer depend on the estimated state variables, i.e., instead of in the fuzzy observer, the situation becomes more complicated. In this case, it is difficult to directly find control and observer gains.

IV. Stability analysis

In this section, it is shown that the proposed fuzzy control scheme with fuzzy observer in (12) can achieve the H_∞ tracing control performance in (13).

Theorem 1: Consider the nonlinear system (13). If there exists $\tilde{\mathbf{P}} = \tilde{\mathbf{P}}^\top > 0$ such that the following conditions hold:

$$\tilde{\mathbf{P}}^\top\mathbf{E} = \mathbf{E}^\top\tilde{\mathbf{P}} \ge 0 \tag{18}$$

and

$$\tilde{\mathbf{A}}_{ij}^{\top}\tilde{\mathbf{P}}+\tilde{\mathbf{P}}\tilde{\mathbf{A}}_{ij}+\frac{1}{\rho^2}\tilde{\mathbf{P}}\tilde{\mathbf{\Xi}}_i\tilde{\mathbf{\Xi}}_i^{\top}\tilde{\mathbf{P}}+\tilde{\mathbf{Q}}+\tilde{\mathbf{P}}\tilde{\mathbf{P}}^{\top}+\delta_i^2\mathbf{I}<0 \quad (19)$$

for $\sum_{i=1}^{r}\lambda_i(\boldsymbol{\xi})\sum_{j=1}^{r}\lambda_j(\boldsymbol{\xi})\neq 0$ and $i,j=1,2,\cdots,r$, then the H_∞ tracking control performance in (16) is guaranteed for a prescribed ρ^2.

Proof: From (16), we can obtain

$$\begin{aligned}
&\int_0^{t_f}(\mathbf{x}(t)-\mathbf{x}_m(t))^{\top}\mathbf{Q}(\mathbf{x}(t)-\mathbf{x}_m(t))dt\\
&=\int_0^{t_f}\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{Q}}\tilde{\mathbf{x}}(t)\\
&=\int_0^{t_f}\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{Q}}\tilde{\mathbf{x}}(t)+\int_0^{t_f}\frac{d}{dt}(\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{E}}^{\top}\tilde{\mathbf{P}}\tilde{\mathbf{x}}(t))dt\\
&\quad+\tilde{\mathbf{x}}^{\top}(0)\tilde{\mathbf{E}}^{\top}\tilde{\mathbf{P}}\tilde{\mathbf{x}}(0)-\tilde{\mathbf{x}}^{\top}(t_f)\tilde{\mathbf{E}}^{\top}\tilde{\mathbf{P}}\tilde{\mathbf{x}}(t_f)\\
&\le\tilde{\mathbf{x}}^{\top}(0)\tilde{\mathbf{E}}^{\top}\tilde{\mathbf{P}}\tilde{\mathbf{x}}(0)+\int_0^{t_f}(\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{Q}}\tilde{\mathbf{x}}(t)+\dot{\tilde{\mathbf{x}}}^{\top}(t)\tilde{\mathbf{E}}^{\top}\tilde{\mathbf{P}}\tilde{\mathbf{x}}(t)\\
&\quad+\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{P}}\tilde{\mathbf{E}}\dot{\tilde{\mathbf{x}}}(t))dt\\
&=\tilde{\mathbf{x}}^{\top}(0)\tilde{\mathbf{E}}^{\top}\tilde{\mathbf{P}}\tilde{\mathbf{x}}(0)+\int_0^{t_f}(\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{Q}}\tilde{\mathbf{x}}(t)\\
&\quad+\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\boldsymbol{\xi})\lambda_j(\boldsymbol{\xi})((\tilde{\mathbf{A}}_{ij}\tilde{\mathbf{x}}(t))^{\top}\tilde{\mathbf{P}}\tilde{\mathbf{x}}(t)\\
&\quad+\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{P}}\tilde{\mathbf{A}}_{ij}\tilde{\mathbf{x}}(t)+\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{P}}\tilde{\mathbf{\Xi}}_i\tilde{\mathbf{w}}(t)+\tilde{\mathbf{\Xi}}_i\tilde{\mathbf{w}}^{\top}(t)\tilde{\mathbf{P}}\tilde{\mathbf{x}}(t)\\
&\quad+\Delta\mathbf{A}_i^{\top}\mathbf{x}^{\top}(t)\tilde{\mathbf{P}}\tilde{\mathbf{x}}(t)+\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{P}}\Delta\mathbf{A}\tilde{\mathbf{x}}(t))dt\\
&=\tilde{\mathbf{x}}^{\top}(0)\tilde{\mathbf{E}}^{\top}\tilde{\mathbf{P}}\tilde{\mathbf{x}}(0)+\int_0^{t_f}(\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{Q}}\tilde{\mathbf{x}}(t)\\
&\quad+\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\boldsymbol{\xi})\lambda_j(\boldsymbol{\xi})((\tilde{\mathbf{A}}_{ij}\tilde{\mathbf{x}}(t))^{\top}\tilde{\mathbf{P}}\tilde{\mathbf{x}}(t)\\
&\quad+\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{P}}\tilde{\mathbf{A}}_{ij}\tilde{\mathbf{x}}(t)+\tilde{\mathbf{w}}^{\top}(t)\tilde{\mathbf{\Xi}}_i^{\top}\tilde{\mathbf{P}}\tilde{\mathbf{x}}(t)+\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{P}}\tilde{\mathbf{\Xi}}_i\tilde{\mathbf{w}}(t)\\
&\quad-\rho^2\tilde{\mathbf{w}}^{\top}(t)\tilde{\mathbf{w}}(t)\\
&\quad-\frac{1}{\rho^2}\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{P}}\tilde{\mathbf{\Xi}}_i\tilde{\mathbf{\Xi}}_i^{\top}\tilde{\mathbf{P}}\tilde{\mathbf{x}}(t)+\rho^2\tilde{\mathbf{w}}^{\top}(t)\tilde{\mathbf{w}}(t)+\frac{1}{\rho^2}\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{P}}\tilde{\mathbf{\Xi}}_i\\
&\quad+\Delta\tilde{\mathbf{A}}_i^{\top}\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{P}}\tilde{\mathbf{x}}(t)+\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{P}}\Delta\tilde{\mathbf{A}}_i\tilde{\mathbf{x}}(t)))dt\\
&=\tilde{\mathbf{x}}^{\top}(0)\tilde{\mathbf{E}}^{\top}\tilde{\mathbf{P}}\tilde{\mathbf{x}}(0)+\int_0^{t_f}(\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{Q}}\tilde{\mathbf{x}}(t)+\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\boldsymbol{\xi})\lambda_j(\boldsymbol{\xi})\\
&\quad\times((\tilde{A}_{ij}\tilde{\mathbf{x}}(t))^{\top}\tilde{\mathbf{P}}\tilde{\mathbf{x}}(t)+\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{P}}\tilde{\mathbf{A}}_{ij}\tilde{\mathbf{x}}(t)-\big(\frac{1}{\rho}\tilde{\mathbf{\Xi}}_i^{\top}\tilde{\mathbf{P}}\tilde{\mathbf{x}}(t)-\rho\tilde{\mathbf{w}}(t)\big)^{\top}\\
&\quad\times(\frac{1}{\rho}\tilde{\mathbf{\Xi}}_i^{\top}\tilde{\mathbf{P}}\tilde{\mathbf{x}}(t)-\rho\tilde{\mathbf{w}}(t))+\rho^2\tilde{\mathbf{w}}^{\top}(t)\tilde{\mathbf{w}}(t)+\frac{1}{\rho^2}\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{P}}\tilde{\mathbf{\Xi}}_i^{\top}\tilde{\mathbf{\Xi}}_i\tilde{\mathbf{P}}\mathbf{x}(t)\\
&\quad+\Delta\tilde{\mathbf{A}}_i^{\top}\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{P}}\tilde{\mathbf{x}}(t)+\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{P}}\Delta\tilde{\mathbf{A}}_i\tilde{\mathbf{x}}(t))dt\\
&\le\tilde{\mathbf{x}}^{\top}(0)\tilde{\mathbf{E}}^{\top}\tilde{\mathbf{P}}\tilde{\mathbf{x}}(0)+\int_0^{t_f}(\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{Q}}\tilde{\mathbf{x}}(t)\\
&\quad+\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\boldsymbol{\xi})\lambda_j(\boldsymbol{\xi})((\tilde{\mathbf{A}}_{ij}\tilde{\mathbf{x}}(t))^{\top}\tilde{\mathbf{P}}\tilde{\mathbf{x}}(t)\\
&\quad+\rho^2\tilde{\mathbf{w}}^{\top}(t)\tilde{\mathbf{w}}(t)+\frac{1}{\rho^2}\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{P}}\tilde{\mathbf{\Xi}}_i^{\top}\tilde{\mathbf{\Xi}}_i\tilde{\mathbf{P}}\mathbf{x}(t)+\Delta\tilde{\mathbf{A}}_i^{\top}\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{P}}\tilde{\mathbf{x}}(t)\\
&\quad+\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{P}}\Delta\tilde{\mathbf{A}}_i\tilde{\mathbf{x}}(t))dt\\
&\le\tilde{\mathbf{x}}^{\top}(0)\tilde{\mathbf{E}}^{\top}\tilde{\mathbf{P}}\tilde{\mathbf{x}}(0)+\int_0^{t_f}(\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{Q}}\tilde{\mathbf{x}}(t)\\
&\quad+\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\boldsymbol{\xi})\lambda_j(\boldsymbol{\xi})((\tilde{\mathbf{A}}_{ij}\tilde{\mathbf{x}}(t))^{\top}\tilde{\mathbf{P}}\tilde{\mathbf{x}}(t)\\
&\quad+\rho^2\tilde{\mathbf{w}}^{\top}(t)\tilde{\mathbf{w}}(t)+\frac{1}{\rho^2}\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{P}}\tilde{\mathbf{\Xi}}_i^{\top}\tilde{\mathbf{\Xi}}_i\tilde{\mathbf{P}}\mathbf{x}(t)+\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{P}}\tilde{\mathbf{P}}\tilde{\mathbf{x}}(t)\\
&\quad+\tilde{\mathbf{x}}(t)\Delta\tilde{\mathbf{A}}_i\Delta\tilde{\mathbf{A}}_i\tilde{\mathbf{x}}(t))dt\\
&\le\tilde{\mathbf{x}}^{\top}(0)\tilde{\mathbf{E}}^{\top}\tilde{\mathbf{P}}\tilde{\mathbf{x}}(0)+\int_0^{t_f}(\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{Q}}\tilde{\mathbf{x}}(t)+\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t))\\
&\quad\times((\tilde{\mathbf{A}}_{ij}\tilde{\mathbf{x}}(t))^{\top}\tilde{\mathbf{P}}\tilde{\mathbf{x}}(t)+\rho^2\tilde{\mathbf{w}}^{\top}(t)\tilde{\mathbf{w}}(t)+\frac{1}{\rho^2}\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{P}}\tilde{\mathbf{\Xi}}_i^{\top}\tilde{\mathbf{\Xi}}_i\tilde{\mathbf{P}}\mathbf{x}(t)\\
&\quad+\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{P}}\tilde{\mathbf{P}}\tilde{\mathbf{x}}(t)+\tilde{\mathbf{x}}^{\top}(t)\delta_i^2\tilde{\mathbf{x}}(t))dt\ (\text{since }\|\Delta\tilde{\mathbf{A}}_i\|<\delta_i)\\
&\le\tilde{\mathbf{x}}^{\top}(0)\tilde{\mathbf{E}}^{\top}\tilde{\mathbf{P}}\tilde{\mathbf{x}}(0)+\int_0^{t_f}(\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\boldsymbol{\xi})\tilde{\mathbf{x}}^{\top}(t)(\tilde{\mathbf{A}}_{ij}^{\top}\tilde{\mathbf{P}}\\
&\quad+\tilde{\mathbf{P}}\tilde{\mathbf{A}}_{ij}+\frac{1}{\rho^2}\tilde{\mathbf{P}}\tilde{\mathbf{\Xi}}_i\tilde{\mathbf{\Xi}}_i^{\top}\tilde{\mathbf{P}}+\tilde{\mathbf{Q}}+\tilde{\mathbf{P}}\tilde{\mathbf{P}}^{\top}+\delta_i^2\mathbf{I})\tilde{\mathbf{x}}(t)\\
&\quad+\rho^2\tilde{\mathbf{w}}^{\top}(t)\tilde{\mathbf{w}}(t))dt
\end{aligned}\quad (20)$$

From (19), we can obtain

$$\int_0^{t_f}\tilde{\mathbf{x}}^{\top}\tilde{\mathbf{Q}}\tilde{\mathbf{x}}dt\le\tilde{\mathbf{x}}^{\top}(0)\tilde{\mathbf{Q}}\tilde{x}(0)+\rho^2\int_0^{t_f}\tilde{\mathbf{w}}^{\top}(t)\tilde{\mathbf{w}}(t)dt \quad (21)$$

Therefore, the H_∞ tracking control performance is achieved within a prescribed ρ^2. ■

To obtain better tracking performance, the tracking control problem can be formulated as the following minimization problem:

$$\min_{\tilde{\mathbf{P}}}\rho^2$$
$$\text{subject to }\tilde{\mathbf{P}}>0\text{ and (19)} \quad (22)$$

To prove the closed-loop system in (17) is quadratically stable, we have the following theorem.

Theorem 2 : Consider the nonlinear closed-loop system (17). If there exists $\tilde{\mathbf{P}}=\tilde{\mathbf{P}}^{\top}>0$ such that $\tilde{\mathbf{P}}^{\top}\mathbf{E}=\mathbf{E}^{\top}\tilde{\mathbf{P}}\ge 0$ for the minimization problem in (22), then the closed-loop system in (17) is quadratically stable.

Proof: Let the Lyapunov function for the system (17) be defined as

$$\mathbf{V(t)}=\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{E}}^{\top}\tilde{\mathbf{P}}\tilde{\mathbf{x}}(t) \quad (23)$$

where the weighting matrix $\tilde{\mathbf{P}}$ is the same as that in (16). By differentiating (23), we obtain

$$\begin{aligned}
\dot{\mathbf{V}}(t)&=\dot{\tilde{\mathbf{x}}}^{\top}(t)\tilde{\mathbf{E}}^{\top}\tilde{\mathbf{P}}\tilde{\mathbf{x}}(t)+\tilde{\mathbf{x}}^{\top}(t)\tilde{\mathbf{P}}\tilde{\mathbf{E}}\dot{\tilde{\mathbf{x}}}(t)\\
&=\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\boldsymbol{\xi})\lambda_j(\boldsymbol{\xi})\tilde{\mathbf{x}}^{\top}(t)\Big(\tilde{\mathbf{A}}_{ij}^{\top}\tilde{\mathbf{P}}+\tilde{\mathbf{P}}\tilde{\mathbf{A}}_{ij}\Big)\tilde{\mathbf{x}}(t)
\end{aligned}\quad (24)$$

By (19), we get $\dot{\mathbf{V}}(t)<0$. This completes the proof. ■

Note that if the input matrices in all the fuzzy plant rules are equal, then Theorem 1 reduces to the following result.

Corollary 1 : Suppose that $\mathbf{B}_i=\mathbf{B}$, $i=1,2,\ldots,r$. Then the closed-loop fuzzy descriptor system (13) under fuzzy observer and control laws in (8) and (12) is regular, impulse free and stable if there exist a matrix $\mathbf{P}>0$ satisfying (19).

Proof: The proof follows immediately by noting that, under the conditions of $\mathbf{B}_i=\mathbf{B}$, $i=1,2,\ldots,r$, the closed-loop fuzzy descriptor system (13) can be rewritten as

$$\mathbf{E}\dot{\tilde{\mathbf{x}}}(t)=\sum_{i=1}^{r}\lambda_i(\boldsymbol{\xi})\Big((\tilde{\mathbf{A}}_i+\Delta\tilde{\mathbf{A}}_i)\tilde{\mathbf{x}}(t)+\tilde{\mathbf{\Xi}}_i\tilde{\mathbf{w}}(t)\Big) \quad (25)$$

where

$$\tilde{\mathbf{A}}_i = \begin{bmatrix} \mathbf{A}_i - \mathbf{L}_i\mathbf{C}_i & 0 & 0 \\ -\mathbf{B}\mathbf{K}_i & \mathbf{A}_i + \mathbf{B}\mathbf{K}_i & -\mathbf{B}\mathbf{K}_i \\ 0 & 0 & \mathbf{A}_m \end{bmatrix} \tag{26}$$

and $\Delta\tilde{\mathbf{A}}_i$ and $\tilde{\mathbf{\Xi}}_i$ are the same as in (15). ■

Remark 2 : If we let $\mathbf{E} = diag\{\mathbf{0}, I_{n_1}\} \in \mathbb{R}^{n\times n}$ in (1). Then the conditions in (18) and (19) can be applied to guarantee that the closed-loop fuzzy descriptor system (13) under fuzzy observer and control laws in (8) and (12) is regular, impulse free and stable, and the H_∞ tracking control performance in (16) is achieved.

Theorems 1 and 2 provide sufficient conditions in terms of LMIs for fuzzy controller design of fuzzy descriptor systems. We remark that the stabilization result for the augmented system considered in [4] is valid only for the case that all the local pairs $(\mathbf{E}, \mathbf{A}_i)$ are regular and impulse-free (due to invertible $\mathbf{E}_i$'s implied by the given results). Our results of Theorems 1 and 2 do not require such constraints.

From the analysis above, the most important task of the fuzzy observer-based state feedback tracking control problem is how to solve the common solution $\tilde{\mathbf{P}} = \tilde{\mathbf{P}}^\top > 0$ from the minimization problem (22). In general, it is not easy to analytically determine the common solution $\tilde{\mathbf{P}} = \tilde{\mathbf{P}}^\top > 0$ for (22). Fortunately, (22) can be transferred into a minimization problem subject to some linear matrix inequalities (LMIs). The LMIP can be solved in a computationally efficient manner using a convex optimization technique such as the interior point method.

For the convenience of design, we assume

$$\tilde{\mathbf{P}} = \begin{bmatrix} \tilde{\mathbf{P}}_{11} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{P}}_{22} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \tilde{\mathbf{P}}_{33} \end{bmatrix} \tag{27}$$

where $\tilde{\mathbf{P}}_{11} = \tilde{\mathbf{P}}_{11}^\top > 0$, $\tilde{\mathbf{P}}_{22} = \tilde{\mathbf{P}}_{22}^\top > 0$, and $\tilde{\mathbf{P}}_{33} = \tilde{\mathbf{P}}_{33}^\top > 0$. This choice is suitable for the separate design of the fuzzy controller and fuzzy observer. By substituting (27) into (19), we obtain

$$\begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} & \mathbf{0} \\ \mathbf{S}_{21} & \mathbf{S}_{22} & \mathbf{S}_{23} \\ \mathbf{0} & \mathbf{S}_{32} & \mathbf{S}_{33} \end{bmatrix} < 0$$

where

$$\begin{aligned}
\mathbf{S}_{11} &= (\mathbf{A}_i - \mathbf{L}_i\mathbf{C}_j)^\top\tilde{\mathbf{P}}_{11} + \tilde{\mathbf{P}}_{11}(\mathbf{A}_i - \mathbf{L}_i\mathbf{C}_j) \\
&\quad + \frac{1}{\rho^2}\tilde{\mathbf{P}}_{11}(\mathbf{L}_i\mathbf{L}_i^\top + \mathbf{I})\tilde{\mathbf{P}}_{11} + \tilde{\mathbf{P}}_{11}\tilde{\mathbf{P}}_{11} + \delta_i^2\mathbf{I} \\
\mathbf{S}_{12} &= \mathbf{S}_{21}^\top = -\tilde{\mathbf{P}}_{22}\mathbf{B}_i\mathbf{K}_j + \frac{1}{\rho^2}\tilde{\mathbf{P}}_{11}\tilde{\mathbf{P}}_{22} \\
\mathbf{S}_{22} &= (\mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j)^\top\tilde{\mathbf{P}}_{22} + \tilde{\mathbf{P}}_{22}(\mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j) \\
&\quad + \frac{1}{\rho^2}\tilde{\mathbf{P}}_{22}\tilde{\mathbf{P}}_{22} + \mathbf{Q} + \tilde{\mathbf{P}}_{22}\tilde{\mathbf{P}}_{22} + \delta_i^2\mathbf{I} \\
\mathbf{S}_{23} &= \mathbf{S}_{32}^\top = -\tilde{\mathbf{P}}_{22}\mathbf{B}_i\mathbf{K}_j - \mathbf{Q} \\
\mathbf{S}_{33} &= \mathbf{A}_r^\top\tilde{\mathbf{P}}_{33} + \tilde{\mathbf{P}}_{33}\mathbf{A}_m + \frac{1}{\rho^2}\tilde{\mathbf{P}}_{33}\tilde{\mathbf{P}}_{33} + \mathbf{Q} + \tilde{\mathbf{P}}_{33}\tilde{\mathbf{P}}_{33} + \delta_i^2\mathbf{I}
\end{aligned}$$

With $\mathbf{Z}_i = \tilde{\mathbf{P}}_{11}\mathbf{L}_i$, we obtain

$$\begin{bmatrix} \mathbf{M}_{11} & \tilde{\mathbf{P}}_{11} & \mathbf{Z}_i & \mathbf{M}_{41}^\top & \mathbf{0} & \mathbf{0} \\ \tilde{\mathbf{P}}_{11} & -(\rho^2+1)\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{Z}_i^\top & \mathbf{0} & -\rho^2\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{M}_{41} & \mathbf{0} & \mathbf{0} & \mathbf{M}_{44} & \mathbf{M}_{45} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{M}_{45}^\top & \mathbf{M}_{55} & \tilde{\mathbf{P}}_{33} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \tilde{\mathbf{P}}_{33} & -(\rho^2+1)\mathbf{I} \end{bmatrix} < 0 \tag{28}$$

where

$$\begin{aligned}
\mathbf{M}_{11} &= \mathbf{A}_i^\top\tilde{\mathbf{P}}_{11} + \tilde{\mathbf{P}}_{11}\mathbf{A}_i - \mathbf{Z}_i\mathbf{C}_j - (\mathbf{Z}_i\mathbf{C}_j)^\top + \delta_i^2\mathbf{I} \\
\mathbf{M}_{41} &= -(\mathbf{B}_i\mathbf{K}_j)^\top\tilde{\mathbf{P}}_{22} + \frac{1}{\rho^2}\tilde{\mathbf{P}}_{22}\tilde{\mathbf{P}}_{11} \\
\mathbf{M}_{44} &= (\mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j)^\top\tilde{\mathbf{P}}_{22} + \tilde{\mathbf{P}}_{22}(\mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j) \\
&\quad + \frac{1}{\rho^2}\tilde{\mathbf{P}}_{22}\tilde{\mathbf{P}}_{22} + \mathbf{Q} + \tilde{\mathbf{P}}_{22}\tilde{\mathbf{P}}_{22} + \delta_i^2\mathbf{I} \\
\mathbf{M}_{45} &= -\tilde{\mathbf{P}}_{22}\mathbf{B}_i\mathbf{K}_j - \mathbf{Q} \\
\mathbf{M}_{55} &= \mathbf{A}_r^\top\tilde{\mathbf{P}}_{33} + \tilde{\mathbf{P}}_{33}\mathbf{A}_r + \mathbf{Q} + \delta_i^2\mathbf{I}
\end{aligned}$$

Corollary 2 : Fuzzy system (1) is stable and the performance index (7) is achieved for all nonzero $\tilde{\mathbf{w}}$ if there exist $\tilde{\mathbf{P}}_{11} > 0$, $\tilde{\mathbf{P}}_{22} > 0$, and $\tilde{\mathbf{P}}_{33} > 0$ that satisfy (28). ■

Since five parameters $\tilde{\mathbf{P}}_{11}, \tilde{\mathbf{P}}_{22}, \tilde{\mathbf{P}}_{33}, \mathbf{K}_j$, and $\mathbf{L}_i$ should be determined from (28), there are no effective algorithms for solving them simultaneously until now. However, by the choice of (27), the control and observer problems can be decoupled and can be solved separately by the following two-stage procedures which solve the fuzzy control parameters first and then solve the fuzzy observer parameters. In the first step, note that (28) implies that $\mathbf{M}_{44} < 0$, and

$$\begin{aligned}
&(\mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j)^\top\tilde{\mathbf{P}}_{22} + \tilde{\mathbf{P}}_{22}(\mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j) + \frac{1}{\rho^2}\tilde{\mathbf{P}}_{22}\tilde{\mathbf{P}}_{22} + \mathbf{Q} \\
&\quad + \tilde{\mathbf{P}}_{22}\tilde{\mathbf{P}}_{22} + \delta_i^2\mathbf{I} < \mathbf{0}
\end{aligned} \tag{29}$$

With $\tilde{\mathbf{W}}_{22} = \tilde{\mathbf{P}}^{-1}$ and $\mathbf{Y}_j = \mathbf{K}_j\tilde{\mathbf{W}}_{22}$, (29) is equivalent to

$$\begin{aligned}
&\tilde{\mathbf{W}}_{22}\mathbf{A}_i^\top + \mathbf{A}_i\tilde{\mathbf{W}}_{22} + \mathbf{B}_i\mathbf{Y}_j + (\mathbf{B}_i\mathbf{Y}_j)^\top + \frac{1}{(\rho^2+1)}\mathbf{I} \\
&\quad + \tilde{\mathbf{W}}_{22}\mathbf{Q}\tilde{\mathbf{W}}_{22} + \delta_i^2\mathbf{I} < 0
\end{aligned} \tag{30}$$

By the Schur complements, (30)

$$\begin{bmatrix} \mathbf{H}_{11} & \tilde{\mathbf{W}}_{22} \\ \tilde{\mathbf{W}}_{22} & -\mathbf{Q}^{-1} \end{bmatrix} < 0 \tag{31}$$

where $\tilde{\mathbf{W}}_{22}\mathbf{A}_i^\top + \mathbf{A}_i\tilde{\mathbf{W}}_{22} + \mathbf{B}_i\mathbf{Y}_j + (\mathbf{B}_i\mathbf{Y}_j)^\top + \frac{1}{(\rho^2+1)}\mathbf{I} + \delta_i^2\mathbf{I} < 0$. The parameters $\tilde{\mathbf{W}}_{22}$ and $\mathbf{Y}_j$ (thus $\tilde{\mathbf{P}}_{22} = \tilde{\mathbf{W}}_{22}^{-1}$ and $\mathbf{K}_j = \mathbf{Y}_j\tilde{\mathbf{W}}_{22}^{-1}$) can be obtained by solving the LMIP in (31) for a prescribed attenuation level ρ^2. In the second step, by substituting $\tilde{\mathbf{P}}_{22}$ and $\mathbf{K}_j$ into (28), (28) becomes standard LMIs. Similarly, we can easily solve $\tilde{\mathbf{P}}_{11}$, $\tilde{\mathbf{P}}_{33}$ and $\mathbf{Z}_i$ (thus $\mathbf{L}_i = \tilde{\mathbf{P}}_{11}^{-1}\mathbf{Z}_i$) from (28). Recall that the attenuation level ρ^2 can be minimized so that the H_∞ tracking performance in (16) is reduced as small as possible

$$\begin{aligned}
&\min_{(\tilde{\mathbf{P}}_{11}, \tilde{\mathbf{P}}_{22}, \tilde{\mathbf{P}}_{33})} \rho^2 \\
&\text{subject to } \tilde{\mathbf{P}}_{11} > 0, \tilde{\mathbf{P}}_{22} > 0, \tilde{\mathbf{P}}_{33} > 0, \text{ and } (28)
\end{aligned} \tag{32}$$

The problem now becomes one of finding the matrix $\mathbf{P}$ such that the observer-based controller in (8) and (12) that will ensure the stability of the system (13) and that the H_∞-norm of the transference from $\mathbf{x} - \mathbf{x}_m$ to $\tilde{\mathbf{w}}$ is less than ρ.

The design algorithm of the proposed tracking control for solving this problem via fuzzy observer-based state feedback is delineated as follows:

input M, r, g
input $\mathbf{A}_i$, $\Delta\mathbf{A}_i$, δ_i, $\mathbf{B}_i$, $\mathbf{K}_i$, $\mathbf{L}_i$, $i=1,2,\ldots,r$, **v**, **w**, **r**
input fuzzy inference rules M_{ij}, $i=1,2,\ldots,r$, $j=1,2,\ldots,g$
input initial conditions $\mathbf{x}(0)$, $\hat{\mathbf{x}}(0)$
for k=1 to M do
$\beta_1=1$
$\mathbf{A}=0$, $\mathbf{B}=0$, $\mathbf{C}=0$, $\mathbf{K}=0$, $\mathbf{L}=0$
for i=1 to r do
if ξ_1 is M_{i1} and $\cdots$ and ξ_g is M_{ig} **then**
for j=1 to g do
$\beta_i \leftarrow \beta_i \times M_{ij}$
end do
$temp \leftarrow \beta_i$
$\lambda_1=0$
for i=1 to r do
$\lambda_i \leftarrow \lambda_i+\beta_i$
end do
$\lambda_i \leftarrow temp/\lambda_i$
$\mathbf{A} \leftarrow \mathbf{A}+\mathbf{A}_i\times\lambda_i$
$\mathbf{B} \leftarrow \mathbf{B}+\mathbf{B}_i\times\lambda_i$
$\mathbf{C} \leftarrow \mathbf{C}+\mathbf{C}_i\times\lambda_i$
$\mathbf{K} \leftarrow \mathbf{K}+\mathbf{K}_i\times\lambda_i$
$\mathbf{L} \leftarrow \mathbf{L}+\mathbf{L}_i\times\lambda_i$
end **if**
end do
Find ρ, $\tilde{\mathbf{P}}_{11}$, $\tilde{\mathbf{P}}_{22}$, $\tilde{\mathbf{P}}_{33}$ from (32) by LMI toolbox
Solve $\mathbf{x}$, y from (3) by numerical differentiation algorithm
Solve $\mathbf{x}_m$, y_m from (6) by numerical differentiation algorithm
Solve $\hat{\mathbf{x}}$, $\hat{y}$ from (9) by numerical differentiation algorithm
Obtain $\mathbf{e}$ from (10)
Obtain $\mathbf{u}$ from (12)
end do

V. SIMULATIONS

Consider the following nonlinear system [4]

$$(1+a\cos(\theta))\ddot{\theta}(t)=-b\dot{\theta}^3(t)+c\theta(t)+du(t)+w(t) \qquad (33)$$

where the range of $\dot{\theta}(t)$ is assumed to satisfy $|\dot{\theta}(t)|<\phi$ and the external disturbance is assumed to be $w(t)=0.1\sin(4t)$. We use descriptor expressions to achieve the stabilization for the example using fuzzy observer-based state feedback control scheme. Let $\mathbf{x}(t)=[x_1(t)\ x_2(t)\ x_3(t)]^\top$ with $x_1(t)=\theta(t)$, $x_2(t)=\dot{\theta}(t)$, and $x_3(t)=\ddot{\theta}(t)$. Then, the system is described by

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix}\dot{\mathbf{x}}(t)=\begin{bmatrix}0&1&0\\0&0&1\\c&-bx_2^2(t)&-1-a\cos(x_1)\end{bmatrix}\mathbf{x}(t)+\begin{bmatrix}0\\0\\d\end{bmatrix}\mathbf{u}(t)+\begin{bmatrix}0\\0\\w\end{bmatrix}$$
$$\mathbf{y}(t)=[1\ 0\ 0]\mathbf{x}(t)$$

This can be expressed exactly by the following fuzzy descriptor form:

$$\mathbf{E}\dot{\mathbf{x}}(t)=\sum_{i=1}^{3}\lambda_i(\boldsymbol{\xi})\Big((\mathbf{A}_i+\Delta\mathbf{A}_i)\mathbf{x}(t)+\mathbf{B}_i\mathbf{u}(t)\Big)+\mathbf{w}(t)$$
$$\mathbf{y}(t)=\sum_{i=1}^{3}\lambda_i(\boldsymbol{\xi})\mathbf{C}_i\mathbf{x}(t) \qquad (34)$$

where

$$\mathbf{E}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix},\quad \mathbf{A}_1=\begin{bmatrix}0&1&0\\0&0&1\\c&-b(\phi^2+2)&a-1\end{bmatrix}$$
$$\mathbf{A}_2=\begin{bmatrix}0&1&0\\0&0&1\\c&0&-a-1-a\phi^2\end{bmatrix},\quad \mathbf{A}_3=\begin{bmatrix}0&1&0\\0&0&1\\c&0&a-1\end{bmatrix}$$
$$\mathbf{B}_1=\mathbf{B}_2=\mathbf{B}_3=\begin{bmatrix}0\\0\\d\end{bmatrix},\quad \mathbf{C}_1=\mathbf{C}_2=\mathbf{C}_3=\begin{bmatrix}1&0&0\end{bmatrix}$$

$$\Delta\mathbf{A}_i = 0.1\mathbf{A}_i,\ \lambda_1=\frac{x_2^2(t)}{\phi^2+2}$$
$$\lambda_2 = \frac{1+\cos(x_1)}{\phi^2+2},\ \lambda_3=\frac{\phi^2-x_2^2(t)+1-\cos(x_1)}{\phi^2+2}.$$

It is seen that $0\le\lambda_i\le1$, $\sum_{i=1}^{3}\lambda_i$=1. For simulation purposes, we set $a=-1$, $b=c=d=1$, and $\phi=2$. Now, following the design algorithm in the above section, the fuzzy tracking control design is given by the following steps:

Steps 1-2: To use the fuzzy control approach, we must have a fuzzy model that represents the dynamics of the nonlinear plant. To minimize the design effort and complexity, we try to use as few fuzzy rules as possible. Hence, we approximate the system by the following three-rule fuzzy model:

$$\begin{aligned}\text{Rule (1) If } \lambda_1 &= \frac{x_2^2(t)}{\phi^2+2},\\ \text{then } \mathbf{E}\dot{\mathbf{x}} &= (\mathbf{A}_1+\Delta\mathbf{A}_1)\mathbf{x}+\mathbf{B}_1\mathbf{u}\\ \mathbf{y} &= \mathbf{C}_1\mathbf{x}\end{aligned}$$

$$\begin{aligned}\text{Rule (2) If } \lambda_2 &= \frac{1+cos x_1(t)}{\phi^2+2},\\ \text{then } \mathbf{E}\dot{\mathbf{x}} &= (\mathbf{A}_2+\Delta\mathbf{A}_2)\mathbf{x}+\mathbf{B}_2\mathbf{u}\\ \mathbf{y} &= \mathbf{C}_2\mathbf{x}\end{aligned}$$

$$\begin{aligned}\text{Rule (3) If } \lambda_3 &= \frac{\phi^2-x_2^2(t)+1-\cos(x_1)}{\phi^2+2},\\ \text{then } \mathbf{E}\dot{\mathbf{x}} &= (\mathbf{A}_3+\Delta\mathbf{A}_3)\mathbf{x}+\mathbf{B}_3\mathbf{u}\\ \mathbf{y} &= \mathbf{C}_3\mathbf{x}\end{aligned}$$

where

$$\mathbf{A}_1=\begin{bmatrix}0&1&0\\0&0&1\\1&-6&-2\end{bmatrix},\ \mathbf{A}_2=\begin{bmatrix}0&1&0\\0&0&1\\1&0&4\end{bmatrix},\ \mathbf{A}_3=\begin{bmatrix}0&1&0\\0&0&1\\1&0&-2\end{bmatrix}$$

and $\delta_1=0.64789$, $\delta_2=0.42361$, and $\delta_3=0.24142$.

Step 3: Solve LMIP using the LMI optimization toolbox in Matlab and we can obtain the minimum value of $\rho=0.1105$, and

$$\tilde{P}_{11}=\begin{bmatrix}0.8018&0.0016&0.0330\\0.0016&0.0035&-0.0075\\0.0330&-0.0075&0.7636\end{bmatrix}$$
$$\tilde{W}_{22}=\begin{bmatrix}-39.4&-3973.9&-6661.4\\2.6&7.4&1.1\\-0.32&-4.43&-7.1\end{bmatrix}$$
$$\tilde{P}_{22}=\begin{bmatrix}0.0025&0.00261&0.0091\\0.00026&0.0074&0.0011\\-0.00032&-0.00443&-0.00071\end{bmatrix}$$

Step 4: The observer parameters are found to be

$$L_1=\begin{bmatrix}3.2956\\1.3549\\-14.9536\end{bmatrix},\ L_2=\begin{bmatrix}2.2837\\5.2610\\-25.6429\end{bmatrix},\ L_3=\begin{bmatrix}5.84221\\3.5191\\-2.2662\end{bmatrix}$$

Then, we can obtain the H_∞ fuzzy observer as

$$\mathbf{E}\dot{\hat{\mathbf{x}}}(t)=\sum_{i=1}^{3}\lambda_i\Big(\mathbf{A}_i\hat{\mathbf{x}}(t)+\mathbf{B}_i\mathbf{u}(t)+\mathbf{L}_i(\mathbf{y}(t)-\hat{\mathbf{y}}(t))\Big)$$

Next, the control parameters are found to be

$$K_1=\begin{bmatrix}-611.1951&-621.2145&-321.6149\end{bmatrix}$$
$$K_2=\begin{bmatrix}-511.18995&-221.12814&-262.6323\end{bmatrix}$$
$$K_3=\begin{bmatrix}-121.18854&-221.11344&-311.6069\end{bmatrix}$$

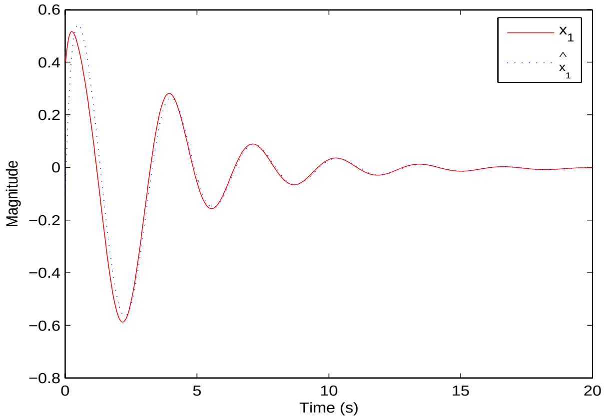

Figure 1. The trajectories of the state variable x_1 (solid line) and estimation state variable $\hat{x}_1$ (dashed line) for case 1.

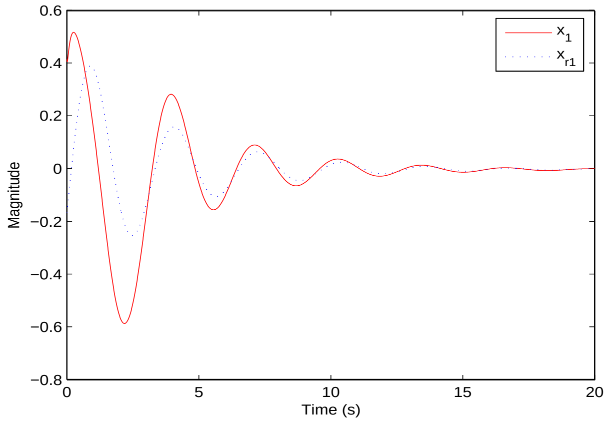

Figure 2. The trajectories of the state variable x_1 (solid line) and reference state variable x_{r1} (dashed line) for case 1.

Step 5: Obtain the H_∞ fuzzy control law in (12) as follows

$$\mathbf{u}(t) = \sum_{i=1}^{3} \lambda_i \Big(\mathbf{K}_i (\hat{\mathbf{x}}(t) - \mathbf{x}_m(t)) \Big)$$

Initial conditions are assumed to be $\mathbf{x}(0) = [0.24 \quad -0.85 \quad 0]^\top$ and $\hat{\mathbf{x}}(0) = [0 \quad -0.8 \quad 0]^\top$. Two cases of regulation (case 1) and tracking (case 2) performances are given to show the effectiveness of the proposed control scheme. Figs. 1-5 show the regulation responses for θ (i.e., x_1) and Figs. 6-10 the responses of tracking a unit step signal. The regulation trajectories of the state variables x_1, $\hat{x}_1$ and x_1, $\hat{x}_{r1}$ are shown in Figs. 1 and 2, respectively. The tracking trajectories of the state variables x_1, $\hat{x}_1$ and x_1, $\hat{x}_{r1}$ are shown in Figs. 6 and 7, respectively. The regulation trajectories of the state variables x_2, $\hat{x}_2$ and x_2, x_{r2} are shown in Fig. 3 and 4, respectively. The tracking trajectories of the state variables x_2, $\hat{x}_2$ and x_2, x_{r2} are shown in Fig. 8 and 9, respectively. The control inputs for regulation and tracking responses are shown in Figs. 5 and 10, respectively. It is clear from Figs. 6 and 7 for tracking responses, the estimation error $(x_1 - \hat{x}_1)$ and tracking error $(x_1 - x_{r1})$ converge to zero neighborhood at about 14 sec. and 15 sec., respectively, and the tracking error is about 1.01%. As for regulation, the estimation errors of $(x_2 - \hat{x}_2)$ and $(x_2 - x_{r2})$ shown in Figs. 1 and 2 converge to zero neighborhood at about 14 sec. and 13sec., respectively, and the tracking error is 1.05%. Hence, from Figs. 1-4 and Figs. 6-9, the performance of the proposed control scheme is satisfactory and H_∞ tracking purpose for the fuzzy descriptor system can be achieved effectively.

VI. Conclusion

In this paper, we have proposed an observer-based fuzzy control scheme for a class of nonlinear descriptor systems with H_∞ tracking performance. The nonlinear descriptor system is represented by an equivalent Takagi-Sugeno type fuzzy model. Then, a fuzzy controller is proposed to override the effect of external disturbances such that the H_∞ model reference tracking performance is achieved subject to external disturbances. Furthermore, If states are not all available, a fuzzy observer is also proposed to estimate the states of the descriptor control system. Based on the Lyapunov stability theorem and linear matrix inequalities (LMIs), the proposed fuzzy control system can guarantee the stability of the whole closed-loop system and obtain good tracking performance within a prescribed attenuation level ρ^2. The problem of H_∞ decentralized fuzzy tracking control design for nonlinear descriptor systems is characterized in terms of solving a linear matrix inequality problem using convex optimization techniques. The proposed fuzzy observer-based decentralized control scheme is simple without complex control algorithms. Therefore, it is suitable for practical applications.The advantage of proposed tracking

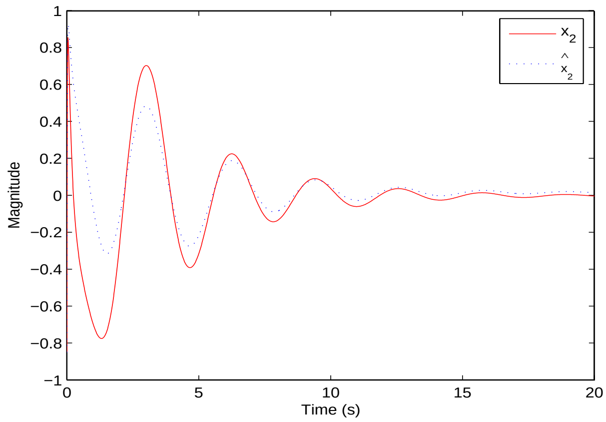

Figure 3. The trajectories of the state variable x_2 (solid line) and estimation state variable $\hat{x}_2$ (dashed line) for case 1

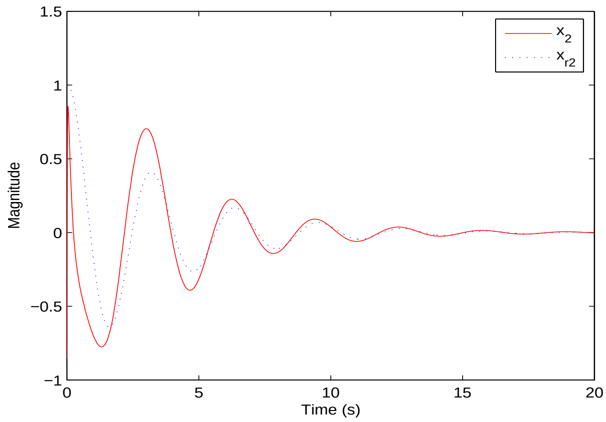

Figure 4. The trajectories of the state variable x_2 (solid line) and reference state variable x_{r2} (dashed line) for case 1.

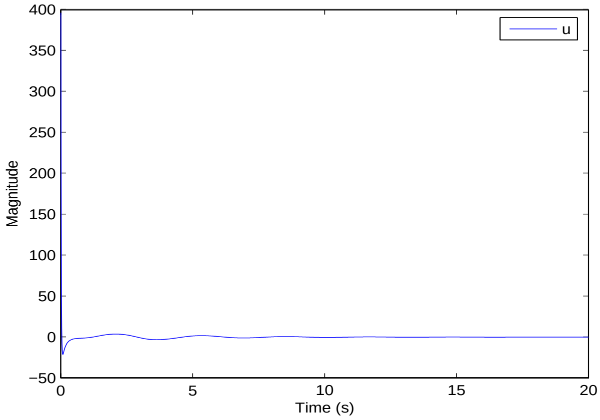

Figure 5. The trajectories of the control input $\mathbf{u(t)}$ for case 1.

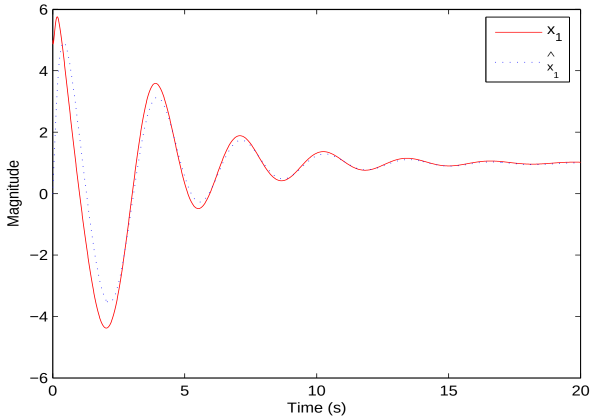

Figure 6. The trajectories of the state variable x_1 (solid line) and estimation state variable $\hat{x}_1$ (dashed line) for case 2.

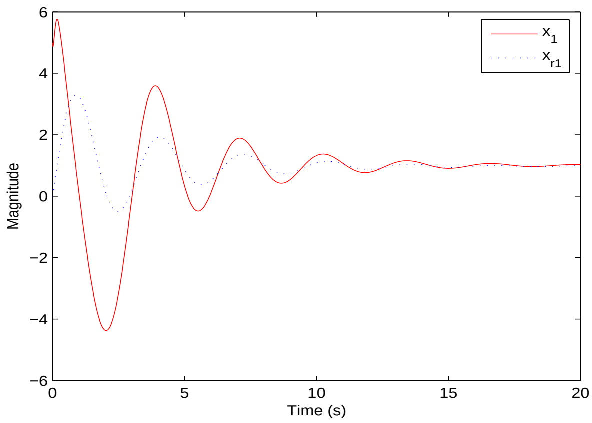

Figure 7. The trajectories of the state variable x_1 (solid line) and reference state variable x_{r1} (dashed line) for case 2.

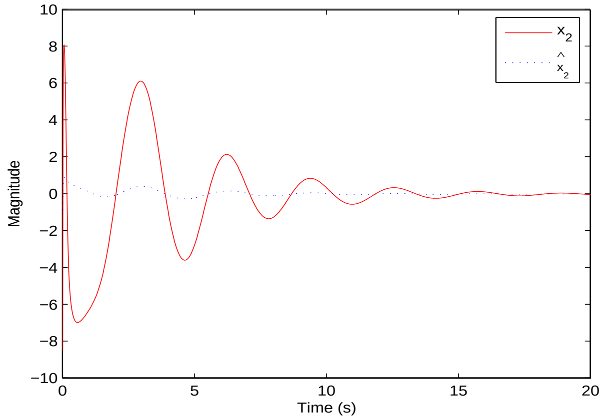

Figure 8. The trajectories of the state variable x_2 (solid line) and estimation state variable $\hat{x}_2$ (dashed line) for case 2.

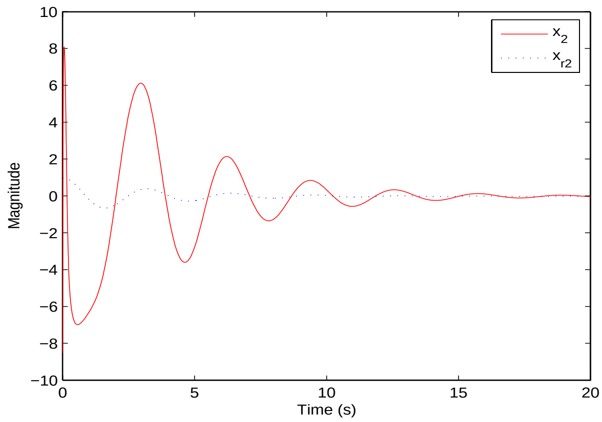

Figure 9. The trajectories of the state variable x_2 (solid line) and reference state variable x_{r2} (dashed line) for case 2.

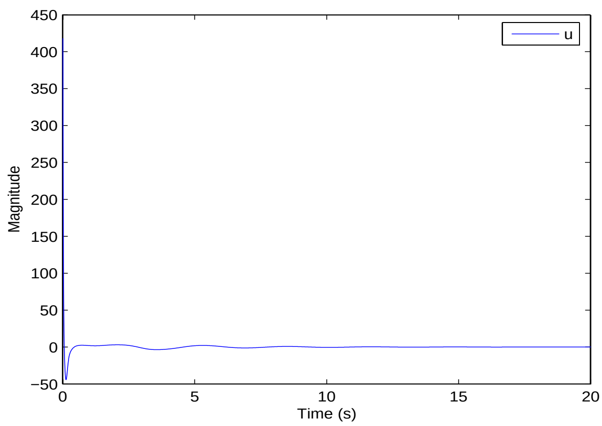

Figure 10. The trajectory of the control input $\mathbf{u(t)}$ for case 2.

control design is that only simple fuzzy observer and controller are used in our approach without feedback linearization technique. The simulation results show that the estimation state and tracking performance are satisfactory.

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