Mining Frequent Subgraph by Incidence Matrix Normalization

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***Abstract*—Existing frequent subgraph mining algorithms can operate efficiently on graphs that are sparse, have vertices with low and bounded degrees, and contain well-labeled vertices and edges. However, there are a number of applications that lead to graphs that do not share these characteristics, for which these algorithms highly become inefficient. In this paper we propose a fast algorithm for mining frequent subgraphs in large database of labeled graphs. The algorithm uses incidence matrix to represent the labeled graphs and to detect their isomorphism. Starting from the frequent edges from the graph database, the algorithm searches the frequent subgraphs by adding frequent edges progressively. By normalizing the incidence matrix of the graph, the algorithm can effectively reduce the computational cost on verifying the isomorphism of the subgraphs. Experimental results show that the algorithm has higher speed and efficiency than that of other similar ones.**

***Index Terms*—graph; incidence matrix; isomorphism; data mining**

I. INTRODUCTION

Data mining is the process of automatically extracting new and useful knowledge hidden in large datasets. In recent years, the study on semi-structured and structured data is showing more and more practical value [1] and there has been an increased interest in developing data mining algorithms on such data like graphs. The problem of frequent subgraph mining arises naturally in a number of different application domains including network intrusion [2,3], semantic web [4], behavioral modeling [5,6] , VLSI reverse engineering [7], link analysis [8,9.10,11] , and chemical compound classification [12-15]. Moreover, they can be used to effectively model the structural and relational characteristics of a variety of datasets arising in areas of physical sciences and chemistry such as fluid dynamics, astronomy, structural mechanics, and ecosystem modeling, life sciences such as genomics, proteomics, pharmacogenomics, and health informatics, and information security such as information assurance, infrastructure protection, and terrorist-threat prediction/identification. We can solve these problems by graph mining method.

Frequent subgraphs mining is to choose subgraphs that meet a certain support threshold from graph data set and it is the base for the research of graph clustering, graph classification, graph query and restricted graph mining. However, the traditional mining methods that based on non-structured data such as frequent-item set mining and sequential-pattern mining have not been equal to the mining work on semi-structured and structured data such like graph. Algorithm studies on frequent graph mining were never stopped. Earlier frequent subgraph mining algorithms like SUBDUE [16] approximate method which can only get approximate result. In SUBDUE, since graphs are compressed as minimum description length and adopting heuristic searching method to mine frequent subgraphs, it can't guarantee the integrality of result. In 2000, Inokuchi and Kuramochi brought Apriori method into subgraph mining and therefore AGM [17] and FSG [18, 19] were proposed. AGM and FSG can enumerate all the subgraphs by increasing vertexes and edges progressively. This approach opened the precedent of putting traditional data mining methods into subgraph mining research. In recent years, algorithms that based on FP-growth like gSpan [20] and FFSM [21] have achieved higher efficiency. Each algorithm has respectively made distinct improvement on the two efficiency-related aspects, that is, subgraph isomorphism judgment and the strategy of frequent subgraph extension. In AGM, FSG, FFSM and their improvements [22,23], graphs are denoted by adjacency matrix and find isomorphic graphs by adjacency matrix code. However, even the most efficient algorithm FFSM still need matrix join and extension operations which made the time complexity for subgraph extension be $O(m^2 \cdot n \cdot 2^n)$ [24]. The algorithm gSpan and its improvements[26] denote graphs in the form of the minimum DFS code, which is the base for its edge extension strategy, yet the time complexity for isomorphic graphs judgment is still as high as $O(2^n)$.

Existing algorithms that mine graph datasets to discover frequent subgraphs can operate efficiently on

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graphs that are sparse, contain a large number of relatively small connected components, have vertices with low and bounded degrees, and contain well-labeled vertices and edges. However, there are a number of applications that lead to graphs that do not share these characteristics, for which these algorithms highly become inefficient. In order to overcome these shortages, we propose a new frequent subgraph mining algorithm *FSMA* (frequent subgraph mining algorithm) in this paper. The algorithm uses incidence matrix to represent the labeled graphs and to detect their isomorphism. Starting from the frequent edges from the graph database, the algorithm identifies the frequent subgraphs by adding edges progressively. By normalizing the incidence matrix of the graph, the algorithm can effectively reduce the computational cost on verifying the isomorphism of the subgraphs. Experimental results show that the algorithm has higher speed and efficiency than that of other similar ones.

The remainder of the paper is organized as follows. In Section 2, we define the labeled graph and their isomorphism. Section 3 presents the method of normalize the incidence matrixes of labeled graph. Framework of the frequent subgraph mining algorithm *FSMA* is proposed in Section 4. Experimental results are shown and analyzed in Section 5, and Section 6 concludes the paper.

II. Labeled Graphs and Their Isomorphism

Definition 1 (Labeled Graph [25] and Labeled Graph Data Set) A labeled graph G is denoted by a quadruple $G=(V(G),E(G),L(V(G)),L(E(G)))$, where $V(G)=\{v_1,v_2,\ldots,v_n\}$ is the graph vertexes set and $E(G)=\{e_k=(v_i,v_j)|v_i,v_j\in V(G)\}\subseteq V\times V$ is the graph edges set, $L(V(G))=\{lb(v_i)|\ \forall\ v_i\in V(G)\}$ is the graph vertexes' labels set, $L(E(G))=\{lb(e_k)|\ \forall\ e_k\in E(G)\}$ is the graph edges' labels set. Let G_1, G_2, $G_3,\ldots,G_u$ be labeled graphs, set $GDB=\{G_1,G_2,G_3,\ldots,G_u\}$ is called a labeled graph data set.

Definition 2 (Identical Labeled Vertexes Set and Identical Labeled Edges Set) For a vertexes set SV of a labeled graph G, if all the vertexes in SV have the same label lv, i.e. for every $v_i\in SV$, $lb(v_i)=lv$, then SV is called an identical labeled vertexes set and is denoted as $SV(lv)$. Similarly, for an edges set SE of a labeled graph G, if all the edges in SE have the same label le, i.e. for every $e_i\in SE$, $lb(e_i)=le$, then SE is called an identical labeled edges set and is denoted as $SE(le)$.

In this paper, in order to identify the vertexes (edges), we assign an index to each vertex (edge) in an identical labeled vertexes (edges) set.

Definition 3 (Labeled Graph Isomorphism [26, 27]) Given two labeled graphs G and G', if there exists a mapping function $f: V(G)\rightarrow V(G')$, which satisfies: (1) $\forall\ v_i\in V(G)$, $lb(v_i)=lb(f(v_i))$; (2) $\forall\ (v_i,v_j)\in E(G)$, $(f(v_i),f(v_j))\in E(G')$, $lb((v_i,v_j))=lb(f(v_i),f(v_j))$, then we call G and G' isomorphic graphs, and denote them as $G\cong G'$.

Definition 4 (Labeled Isomorphic Subgraph) Given two labeled graph G and H, if H' is a subgraph of H and $H'\cong G$, then G is the labeled isomorphic subgraph of H, and is denoted as $G\subseteq H$.

In Fig. 1, (b) is a labeled isomorphic subgraph of Fig. 1 (a).

Definition 5 (Support and Frequent Subgraph) Given labeled graph data set $GDB=\{G_1,G_2,G_3,\ldots,G_u\}$ and labeled graph G', support of G' in GDB is defined as $sup(G')=|\{H|\ H\in GDB \text{ and } G'\subseteq H\}|$ which is the number of graphs in GDB of which G' is a labeled isomorphic subgraph. A frequent subgraph of GDB is a graph whose support is no less than a predefined threshold *minsup*.

Definition 6 (Maximum Frequent Subgraph) Suppose G' is a frequent subgraph of GDB, if for any labeled graph G which satisfies $G'\subseteq G$, G is not a frequent subgraph of GDB, then we call G′ a maximum frequent subgraph of GDB.

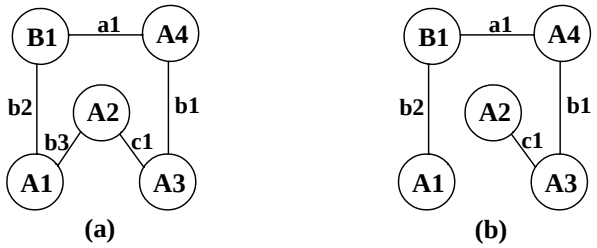

Figure 1. Labeled graphs

The goal for frequent subgraph mining is to find all the maximum subgraphs that satisfy the threshold *minsup* in a graph data set GDB.

III. Graph's Incidence Matrix and Its Normalization

Important part of frequent subgraph mining is the strategy to represent the graphs in appropriate data structure, so that the algorithm could take feasible graph extension strategy to enumerate all frequent subgraphs. Since verifying the isomorphism of graphs is a NP-complete problem, it should make an effective way to normalize these graphs in order to reduce the complexity on isomorphism detecting. As a representation of graphs, incidence matrix has the characteristics of intuitionist and being easy compressed. Therefore, we use incidence matrix to denote graphs in our algorithm FSMA. For easy normalization as well as subgraph extension, we also present the definition of normalized format of incidence matrix.

A. Graph's Incidence Matrix

Given a connected labeled graph $G=(V(G),E(G),L(V(G)),L(E(G)))$, where $L(V(G))=\{A_1,A_2,\ldots,A_r\}(r\le m)$ and $L(E(G))=\{B_1,B_2,\ldots,B_s\}(s\le n)$. Let $SV(A_i)=\{v_{i1},v_{i2},\ldots,v_{imi}\}$ be an identical labeled vertexes set with vertexes of the same label A_i and $\sum_{i=1}^{r} m_i = m$, $SE(B_j)=\{e_{j1},e_{j2},\ldots,e_{jnj}\}$ be an identical labeled edges set with edges of the same

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label B_j and $\sum_{j=1}^{s} n_j = n$. For labeled graph G, its incidence matrix M is an $m \times n$ matrix, where each row represents a vertex and each column represents an edge. The entry for row v and column e is 1 if e is incident with v and 0 otherwise. Namely, matrix element M_e^v can be defined as:

$$M_e^v = \begin{cases} 1 & e \text{ is incident with } v \\ 0 & e \text{ is not incident with } v \end{cases}.$$

The rows of the matrix are divided into several blocks each of which represents the rows of one identical row set. Similarly, the columns of the matrix are also divided into groups representing the identical column sets. Table Ⅰ shows a common format for incidence matrix of a labeled graph.

TABLE Ⅰ.
INCIDENCE MATRIX OF A LABELED GRAPH

	$e_{11} \cdots e_{1n1}$	$e_{21} \cdots e_{2n2}$	…	$e_{s1} \cdots e_{sns}$
v_{11} ⋮ v_{1m1}	$M_{11}^{11} \cdots M_{1n1}^{11}$ … $M_{11}^{1m1} \cdots M_{1n1}^{1m1}$	$M_{21}^{11} \cdots M_{2n2}^{11}$ … $M_{21}^{1m1} \cdots M_{2n2}^{1m1}$	⋱	$M_{s1}^{11} \cdots M_{sns}^{11}$ … $M_{s1}^{1m1} \cdots M_{sns}^{1m1}$
v_{21} ⋮ v_{2m2}	$M_{11}^{21} \cdots M_{1n1}^{21}$ … $M_{11}^{2m2} \cdots M_{1n1}^{2m2}$	$M_{21}^{21} \cdots M_{2n2}^{21}$ … $M_{21}^{2m2} \cdots M_{2n2}^{2m2}$	⋱	$M_{s1}^{21} \cdots M_{sns}^{21}$ … $M_{s1}^{2m2} \cdots M_{sns}^{2m2}$
⋮	⋱	⋱	⋱	⋱
v_{r1} ⋮ v_{rmr}	$M_{11}^{r1} \cdots M_{1n1}^{r1}$ … $M_{11}^{rmr} \cdots M_{1n1}^{rmr}$	$M_{21}^{r1} \cdots M_{2n2}^{r1}$ … $M_{21}^{rmr} \cdots M_{2n2}^{rmr}$	⋱	$M_{s1}^{r1} \cdots M_{sns}^{r1}$ … $M_{s1}^{rmr} \cdots M_{sns}^{rmr}$

For a connected labeled graph G, its incidence matrix is obviously not unique. Since the vertexes (or edges) sharing the same label could be arranged in different orders in the matrix, different forms of incidence matrix for G could be produced. In order to detect the isomorphism of the subgraphs, these incidence matrixes should be transformed into a normalized form which is unique for a connected labeled graph.

B The Algorithm for Incidence Matrix Normalization

Here, we propose an algorithm named IM-Norm to transform an incidence matrix of a labeled graph G into the normalized one. The input for the algorithm is an incidence matrix M of a k-edge candidate subgraph $g(G, k)$ in G, and the output is $g(G, k)$'s normalized incidence matrix MS. The algorithm consists of two steps, the first step performs normalization on the rows of the matrix (namely, graph's vertexes), and then the columns (i.e. graph's edges) normalization. In the row (column) normalization, rows (columns) are rearranged according to their row (column) labeled vectors and the binary numbers they represented.

Definition 7 For a vertex v_k (i.e. row k in the incident matrix) in identical vertex set $SV(A_i)$, its labeled vector is defined as $C_k=(c_{kb1}, c_{kb2}, \ldots, c_{kbs})$, where c_{kbj} is the number of edges in the identical labeled edges set $SE(B_j)$.

Definition 8 For an edge e_k (i.e. column k in the incident matrix) in identical edge set $SE(B_j)$, its labeled vector is defined as $D_k=(d_{ka1}, d_{ka2}, \ldots, d_{kar})$, where d_{kai} is the number of vertexes in the identical labeled vertexes set $SV(A_i)$.

Since each entry in the incidence matrix is a binary bit, each row (column) represents a binary number. We denote the binary number the kth row (column) represented as $rcode_k$ ($ccode_k$).

The algorithm consists of two steps each of which performs rows or columns normalization. In the rows normalization, the algorithm sorts the rows in an identical labeled row set according to their labeled vectors in lexicographic order $\prec$. However, if labeled vectors of two rows in the same identical row set are equivalent, they will be sorted according to the values of the binary numbers they represented.

The algorithm performs the column normalization in a similar way. Suppose labeled graph G has r identical labeled row sets and s identical labeled column sets, framework of the algorithm *IM-_Norm* is as follows:

Algorithm 1 *IM_Norm*
Input: Candidate subgraph's incidence matrix M
Output: Candidate subgraph's normalized incidence matrix MS

```
1:  for round=1 to 2 do
          /* two steps of rows & columns normalization*/
2:    for i= 1 to r do        /* rows normalization */
3:      for k=i_1 to i_mi do
              /* sort rows in row block of SV(A_i) */
4:        for j= k+1 to i_mi do
5:          if C_k ≺ C_j then
6:            exchange rows k and j
7:          else if (C_k = C_{k+1}) and
                    (rcode_k < rcode_{k+1}) then
8:              exchange rows k and j
9:          endif
10:       endif
11:     endfor j
12:   endfor k
13:  endfor i
14:  for i= 1 to s do      /* columns normalization */
15:    for k=j_1 to j_nj do
            /* sort columns in column block of SE(B_j) */
16:      for j= k+1 to j_nj do
17:        if D_k ≺ D_j then
18:          exchange columns k and j
           else if (D_k = D_{k+1}) and
19:                (ccode_k < ccode_{k+1}) then
20:          exchange columns k and j
21:        endif
22:      endif
23:    endfor j
```

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24: endfor *k*
25: endfor *i*
26: endfor *round*
27: return(*MS*)

The first part of IM_Norm (lines 2 ~ 13) is to normalize the rows (i.e. vertexes) of matrix *M* by arranging the rows sharing the same vertex label in lexicographic order. However, if labeled vectors of two rows' in the same identical row set are equivalent (lines 7 ~ line 9), they will be sorted according to the values of the binary numbers they represented.

The second part of *IM_Norm* (lines 14 ~ line 26) is to normalize all the columns (i.e. edges) of matrix *M* in a similar way.

The algorithm reiterates such row and column normalization once more to get the normalized form of the incidence matrix *MS* which will be used in graph isomorphism detection.

Fig. 2 shows the incidence matrix and its normalized form of the labeled graph of Fig. 1(a). The computational time complexity of algorithm *IM_Norm* is $O(m_1^2+m_2^2+m_3^2+\ldots+m_r^2)$, where m_i is the number of rows in the *i*th identical labeled row set. Similarly, time complexity of column normalization is $O(n_1^2+n_2^2+n_3^2+\ldots+n_s^2)$, where n_i is the number of columns in the *i*th identical labeled column set. Since $\sum_{i=1}^{r} m_i = m$, $\sum_{j=1}^{s} n_j = n$, and $m_1^2+m_2^2+m_3^2+\ldots+m_r^2 \leqslant m^2$, $n_1^2+n_2^2+n_3^2+\ldots+n_s^2 \leqslant n^2$, time complexity for algorithm *IM_Norm* is $O(m^2+ n^2)$.

	a_1	b_1	b_2	b_3	c_1
A_1	0	0	1	1	0
A_2	0	0	0	1	1
A_3	0	1	0	0	1
A_4	1	1	0	0	0
B_1	1	0	1	0	0

Normalization

	a_1	b_1	b_3	b_2	c_1
A_4	1	1	0	0	0
A_1	0	0	1	1	0
A_3	0	1	0	0	1
A_2	0	0	1	0	1
B_1	1	0	0	1	0

Figure 2. Incidence matrix normalization

Ⅳ. FREQUENT SUBGRAPH MINING ALGORITHM FSMA

The goal of algorithm *FSMA* is to find all the subgraphs which have a certain minimum support in a given graph data set *GDB*. *FSMA* uses the normalized incidence matrix to present the subgraphs. By scanning *GDB*, *FSMA* first finds all the frequent edges, which is also called 1-edge frequent subgraphs. Then the algorithm extends the 1-edge frequent subgraphs by adding frequent edges on them to get 2-edge frequent subgraphs. Repeating this procedure of subgraph extension until no more frequent subgraphs being generated. The advantage of *FSMA* is that it extends the frequent subgraphs by only adding the frequent edges instead of enumerating all the subgraphs in the graphs. This pruning technique can greatly reduce the time complexity.

Table Ⅱ lists the parameters used in *FSMA*.

TABLE Ⅱ.
PARAMETERS USED IN ALGORITHM FSMA

Parameter	Meaning
$F(k)$	number of k-edge frequent subgraphs
Ne	set of edges $e \notin g(G, k)$ but connected with subgraph $g(G, k)$
$TFG(G, k)$	set of k-edge candidate subgraphs in graph G
$TFGC(K)$	set of k-edge candidate subgraphs
$FG(G, K)$	set of k-edge frequent subgraphs in graph G
$FGS(GDB)$	set of 1-edge to k-edge frequent subgraphs
$gn(G, k)$	normalized incident matrix of a k-edge subgraph in graph G
$Freq\_gn(G, k)$	frequency of $gn(G, k)$

We use "$gn(G_i , k) \diamond e$" to denote the operation of extending the subgraph represented by $gn(G_i , k)$ with edge *e*. Framework of algorithm *FSMA* is as follows:

Algorithm 2 *FSMA*
Input: Graph data set $GDB=\{G_1, G_2, \ldots, G_n\}$, the minimum support threshold *minsup*
Output: Frequent subgraph data set $FGS(GDB)$

1: scan *GDB*, generate frequent edges; number of frequent edges is saved in F(1)
2: for *i*=1 to *u* do /* for each G_i in *GDB* */
3: delete unfrequent edges in G_i
4: $FG(G_i, 1) \leftarrow$ { frequent edges in G_i }
5: endfor
6: $k \leftarrow 1$
7: while $F(k)>0$ do
8: for *i*=1 to *n* do /* for each G_i in *GDB* */
9: $TFG(G_i, k+1)=\Phi$
10: for each *k*-edge frequent subgraph $gn(G_i, k) \in FG(G_i, k)$ do
11: Ne ← { set of edges $e \notin g(G, k)$ but connected with subgraph $g(G, k)$ }
12: for each *e* in *Ne* do
13: $gn(G_i, k+1)= gn(G_i , k) \diamond e$
14: if $gn(G_i, k+1) \notin$ in $TFG(G_i, k+1)$ then
15: $TFG(G_i, k+1)= TFG(G_i, k+1) \cup \{gn(G_i, k+1)\}$
16: endif
17: endfor *e*
18: endfor
19: endfor *i*
20: for *i*=1 to *n* do /* for each G_i in *GDB* */
21: for each $gn(G, k+1)$ in $TFG(G_i, k+1)$ do
22: if $gn(G, k+1) \notin TFGC(k+1)$ then
23: $TFGC(k+1)= TFGC(k+1) \cup \{gn(G, k+1)\}$
24: $Freq\_gn(G, k+1)= 1$
25: else
26: $Freq\_gn(G, k+1)=Freq\_gn(G, k+1)+1$
27: endif
28: endfor

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```
29:       endfor i
30:      F(k+1)=0;
31:      for each gn(G, k+1) in TFGC(k+1) do
32:         if Freq_gn(G, k+1)> minisup then
33:           FGS(GDB)= FGS(GDB) ∪ {gn(G, k+1)}
34:           F(k+1)= F(k+1)+1
35:         endif
36:      endfor
37:      for i=1 to n do   /* for each Gi in GDB */
38:        FG(G, k+1)=Φ
39:        for each gn(Gi, k+1) in TFGC(k+1) do
40:          if gn(Gi, k+1) ∈ FGS(GDB)
41:            FG(G, k+1)=FG(G, k+1) ∪ {gn(G, k+1)}
42:          endif
43:        endfor
44:      endfor i
45:      k←k+1
46:    wend
47:    delete non-maximum frequent subgraphs in
       FGS(GDB)
48:  Return (FGS(GDB))
```

The first part of *FSMA* (lines 1 ~ 5) is the initial stage of the algorithm. In this stage, the algorithm first scans *GDB* and generates frequent edges and computes the number of those frequent edges. For each graph G_i, the algorithm builds a set $FG(G_i, 1)$ consisting of all frequent edges in G_i. Lines 3 and 4 delete all unfrequent edges in each graph and update the graph data set. Lines 6 to 46 perform subgraph extension by progressively adding frequent edges on the existing frequent subgraphs. In each iteration of the extension procedure, a set of (k+1)-edge frequent subgraphs in graph G_i , which is named $FG(G, k+1)$, can be obtained based on $FG(G, k)$. In lines 8 to 19, all the (k+1)-edge candidate subgraphs in each graph G_i are generated by adding frequent edges on the k-edge frequent subgraphs in graph G_i . Line 13 generates a (k+1)-edge candidate subgraph $gn(G_i, k+1)$ by adding an edge e in Ne on k-edge frequent subgraph $gn(G_i , k)$. This extension operation "◇" is implemented by normalizing the incidence matrixes of the subgraphs using algorithm *IM_Norm.* All the k-edge candidate subgraphs of graph G_i are saved in set $TFG(G_i, k)$. In lines 20 to 29, the candidate subgraphs in $TFG(G_i, k)$, i= 1,2,…,n, are jointed into set $TFGC(k+1)$ and their frequencies are calculated. In lines 30 to 36, the frequent (k+1)-edge subgraphs are selected from $TFGC(k+1)$ and saved into the set $FGS(GDB)$. In the meanwhile, number of (k+1)-edge frequent subgraphs, denoted by $F(k+1)$, are calculated. Lines 37 to 46 construct a set $FG(G, k+1)$ for each graph G in *GDB* to record the (k+1)-edge frequent subgraphs in G . Set $FG(G, k+1)$ will be used to generate the (k+2)-edge candidate subgraphs in the next iteration. The iteration of subgraph extension will end when $F(k)$=0 which indicates no frequent subgraph with more edges could be produced. In lines 14, 22 and 40, isomorphism of subgraphs should be detected when searching a subgraph in the graph set. Since the algorithm uses the normalized incidence matrix, it can detect the isomorphism very efficiently. In the algorithm, since the frequent subgraphs are extended progressively from 1-edge subgraphs and their incident matrixes are normalized when they are generated by subgraph extension, it is not necessary to normalize each graph's incidence matrix in the initial stage.

Ⅴ. EXPERIMENT RESULTS AND PERFORMANCE ANALYZE

In this section, we report our experiments that validate the effectiveness and efficiency of our algorithm FSMA. We test algorithm FSMA on a 2.80GHz Intel Pentium Ⅳ PC with 256 MB main memory, running Windows XP Professional SP2. A comparative study also has been conducted between *FSMA* and *SLAGM* [22] on their performance on random graphs data set. Those graphs are generated under certain controlling parameters and are saved in the forms of incidence matrix and adjacency matrix respectively. Both *FSMA* and *SLAGM* are coded using VC++ 6.0. Table Ⅲ lists the parameters used in the experiments.

TABLE Ⅲ.
PARAMETERS IN THE EXPERIMENTS

Parameter	Meaning
$\|GDB\|$	Number of graphs in graph data set *GDB*
$\|T\|$	average number of edges
$\|V\|$	number of vertexes' labels
$\|E\|$	number of edges' labels

All the experimental results show that *FSMA* and *SLAGM* have the same quality of results but *FSMA* has obviously higher processing speed than *SLAGM*. Fig. 3, 4 and 5 show the runtime comparisons of two algorithms. In these experiments, the parameters are set as follows: in Fig. 3, $|GDB|$=200, $|T|$=20, $|V|$=17, $|E|$=11, in Fig. 4, $|GDB|$=200, *minsup*=10%, $|V|$=23, $|E|$=15, in Fig. 5, $|T|$=20, *minsup*=10%, $|V|$=19, $|E|$=13.

Fig. 3 shows runtime comparison between *SLAGM* and *FSMA* under different values of *minsup*. From Fig. 3, we can see that efficiency of *FSMA* is superior to *SLAGM*. When *minsup* decreases, *FSMA* is even more efficient. This is because *SLAGM* extends a (k+1)-vertex subgraph based on two k-vertex subgraphs that share the common (k-1)-vertex subgraphs, it consumes large amount of time to discover such pairs of k-vertex subgraphs. Since *FSMA* extends the subgraph directly by adding a frequent edge in each iteration, it requires much less computation time.

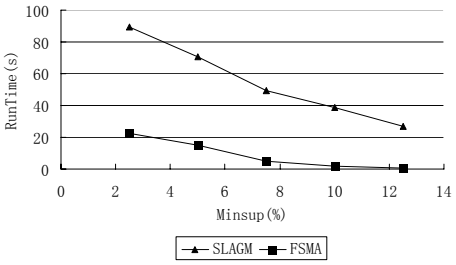

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Figure 3. Runtime comparison between SLAGM and FSMA under different minisup

Fig. 4 shows runtime comparison between *SLAGM* and *FSMA* under different graph scales |*T*| which is measured by the average number of edges of the graphs in *GDB*. From Fig. 4 we can see that *FSMA* is always faster than *SLAGM* under different |*T*| and *FSMA* is even more efficient when |*T*| increases. This is because subgraph extensions on large scaled graphs spend much more computation time in *SLAGM*.

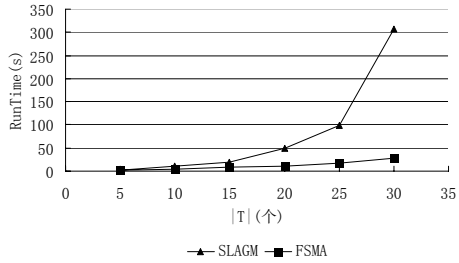

Figure 4. Runtime Comparison between SLAGM and FSMA under Different |T|

Fig. 5 shows runtime comparison between *SLAGM* and *FSMA* under different |*GDB*|, which is the umber of graphs in graph data set *GDB*. From Fig. 5, we can see that the speed by *FSMA* could be 10 ~ 12 times of that by *SLAGM* when the number of graphs increases.

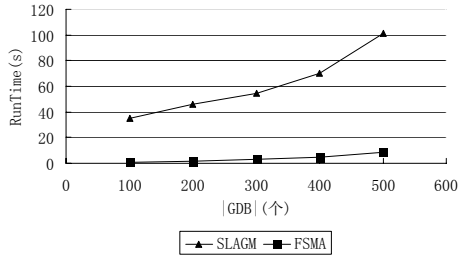

Figure 5. Runtime Comparison between SLAGM and FSMA under Different |GDB|

VI. CONCLUSIONS

An algorithm *FSMA* to find frequent subgraphs in large labeled graph database is presented. *FSMA* uses incidence matrix to represent the labeled graphs and to detect their isomorphism. Starting from the frequent edges from the graph database, the algorithm searches for the frequent subgraphs by progressively adding frequent edges on the existing frequent subgraphs. By normalizing the incidence matrix of the graph, the algorithm can effectively reduce the computational cost on verifying the isomorphism of the subgraphs. Experimental results show that the algorithm has higher speed and efficiency than that of other similar ones.

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