

Two-Dimensional Digital Filters with Variable Magnitude Characteristics Obtained from a 1-D Monotonic Response

Venkat Ramachandran,

Department of Electrical and Computer Engineering, Concordia University, Montreal, QC, Canada, H3G 1M8
E-mail: kamala@ece.concordia.ca

Christian S.Gargour

Department of Electrical and Computer Engineering, École Technologie Supérieure, Montreal, QC, Canada, H3C 1K3
E-mail: Christian.gargour@etsmtl.ca

Ravi P.Ramachandran

Department of Electrical and Computer Engineering, Rowan University, Glassboro, New Jersey, U.S.A. 08028
E-mail: ravi@rowan.edu

Abstract—A method for the generation of discrete-domain two-dimensional (2-D) transfer functions possessing variable magnitude and contour characteristics is presented in this paper. The proposed method is based upon a configuration constituted by two 1-D filters in cascade and a feedback loop. Each of these 1-D filter is designed to have a monotonic magnitude frequency response. This is obtained by performing one or several integrations, either with respect to ω or ω^2 , of the denominator of a magnitude Butterworth low-pass frequency response and obtaining the corresponding modified transfer function. The variable characteristics in each domain are obtained by changing a multiplier either in the forward path or in the feedback path of the proposed general configuration. The use of a generalized bilinear transformation (GBT) on the transfer functions obtained by the above mentioned method permits the generation of a large number of different characteristics. A certain number of these characteristics is examined in some detail. Illustrative examples are provided.

Index Terms—2-D discrete filters, variable magnitude.

I. INTRODUCTION

Various magnitude characteristics will be required in many aspects of signal processing like speech processing, image processing etc., There are different methods of obtaining such variable characteristics. One of them is to introduce a constant in the denominator, which can be varied [1-2]. However, this results in some disadvantages in the actual realization. There are other methods also [3-5]. However, it is advantageous to start with a known filter in one dimension with a feedback constant (which can be varied). This entire unit can be cascaded with another set in the other dimension. This permits us to vary the characteristics in each dimension separately and hence

the overall 2-D magnitude characteristics can be changed.

In this paper, we start with a known filter in the analog domain and apply the generalized bilinear transformation (GBT) [6] to get a discrete filter, whose characteristics can be changed by varying a constant in the feedback path.

II. THE STARTING CONFIGURATION

The starting configuration is shown either in Fig.1(a) or in Fig.1(b).

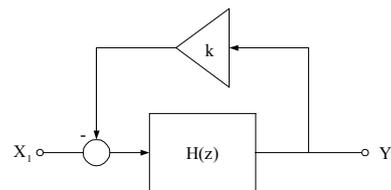


Fig.1(a)

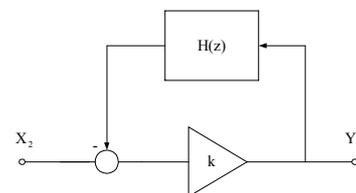


Fig.1(b)

where

$$H(z) = \frac{N(z)}{D(z)} \quad ..(1)$$

Analysis yields

$$\frac{Y_1(z)}{X_1(z)} = \frac{N(z)}{D(z) + kN(z)} \quad ..(2)$$

for the structure of Fig.1(a).

Similarly, for the structure of Fig.1(b), we have

$$\frac{Y_2(z)}{X_2(z)} = \frac{D(z)}{D(z) + kN(z)} \quad ..(3)$$

In both cases, the denominator remains the same. This means that, if we start with a given D(z), N(z) can be associated with it and the required range of values of k has to be computed so that the structure remains stable. It is tacitly assumed that when k = 0, the structure is always stable.

III. GENERATION OF D(z)

The polynomial D(z) can be generated in a large number of ways. It is intended to use a polynomial which gives a monotonic magnitude response. Here also, a number of possibilities exists [7-10]. However, we generate a second-order polynomial by the integration of a first-order Butterworth polynomial. Here also, there are two possibilities which are discussed below:

Category A: Consider

$$d_1(x) = x + 1 \quad ..(4a)$$

where $x = \omega^2 \quad ..(4b)$

Integrating $d_1(x)$ with respect to x once and choosing the constant of integration as unity so that the response at $\omega = 0$ to be unity, we have [11]

$$\int d_1(x) dx = \frac{x^2}{2} + x + 1 \quad ..(5)$$

Substituting (4b) in (5) along with $s = j\omega$ and factorizing, we have four roots with quadrantal symmetry. Selecting the roots in the left-half of the s-plane, the transfer function in the analog domain will be

$$T_{A1}(s) = \frac{1}{0.7071s^2 + 1.5537s + 1} \quad ..(6)$$

Category B: In this category, the integration is carried out twice both with respect to ω . Two integrations have to be carried out, because the highest degree should be of even degree. Specifically, starting with (4a) and (4b), the first integration yields

$$\int (\omega^2 + 1) d\omega = \frac{\omega^3}{3} + \omega \quad ..(7a)$$

The next stage of integration yields

$$\int \left(\frac{\omega^3}{3} + \omega \right) d\omega = \frac{\omega^4}{12} + \frac{\omega^2}{2} + 1 \quad ..(7b)$$

The constant '1' is added so that the response is normalized to unity at $\omega = 0$.

Proceeding as before, we get the transfer function in this case as

$$T_{B1}(s) = \frac{1}{0.2887s^2 + 1.038s + 1} \quad ..(8)$$

Table I gives the denominator polynomials obtained by the above method for both categories A and B, starting from Butterworth polynomials up to order 5. As can be observed, the starting polynomials could be different and such tables can be constructed for each case easily following the above approach.

Now, we can apply the Generalized Bilinear Transformation (GBT) [6] given by

$$s = \alpha \frac{z - a}{z + b} \quad ..(9)$$

If a low pass filter is needed, $b = 1$, in which case, the relationships $0 < a \leq 1$ and $\alpha > 1$ hold, in order that the resulting digital filter is stable. The denominator polynomial can be obtained and any numerator can be associated with it. The transfer function to be considered can be written as

TABLE I

| Starting Butterworth function (only the denominator polynomial is given) | The denominator polynomial obtained in Category A | The denominator polynomial obtained in Category B |
|--|---|--|
| $1 + \omega^2$ | $0.7071 s^2 + 1.5537 s + 1$ | $0.2887 s^2 + 1.038 s + 1$ |
| $1 + \omega^4$ | $0.5774 s^3 + 1.5243 s^2 + 2.0121 s + 1$ | $0.1826 s^3 + 0.7116 s^2 + 1.3868 s + 1$ |
| $1 + \omega^6$ | $0.5 s^4 + 1.6459 s^3 + 2.7092 s^2 + 2.5334 s + 1$ | $0.1336 s^4 + 0.6109 s^3 + 1.3964 s^2 + 1.8146 s + 1$ |
| $1 + \omega^8$ | $0.4472 s^5 + 1.7395 s^4 + 3.3831 s^3 + 4.0433 s^2 + 2.9303 s + 1$ | $0.1054 s^5 + 0.5556 s^4 + 1.4640 s^3 + 2.3626 s^2 + 2.2859 s + 1$ |
| $1 + \omega^{10}$ | $0.4082 s^6 + 1.8914 s^5 + 4.3816 s^4 + 6.4059 s^3 + 6.1656 s^2 + 3.6512 s + 1$ | $0.08704 s^6 + 0.5195 s^5 + 1.5502 s^4 + 2.9210 s^3 + 3.6294 s^2 + 2.7855 s + 1$ |

$$H_d(z) = \frac{N_d(z)}{D_d(z)} \quad ..(10)$$

For the purposes of this paper, only the second-order polynomial in either of the categories A or B will be used and the numerator $N(z)$ as the function $LP_{00} = 1$ which is based on Switched-Capacitor filter low pass classification [12]. Other classifications can be used giving other possibilities. Therefore, we can write

$$D(z) = e_{21} z^2 + e_{11} z + e_{10} \quad ..(11)$$

When such a filter is utilized in either configuration, the value of 'k' required in order to ensure stability is governed by the following two inequalities [13]:

$$\left| \frac{e_{10} + k}{e_{21}} \right| < 1 \quad ..(12a)$$

and
$$\left| \frac{e_{11}}{e_{21} + e_{10} + k} \right| < 1 \quad ..(12b)$$

As can be seen, even with a small class of functions considered, a very large number of possibilities exist. In the next two sections, we shall consider some typical 2-D responses.

IV. SOME TYPICAL 2-D RESPONSES OBTAINED FROM THE SECOND-DEGREE POLYNOMIAL OBTAINED IN CATEGORY A

As remarked earlier, numerable possibilities exist, because when one unit is a function of z_1 and the other unit is a function of z_2 and they are cascaded together. Some representative responses are given below: In this section, we shall consider only second-order functions obtained in Category A. After the application of the GBT, the transfer function to be considered can be written as:

$$H_{A2}(z) = \frac{1}{D_{A2}(z_1, z_2)} \quad ..(13)$$

where

$$D_{A2}(z_1, z_2) = \begin{pmatrix} e_{21}z_1^2 + e_{11}z_1 + e_{01} + k_1 \\ e_{22}z_2^2 + e_{12}z_2 + e_{02} + k_2 \end{pmatrix} \quad (14)$$

with

$$\begin{aligned} e_{21} &= (0.7071\alpha_1^2 + 1.5337\alpha_1 + 1) \\ e_{11} &= -1.4142\alpha_1^2 a_1 + 1.5337\alpha_1 (1 - a_1) + 2 \\ e_{01} &= (0.7071\alpha_1^2 a_1^2 - 1.5337\alpha_1 a_1 + 1) \end{aligned}$$

$$\begin{aligned} e_{22} &= (0.7071\alpha_2^2 + 1.5337\alpha_2 + 1) \\ e_{12} &= -1.4142\alpha_2^2 a_2 + 1.5337\alpha_2 (1 - a_2) + 2 \\ e_{02} &= (0.7071\alpha_2^2 a_2^2 - 1.5337\alpha_2 a_2 + 1) \end{aligned}$$

Case (A1): Let $\alpha_1 = 1, a_1 = 1, \alpha_2 = 1$ and $a_2 = 1$. The case corresponds to the well-known bilinear transformations. The overall transfer function is

$$\left(\frac{1}{3.2607z_1^2 + 0.5858z_1 + (0.1534 + k_1)} \right) \cdot \left(\frac{1}{3.2607z_2^2 + 0.5858z_2 + (0.1534 + k_2)} \right) \quad ..(15)$$

The limits for k_1 and k_2 are obtained as

$$-2.9315 < [k_1, k_2] < 3.1073 \quad ..(16)$$

Figs. 2(a), 2(b) and 2(c) give the magnitude responses and the contour plots for the cases (a) $k_1 = k_2 = 1$, (b) $k_1 = k_2 - 1$ and (c) $k_1 = -k_2 = 1$.

Case (A2): Let $\alpha_1 = 2, a_1 = 0.5, \alpha_2 = 2$ and $a_2 = 0.5$. The overall transfer function of the 2-D filter as

$$\left(\frac{1}{6.9358z_1^2 + 0.7253z_1 + (0.1534 + k_1)} \right) \cdot \left(\frac{1}{6.9358z_2^2 + 0.7253z_2 + (0.1534 + k_2)} \right) \quad (17)$$

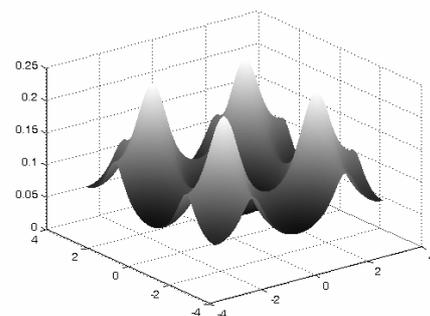
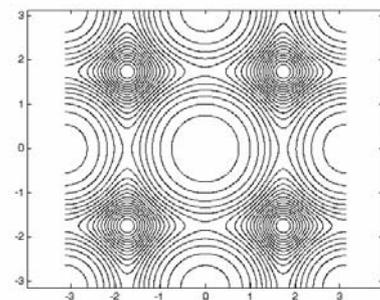


Fig.2(a) : Magnitude and Contour characteristics when $k_1 = 1, k_2 = 1$ for the transfer function given in (15)

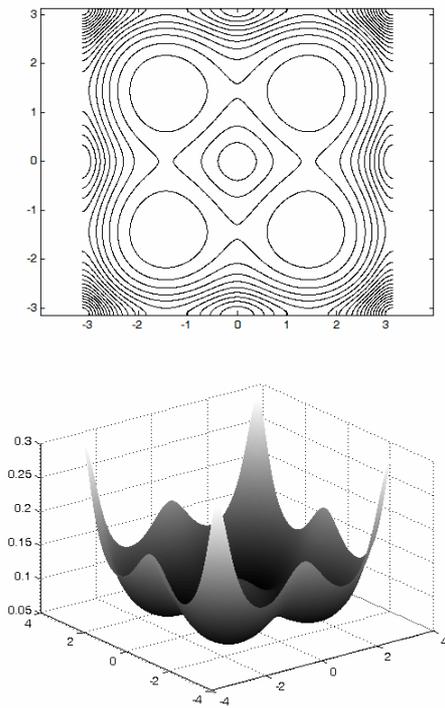


Fig.2(b) : Magnitude and Contour characteristics when $k_1 = -1, k_2 = -1$ for the transfer function given in (15)

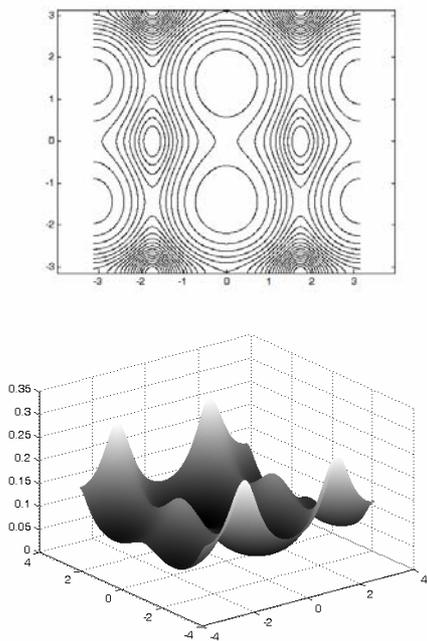


Fig.2(c) : Magnitude and Contour characteristics when $k_1 = 1, k_2 = -1$ for the transfer function given in (15)

The limits for k_1 and k_2 are obtained as

$$-6.3639 < [k_1, k_2] < 6.7824 \quad \dots(18)$$

Figs. 3(a), 3(b) and 3(c) give the magnitude responses and the contour plots for the cases (a) $k_1 = k_2 = 2$, (b) $k_1 = k_2 = -2$ and (c) $k_1 = -k_2 = 3$.

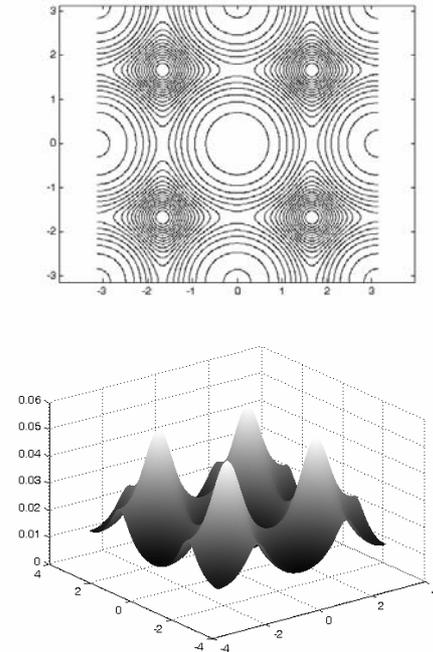


Fig.3(a) : Magnitude and Contour characteristics when $k_1 = 2, k_2 = 2$ for the transfer function given in (17)

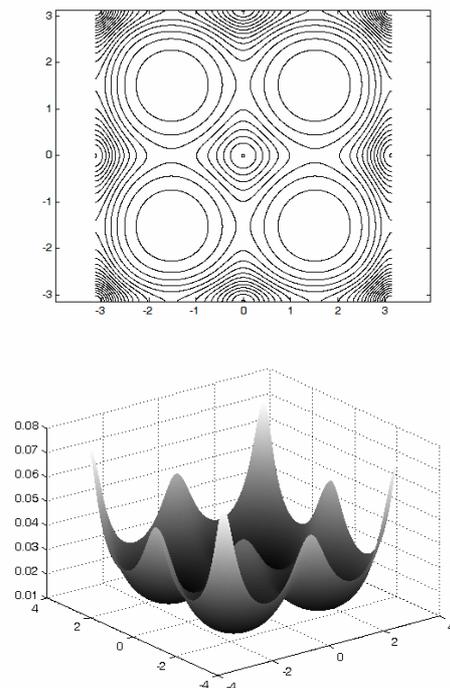


Fig.3(b) : Magnitude and Contour characteristics when $k_1 = -3, k_2 = -3$ for the transfer function given in (17)

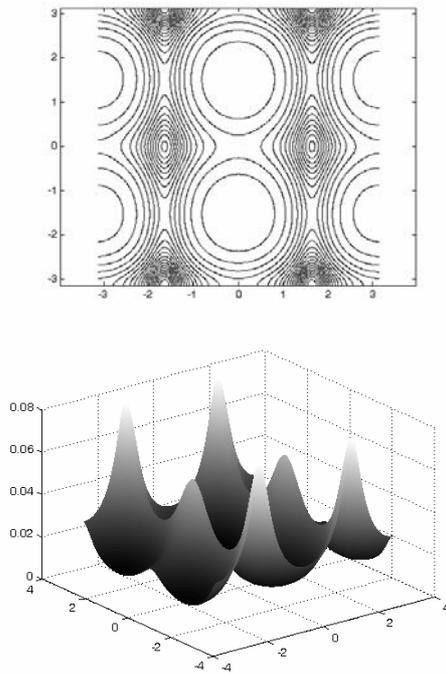


Fig.3(c) : Magnitude and Contour characteristics when $k_1 = 3, k_2 = -3$ for the transfer function given in (17)

V. SOME TYPICAL 2-D RESPONSES OBTAINED FROM THE SECOND-DEGREE POLYNOMIAL OBTAINED IN CATEGORY B

As remarked earlier, numerable possibilities exist, because when one unit is a function of z_1 and the other unit is a function of z_2 and they are cascaded together. Some representative responses are given below: In this section, we shall consider only second-order functions obtained in Category B. After the application of the GBT, the transfer function to be considered can be written as:

$$H_{B2}(z) = \frac{1}{D_{B2}(z_1, z_2)} \quad ..(19)$$

where

$$D_{B2}(z_1, z_2) = \begin{pmatrix} e_{21}z_1^2 + e_{11}z_1 + e_{01} + k_1 \\ e_{22}z_2^2 + e_{12}z_2 + e_{02} + k_2 \end{pmatrix} \quad ..(20)$$

with

$$\begin{aligned} e_{21} &= (0.2887\alpha_1^2 + 1.038\alpha_1 + 1) \\ e_{11} &= -0.5774\alpha_1^2 a_1 + 1.038\alpha_1 a_1 + 2 \\ e_{01} &= (0.2887\alpha_1^2 a_1^2 - 1.038\alpha_1 a_1 + 1) \end{aligned}$$

$$\begin{aligned} e_{22} &= (0.2887\alpha_2^2 + 1.038\alpha_2 + 1) \\ e_{12} &= -0.5774\alpha_2^2 a_2 + 1.038\alpha_2(1 - a_2) + 2 \\ e_{02} &= (0.2887\alpha_2^2 a_2^2 - 1.038\alpha_2 a_2 + 1) \end{aligned}$$

Case (B1): Let $\alpha_1 = 1, a_1 = 1, \alpha_2 = 1$ and $a_2 = 1$. The case corresponds to the well-known bilinear transformations. The overall transfer function of the 2-D filter as

$$\left(\frac{1}{2.3267z_1^2 + 1.4226z_1 + (0.2507 + k_1)} \right) \left(\frac{1}{2.3267z_2^2 + 1.4226z_2 + (0.2507 + k_2)} \right) \quad (21)$$

The limits for k_1 and k_2 are obtained as

$$-1.1548 < [k_1, k_2] < 2.0760 \quad ..(22)$$

Figs. 4(a), 4(b) and 4(c) give the magnitude responses and the contour plots for the cases (a) $k_1 = k_2 = 0.5$ b) $k_1 = k_2 = 0.5$ and (c) $k_1 = -k_2 = 0.5$.

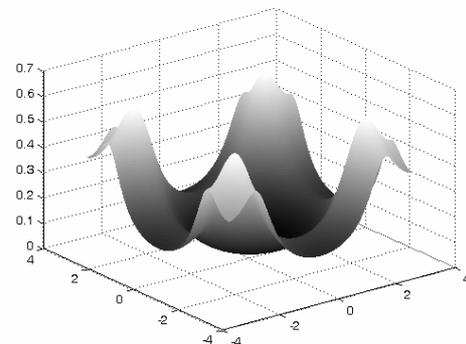
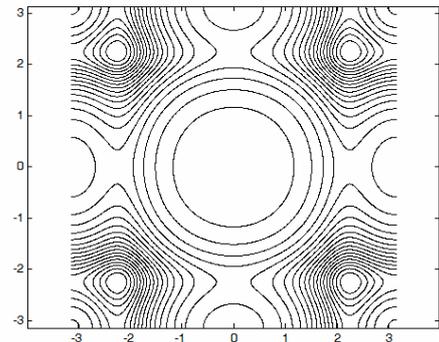


Fig.4(a) : Magnitude and Contour characteristics when $k_1 = 0.5, k_2 = 0.5$ for the transfer function given in (21)

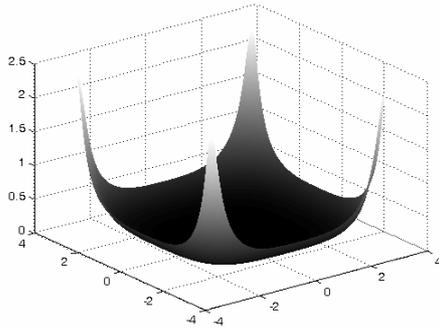
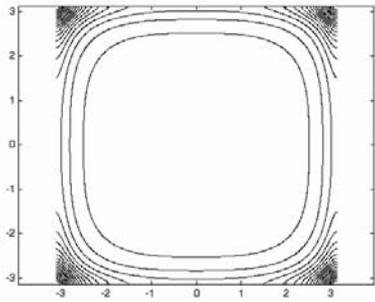


Fig.4(b) : Magnitude and Contour characteristics when $k_1 = -0.5, k_2 = -0.5$ for the transfer function given in (21)

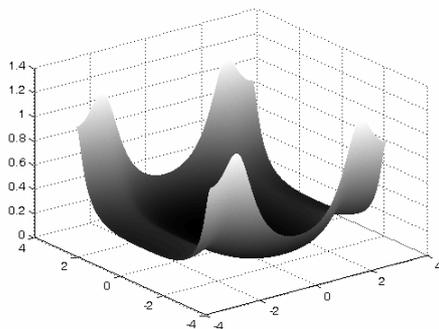
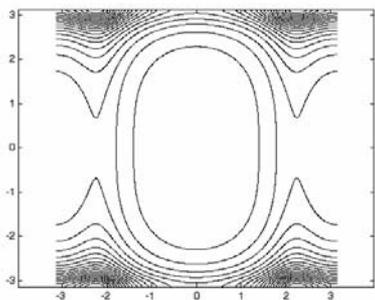


Fig.4(c) : Magnitude and Contour characteristics when $k_1 = 0.5, k_2 = -0.5$ for the transfer function given in (21)

Case (B2): Let $\alpha_1 = 2, a_1 = 0.5, \alpha_2 = 2$ and $a_2 = 0.5$. The overall transfer function of the 2-D filter as

$$\left(\frac{1}{4.2308z_1^2 + 1.3642z_1 + (0.2507 + k_1)} \right) \left(\frac{1}{4.2308z_2^2 + 1.3642z_2 + (0.2507 + k_2)} \right) \quad (23)$$

The limits for k_1 and k_2 are obtained as

$$-3.1173 < [k_1, k_2] < 3.9801 \quad ..(24)$$

Figs. 5(a), 5(b) and 5(c) give the magnitude responses and the contour plots for the cases (a) $k_1 = k_2 = 0.5$, (b) $k_1 = k_2 = -0.5$ and (c) $k_1 = -k_2 = 0.5$.

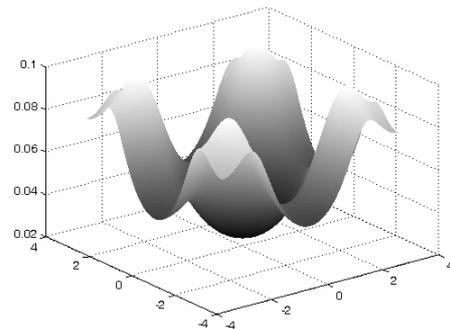
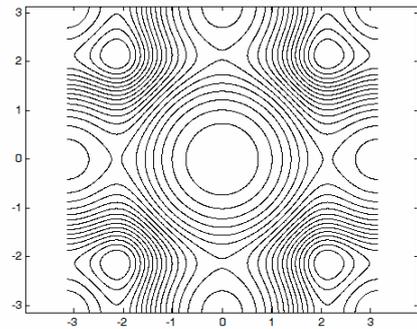


Fig.5(a) : Magnitude and Contour characteristics when $k_1 = 0.5, k_2 = 0.5$ for the transfer function given in (23)

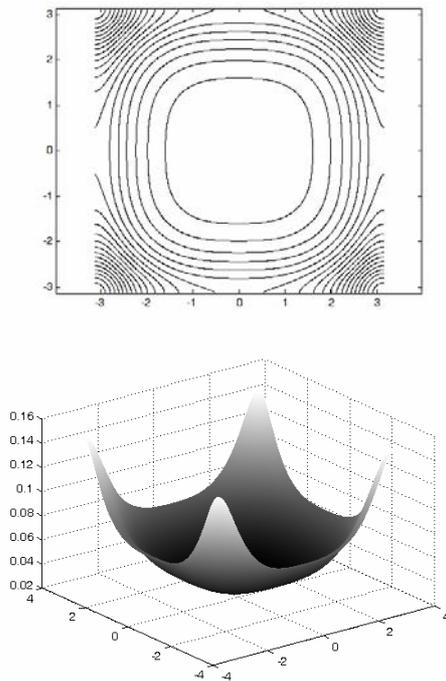


Fig.5(b) : Magnitude and Contour characteristics when $k_1 = -0.5$, $k_2 = -0.5$ for the transfer function given in (23)

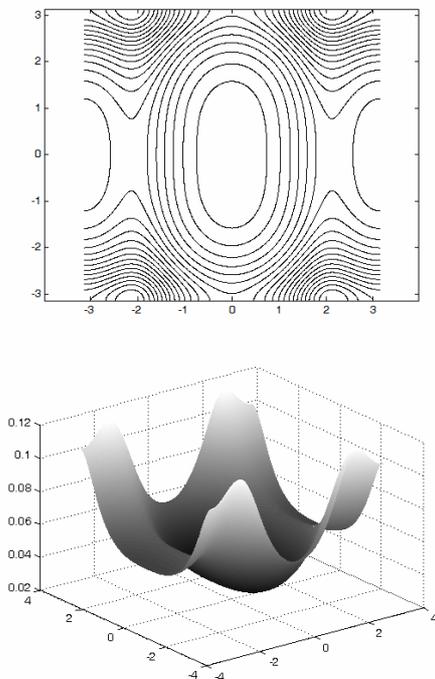


Fig.5(c) : Magnitude and Contour characteristics when $k_1 = 0.5$, $k_2 = -0.5$ for the transfer function given in (23)

VI. SUMMARY AND DISCUSSIONS

Various 2-D filters with different magnitude characteristics have been obtained by connecting two 1-D filters in cascade with a feedback multiplier loop. The characteristics of each of these 1-D filters have been varied independently of the other one. Each starting 1-D filter has been generated such as to have a monotonic magnitude frequency response obtained by the integration of the denominator of the frequency magnitude response of a low-order Butterworth transfer function, called the generating function. Any other type of filter characteristics could be considered also. There are two different possibilities to generate the required two 1-D filters: (a) one of them is obtained by integrating the starting polynomial with respect to ω^2 , and (b) the other one is obtained by double integration of the starting polynomial with respect to ω . Further integrations can be carried out and the two categories can be intermixed in any fashion. By the use of the generalized bilinear transformation, two more variables can be introduced, in addition to the feedback factor for the generation of LP filters. These two parameters together with the feedback factor can generate an infinite number of possibilities in the shapes of the resulting 2-D filter frequency responses. From the given examples (though small in number), the variations in the magnitude characteristics are readily noticeable. It is readily concluded that the nature of the starting filter, the feedback factor and the variables in the generalized bilinear transformations yield different characteristics. It is also observed that the two 1-D filters need not be identical. The only precaution to be taken is that the feedback factor and the variables of the GBT have to be kept within prescribed limits so that the stability is guaranteed. In this paper, only second order factors have been considered. Higher orders could also be considered and appropriate stability conditions obtained. It is also noted that, even though we started with monotonic responses in both the filters, the overall 2-D characteristics need not be monotonic in character. The conditions which ensure the maintenance of the 1-D monotonic frequency response in the 2-D domain requires a separate study.

REFERENCES

- [1] V.Ramachandran, C.S.Gargour and G.Bogdadi, A unified approach for the design of stable 1-D analog and digital filters having variable magnitude characteristics, *Pacific Rim Conference on Communications, Computer and Signal Processing*, Victoria, B.C., Canada, June 1-2, 1999, pp.178-180.
- [2] V.Ramachandran and C.S.Gargour, Variable magnitude 2-D low pass filters having non-separable denominators generated from separable functions, *IEEE International Symposium on Circuits and Systems*, Singapore, June 1991, pp.2963-2966.
- [3] C.S.Gargour and V.Ramachandran, Properties of a class of third-order two-parameter variable magnitude 2-D transfer function, *43rd Midwest symposium on Circuits*

and Systems, New Mexico State university, August 1999, pp.109-112.

- [4] V.Ramachandran and C.S.Gargour, Combination of two 2-D all-pass filters with guaranteed no-delay free-loop producing variable magnitude and contour characteristics, *World Automation Congress*, Maui Islands, Hawaii, June 11-16, 2000, Session MoA2 (Five pages).
- [5] V.Ramachandran, C.S.Gargour and Ravi P.Ramachandran, Generation of a class of two-dimensional 2-D transfer functions yielding variable magnitude and contour characteristics, *IEEE International Symposium on Circuits and Systems*, Sydney, Australia, May 2001, pp.II-797 to II-800.
- [6] C.S.Gargour, V.Ramachandran, Ravi P.Ramachandran and F.Awad, Variable magnitude characteristics of 1-D IIR discrete filters by a generalized bilinear transformation, *43rd Midwest Symposium on Circuits and Systems*, Michigan state university, Michigan, U.S.A., August 8-11, 2000, (Four pages), Session FAP-2.
- [7] I.M.Filanovsky, A generalization of filters with monotonic amplitude frequency response, *IEEE Transactions on Circuits and Systems – I*, Vo.46, No.11, pp.1382-1385, November 1999.
- [8] M.Fukada, Optimum filters of even orders with monotonic response, *IRE Transactions on Circuit Theory*, Vol.CT-6, pp.277-281, 1959.
- [9] W.Selenick and S.C.Burrus, Generalized digital Butterworth filter design, *IEEE Transactions on Signal Processing*, Vol.46, Issue 6, pp.1688-1694, June 1998.
- [10] Ajit Singh Sandhu, Generation of 2-D analog and digital low-pass, high-pass and band-pass filters with monotonic amplitude-frequency response, M.A.Sc.Thesis, April 2005, Concordia University, Montreal, Quebec, Canada.
- [11] V.Ramachandran, C.S.Gargour and Ravi P.Ramachandran, Generation of analog and digital transfer functions having a monotonic magnitude response, *IEEE Canadian Conference on Electrical and Computer Engineering*, Niagara Falls, Canada, May 3-5, 2004 (Four pages).
- [12] P.E.Fleischer and K.R.Laker, A family of active switched-capacitor biquad building blocks, Vol.58, pp.2235-2269, December 1979.
- [13] C.S.Gargour and V.Ramachandran, Implementation of a stability test of 1-D discrete system based on Schussler's theorem and some coefficient conditions, *Journal of Franklin Institute*, Vol.317, No.5, pp.341-358, May 1984.

(The above references contain other references which are not listed here)

Venkat Ramachandran obtained his Ph.D. degree from Indian Institute of Science, Bangalore, India. He is presently a Professor in Concordia University, Montreal, Canada. He is a Fellow of IEEE and is the recipient of a number of teaching excellence awards. He also got the Outstanding Engineering Educator Award from IEEE, Canada. He is author or coauthor of six textbooks and has contributed state-of-the-art articles in multidimensional systems. He has coauthored a research monograph entitled "Relative efficiencies of some Indian languages (A study from information theory point of view)". He received the Myrial B.Reed Best research paper award from the Midwest Symposium on Circuits and Systems. In addition, he has received awards from American Society for Engineering education.

Christian S. Gargour obtained his doctorate degree from Concordia University, Montreal, Canada. He has taught in several institutions and is currently a Professor with the Electrical Engineering Department of École de technologie supérieure (ETS), University of Quebec, Montreal. He is a Senior Member of the IEEE and a Fellow of the IET (IEE). He is a recipient of the Myril B.Reed Best Paper Award in 1985. His research interests include analog and digital signal processing, filters, wavelets and their applications. The results of his research work have appeared in the form of numerous publications and conferences. He has coauthored two books.

Ravi P.Ramachandran received the B.Eng degree (with great distinction) from Concordia University in 1984, the M.Eng degree in 1986 and the Ph.D.degree in 1990, both from McGill University, Montreal. From October 1990 to December 1992, he worked at the Speech research department at AT & T Bell Laboratories. From January 1993 to August 1997, he was a Research Assistant Professor at Rutgers university. He was also a senior speech scientist at T-Netrix from July 1996 to August 1997. Since September 1997, he is with the Department of Electrical and Computer Engineering at Rowan University, where he has been a Professor since September 2006. He has served as a consultant to T-Netrix, Avenir Inc. and Motorola. From Sept.2002 to Sept.2005, he was an Associate Editor for the IEEE Trans. On Speech and Audio Processing and was on the Speech technical Committee for the ICASSP Conference. Since Sept.2000, he has been on the Editorial Board of the IEEE Circuits and Systems Magazine. Since May 2002, he has been on the Digital Signal Processing Technical Committee for the ISCAS Conference. His research interests are in digital signal processing, speech processing, pattern recognition and filter design.