Image Encryption Based on Permutation and Substitution Using Clifford Chaotic System and Logistic Map

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Manuscript submitted March 21, 2017; accepted June 25, 2017.

doi: 10.17706/jcp.13.3.309-326

Abstract: Chaos theory is a field of study in mathematics that studies the behavior of dynamical systems that are highly sensitive to initial conditions. Many algorithms have been proposed for image encryption, but the chaotic encryption methods have a good combination of speed and high security. The key space for a good encryption scheme should be big enough to resist brute-force attacks. In this paper, first we investigate Clifford chaotic system for image encryption. In order to optimize the encryption system we increase the space key by using logistic map. The security and performance of the proposed method are analyzed by using the key space, entropy, statistical, differential and performance analysis. It can be concluded that the proposed image encryption system is suitable choice for practical applications.

Key words: Image encryption, chaos, clifford chaotic system, logistic map.

1. Introduction

Chaotic maps have been used in data encryption recently, because chaos is sensitive to initial conditions and control parameters, pseudo randomness and ergodicity [1], [2]. In the late 1980s, Matthews firstly used chaos to encrypt information and produced the key stream based on logistic map [3].

Habutsu *et al* proposed the first chaotic block cipher algorithm in 1991 [4]. Baptista [5] published a paper about chaotic encryption algorithm in 1998. Some of the 1D chaotic maps have been used in data encryption. 2D or higher dimensional chaotic maps are usually employed for image encryption.

In 1998, Friedrich has suggested that an image encryption system need to be repeated in two steps: diffusion and confusion [6], [7]. The chaos-based encryption schemes are composed of two processes generally: chaotic confusion of pixel positions by permutation process and diffusion of pixel grey values by a diffusion process. The pixels of plain image permute with a 2D chaotic map, while the latter changes the pixel values of each pixel one by one so that a small change for one pixel can spread out to almost all pixels in the whole image.

In 2004, Chen *et al* and Mao *et al* employed 3D chaotic Cat map and Baker map in permutation step of image encryption [8], [9].

Guam *et al* used a 2D Cat map and Chen's chaotic system for pixel position permutation and pixel value masking, in 2005 respectively [1]. Giesl [10] proposed an image encryption system using strange attractor in 2009. Today, because of the properties of chaotic systems, such as sensitivity to initial conditions and control parameters, pseudo randomness and ergodicity, chaos becomes popular in image encryption (Lian *et al*

(2005) [11], Patidar *et al.* (2010) [12], Wang and Lie (2010)[13], Wang *et al.* (2012) [14], Wang and Guo (2014)[15], Hua *et al.* (2015) [16]).

Therefore, in order to improve the image encryption in [10] we proposed a new algorithm for image encryption using Clifford attractor and logistic map. This algorithm consists of two parts: the logistic map shuffle of the pixel position (x_0, y_0) and the pixel value $z_0 = P(x_0, y_0)$. We use k_2 times for shuffling the pixel position and value. Then we generate (x_k, y_k) and z_k .

The rest of the paper is arranged as follows: section 2 describes the 1D logistic map. Section 3 constructs a new image encryption algorithm based on Clifford attractor and logistic map. The experimental results, analysis and comparison of the proposed image encryption system are shown in section 4, and finally section 5 concludes the paper.

2. Logistic Map

It is now widely understood that complex behavior in nature may have a simple underlying cause. Several examples of low-dimensional nonlinear maps have been extensively studied [5], [17], [18], [19]. On the other hand, many familiar processes are known to have regular periodic solutions. One of the most studied examples of a one-dimensional system capable including chaos is the logistic map [2], [20]

$$x_{n+1} = r x_n (1 - x_n) \quad (1)$$

where r is the control parameter.

Control parameter r has a crucial role in behavior of the map and we can recognize the qualitative changes in the dynamics of map by varying the value r .

Fig. 1. First 100 points of logistic map with $x_0 = 0.4$ and different values of r (a) $r = 0.95$, (b) $r = 2.95$, (c) $r = 3.5$, (d) $r = 3.9$.

Upon repeated iteration, the solution will be one of the following cases depending upon the initial condition and its value: (1) it will converge to a stable fixed point (a point attractor), (2) it will converge to a periodic series of distinct values (a limit cycle), (3) it will yield a non periodic series of values within some bounded range of X (a chaotic strange attractor), or (4) it will diverge (attract to infinity).

Fig. 2. (a) Behavior of logistic map with $x_0 = 0.4$ and $r = 0.95$, (b) The bifurcation diagram for logistic map.

A feature of a chaotic system is sensitivity to initial conditions. If two trajectories which start off close to each other deviate more and more within creasing time, the system is said to be chaotic. The rate at which nearby trajectories deviate from each other with time is characterized by a quantity called the Lyapunov exponent. The Lyapunov exponent for the discrete chaotic system can be described as below

$$h = \lim_{n \rightarrow \infty} \frac{1}{n} \ln |f'(x)| \quad (2)$$

where the parameter n is the length of the sequence and f is the chaos map. If $h > 0$, it means that the chaotic system is currently chaotic with the certain condition. If the chaotic system is not chaotic under current condition.

Fig. 3. Lyapunov exponent for logistic map.

For this case, the stable solutions are all in the range of $3.5 \leq r \leq 4$, and it is only within this range that the logistic equation represents a candidate model for a real physical process [21].

3. Methodology

Strange attractors [2], [10], [22]-[25] are complicated sets with a fractal structure to which chaotic dynamical systems evolve after enough long time. These attractors can be generated in several ways.

In general, a map is said to be dissipative if it does not preserve phase space volumes on each iterate. Therefore, the bounded region on the phase place will certainly shrink for the reason that map can be iterated and this is associated with the presence of an attractor within that region.

The proposed image encryption is a modification of the one suggested by Giesl in [10]. The algorithm in [10] does not create any encryption key, but uses attractor map for pixel permutation and diffusion directly. Parameters of attractor map play the role of encryption keys here and key space is very large due to their non-integer character. Chaos-based encryption has been done by means of so called Clifford system (attractor) [20],

$$\begin{aligned}x_{n+1} &= \sin(ay_n) + c \cos(ax_n) \\ y_{n+1} &= \sin(bx_n) + d \cos(by_n)\end{aligned}\quad (3)$$

$$\begin{aligned}x_{n+1} &= \sin(ay_n) + c \cos(ax_n) \\ y_{n+1} &= \sin(bx_n) + d \cos(by_n) \\ z_{n+1} &= \sin(ey_n) + f \cos(ez_n)\end{aligned}\quad (4)$$

Fig. 4. Clifford attractor based on (4).

Implementation of image encryption is in two steps: the first step is image scrambling that the position of the pixels in the image is changed and the second step is pixel substitution. An image can be described by the position and the pixel value. This method can change the position and value of the image pixel and it converts the image from a plain digital image into a noise-polluted image.

Pixel substitution is implemented by XOR or other operation with other sequence or matrix to change plain image pixel value. The main parameters of the Clifford attractor are keys for the process of encryption. Pixel of image is used as the initial value of Clifford system. Equation in (4) is 3D extension which can be used for encryption of coordinates (pixel position) (x_0, y_0) and the value of each pixel $P(x_0, y_0)$. New positions and modification value are gained after iterations and quantization. These positions are then used for pixel permutation and the modification value is XOR operated with original pixel value and the value of the previous pixel and encrypted pixel is gained this way [10]. According to Fig. 5, the previous algorithm was changed as follows:

Fig. 5. Flowchart of encryption process suggested in [10].

Fig. 6. Flowchart of proposed encryption process.

In this paper, encryption with Clifford chaotic system and the proposed Clifford chaotic system have been tested. The tested “message” for encryption is picture. We use the trigonometric strange attractor which is

described by (4). In the new algorithm, first coordinates (x_0, y_0) and value $P(x_0, y_0)$ of pixels put into a Clifford system. Next Logistic map iterate k_2 times with r_1, r_2, r_3 parameters. After quantization step, positions and modification value are gained. Encryption system contains 7 steps as follows:

$$P(x_0, y_0) \leftrightarrow P(x_k, y_k) \oplus z_k \oplus Q$$

Pseudo code implementation of Proposed image encryption

```

Initialization ( a,b,c,d,e,f,k,r1,r2,r3,lgst_round)
Pic ← input_image
r_im ← pic.row
c_im ← pic.column
for i → r_im do
    for j → c_im do
        x ← i
        y ← j
        z ← pic [i] [j]
        for n → k do
            for p → lgst_round do
                x ← r1 * x * (1-x)
                y ← r2 * y * (1-y)
                z ← r3 * z * (1-z)
            end for
            xk [i] [j] ← sin (a * y) + c * cos (a * x)
            yk [i] [j] ← sin (b * x) + d * cos (b * y)
            zk [i] [j] ← sin (e * y) + f * cos (e * z)
        end for
    end for
end for
Q ← 0
for i → r_im do
    for j → c_im do
        x_step ← minmax ( xk) / ( r_im -1)
        y_step ← minmax ( yk) / ( c_im -1)
        z_step ← minmax ( zk) / ( 256)
        Xk ← xk [i] [j] - min (xk) / x_step
        Yk ← yk [i] [j] - min (yk) / y_step
        Zk ← zk [i] [j] - min (zk) / z_step
        Q_new ← XOR (pic [ Xk[i] [j] ] [ Yk [i] [j] ], Zk [i] [j] )
        Q_new ← XOR ( Q_new , Q )
        New_pic [i] [j] ← Q_new
        Q ← Q_new
    end for
end for

```

Step1. Key generation: Select parameters a, b, c, d, e, f of Clifford attractor and r_1, r_2, r_3, k_2 of Logistic map.

Step2. Image conversion matrix: Converts the tested image to a matrix. For grayscale images, the dimension of matrix is. n is the height of the image while m is the width. A color image can be converted into grayscale image.

Step3. Initial selection: as the initial process of encryption, select a pixel position (x_0, y_0) and a pixel value $z_0 = P(x_0, y_0)$.

Step4. New Clifford system: perform Logistic map and Clifford stranger to disorder the position and value pixel of image. Put x_0, y_0 and $z_0 = P(x_0, y_0)$ into the Logistic map with k_2 iterations.

$$x_0 = r_1 x_0 (1 - x_0)$$

$$y_0 = r_1 y_0 (1 - y_0)$$

$$z_0 = r_1 z_0 (1 - z_0)$$

Then Clifford attractor is used to shuffle pixel position and pixel value. The obtained x_0, y_0, z_0 are the initial values for Clifford equation.

$$x_k = \sin(ay_0) + c \cos(ax_0)$$

$$y_k = \sin(bx_0) + d \cos(by_0)$$

$$z_k = \sin(ey_0) + f \cos(ez_0)$$

Set $x_0 = x_k, y_0 = y_k, z_0 = z_k$ and iterate the above equation k times. The original pixel at coordinates (x_0, y_0) is swapped with the pixel at coordinates.

Step5. Reshaping matrices: Convert 2D matrices x_k, y_k, z_k into 1D matrices.

Step6. Quantization: For the obtained vectors in step 5, determine minimum and maximum value of vectors. Then the range is divided between the minimum and maximum.

Step7. Changing values of pixels: Pixel value at coordinates (x_k, y_k) is modified by XOR operation of value z_k and then XOR operated with the value of the previous processed pixel Q. This process must be done for every pixel in the image.

4. Simulation Results and Statistical Analysis

The following tests are realized by MATLAB software on Intel Core (TM) i7, 2.2 GHz PC.

$$a = -1.85, b = 1.48, c = -1.55, d = -1.87, e = -4.32, f = 0.63, r_1 = 4, r_2 = 4, r_3 = 3.95, k_2 = 5$$

(a)

(b)

(c)

(d)

Fig. 7. (a) Plain image, (b) Histogram of plain image, (c) Cameraman image by proposed encryption system, (d) Histogram of Cameraman image by proposed encryption system.

4.1. Key Space Analysis

A good encryption algorithm should have large key space to prevent brute-force attacks which is defined to exhaust all the possible keys until the correct one. In the proposed method we increase the space key by using logistic map. So the number of parameters in the Clifford system increases from six to nine parameters. Therefore the size of the key space for the proposed system is larger than Clifford system.

4.2. Encryption and Histogram Test

Some experimental results are given in this section to demonstrate the efficiency of our scheme. The plain images are “Cameraman” and “Lena” with the size 256×256 . Their cipher images are shown in Fig. 4(c) and Fig. 5(c). The histogram of the cipher images is shown in in Fig. 4 (d) and Fig. 5(d). As we can see, the pixel distribution of the cipher images is fairly uniform, which can greatly reduce the correlation between the pixel values. Both kinds of pictures show that the picture is really well encrypted.

Fig. 8. (a) Plain image, (b) Histogram of plain image, (c) Lena image by proposed encryption system, (d) Histogram of Lena image by proposed encryption system.

4.3. Chi-square Test

The uniformity is justified by chi-square test, which is described by the following expression:

$$\chi^2 = \sum_{k=1}^{256} \frac{(O_k - E_k)^2}{E_k} \quad (5)$$

O_k = the observed frequencies of each gray level (0–255)

E_k = the expected frequencies of each gray level (0–255)

The following hypothesis is tested.

H_0 : The histogram of encrypted image has uniform distribution.

H_1 : The histogram of encrypted image has not uniform distribution.

Assume significant level of 0.05, $\chi^2(255, 0.05) = 291$. If $\chi^2_{test} < \chi^2(255, 0.05)$ then the null hypothesis is not rejected and the distribution of the encrypted histogram is uniform. Following table shows chi-square values of different cipher images.

Table 1. Chi-square Test for Cipher Images

image	size	Chi-square	
		χ^2_{test}	H_0
Monalisa	128×128	228.375000	Accepted
Elaine	225×225	221.597754	Accepted
Lena	256×256	262.940492	Accepted
Cameraman	256×256	250.085937	Accepted
Puppy	256×256	242.525729	Accepted
Clock	256×256	246.960937	Accepted
Moon surface	256×256	280.6328125	Accepted
Boat	512×512	252.0312500	Accepted
Mandrill	512×512	216.731558	Accepted
Barbara	512×512	211.583984	Accepted

4.4. Analysis for Correlation of Adjacent Pixels

Correlation analysis of adjacent pixels is an important way to test the gray value relationship between adjacent pixels in cipher image. Adjacent pixels in plain image have high correlation and in ciphered image should have low correlation. We calculate the correlation coefficient of the plain and encrypted image $r_{P,C}$ by using the following formulas [26],

$$E(P) = \frac{1}{M \cdot N} \sum_{i=1}^M \sum_{j=1}^N P(i, j), \quad E(C) = \frac{1}{M \cdot N} \sum_{i=1}^M \sum_{j=1}^N C(i, j) \quad (6)$$

$$D(P) = \frac{1}{M \cdot N} \sum_{i=1}^M \sum_{j=1}^N [P(i, j) - E(P)]^2 \quad (7)$$

$$\text{cov}(P, C) = \frac{1}{M \cdot N} \sum_{i=1}^M \sum_{j=1}^N [P(i, j) - E(P)][C(i, j) - E(C)] \quad (8)$$

$$r_{P,C} = \frac{\text{cov}(P, C)}{\sqrt{D(P)}\sqrt{D(C)}} \quad (9)$$

In these formulas $P(i, j)$ and $C(i, j)$ are gray values of the plain pixel and the encrypted one. $E(P)$, $E(C)$, $D(P)$ and $D(C)$ are mathematical expectation of plain pixels $P(i, j)$, mathematical expectation of cipher pixels $C(i, j)$, variance of plain pixels $P(i, j)$ and variance of cipher pixels $C(i, j)$, respectively. In order to test the correlation of adjacent pixels in the cipher image which is encrypted according to the proposed scheme, a test is designed. 2000 pixels are chosen randomly from Lena image, and their correlation

distribution of gray value is plotted.

Fig. 9. Distribution of correlation coefficients: (a) distribution of the horizontal correlation coefficients of adjacent pixels of plain image, (b) distribution of the horizontal correlation coefficients of adjacent pixels of cipher image, (c) distribution of the vertical correlation coefficients of adjacent pixels of plain image, (d) distribution of the vertical correlation coefficients of adjacent pixels of cipher image, (e) distribution of the diagonal correlation coefficients of adjacent pixels of plain, (f) distribution of the diagonal correlation coefficients of adjacent pixels of cipher image.

The correlation coefficients of the adjacent pixels in horizontal, vertical and in diagonal are listed in Table 2.

Table 2. Comparison of Correlation Coefficient of the Cipher Images in Three Algorithms

Method	Correlation		
	Horizontal	Vertical	Diagonal
Plain image	0.891095	0.937048	0.873472
Proposed method	-0.009264	-0.000945	-0.000284
Clifford system	0.029080	-0.003002	-0.000655
Ref.[27]	0.005336	-0.002761	0.001662
Ref.[13]	0.02046	0.01748	0.002317
Ref.[28]	0.0368	-0.0392	0.0068

4.5. Analysis for Correlation of Plain Image and Cipher Image

We analyzed the correlation between various pairs of plain image and cipher image. Table 3 shows the results of correlation coefficient between the plain image and corresponding cipher image.

Table 3. Comparison of Correlation Coefficient of the Plain and Cipher Images by Using Two Methods

image	Correlation coefficient	
	Proposed method	Clifford system
Lena	-0.000138	0.000809
Cameraman	-2.821731×10^{-5}	0.004672
Puppy	-6.908934×10^{-5}	0.002807
Mona_lisa	0.000700	-0.007508
Elaine	-0.002077	-0.002793
Moon surface	-0.004275	0.004224
Clock	-0.000867	0.007334
Boat	0.001827	0.003237
Mandrill	-7.285256×10^{-5}	0.000911
Barbara	-0.000140	0.001239

The correlation coefficient value ranges from -1 to $+1$. The correlation coefficients measured by using the proposed encryption system are very close to 0. Therefore, the cipher and plain images are significantly different.

4.6. Information Entropy Analysis

Image information entropy can measure the distribution of image gray values. Image entropy is defined as [29],

$$H = - \sum_{i=1}^{256} p_i \log p_i \quad (10)$$

where p_i represents the probability of occurrence of each pixel and $\log$ denotes the base 2 logarithm. The ideal value of the cipher information entropy is 8. Table 4 and Table 5 demonstrate that the entropy of cipher images is very close to the ideal value by using the proposed method.

Table 4. Comparison of Entropy Test for the Cipher Images

Image	Entropy		
	Plain image	Proposed method	Clifford system
Lena	7.651765	7.999233	7.998972
Cameraman	7.009716	7.997500	7.997452
puppy	7.453406	7.999414	7.999402
Mona_lisa	7.345221	7.997861	7.997836
Clock	6.705667	7.997284	7.997380
Moon surface	6.709312	7.996914	7.997010
Boat	7.191370	7.999385	7.999351
Mandrill	7.284855	7.999399	7.999411
Elaine	7.498110	7.998947	7.998888
Barbara	7.632119	7.999417	7.999282

Table 5. Comparison of the Cipher Image Entropy Using Five Algorithms

Image	Proposed	Clifford system	Ref.[27]	Ref.[13]	Ref.[28]
Lena	7.999233	7.998972	7.997116	7.9990	7.9880

Recently, the block entropy test was derived. In the following block entropy test [29], 100 non-overlapped blocks of the size 16×16 are randomly selected from each cipher image. The information entropy of each block is recorded via (10) and the average entropy is calculated.

Table 6. Entropy Test of Proposed Algorithm for Different Cipher Images

Image	Entropy test			
	Global entropy	Actual block entropy	Theoretical block entropy	
			$\alpha = 0.01$	$\alpha = 0.05$
			7.16276745	7.16634107
Lena	7.999233	7.18390012	Pass	Pass
Cameraman	7.997500	7.17484195	Pass	Pass
Puppy	7.999414	7.17699590	Pass	Pass
Mona_lisa	7.997861	7.17077829	Pass	Pass
Mandrill	7.999399	7.17270896	Pass	Pass
Clock	7.997284	7.17073254	Pass	Pass
Moon surface	7.996914	7.17263831	Pass	Pass
Boat	7.999305	7.17555463	Pass	Pass
Elaine	7.998947	7.18004111	Pass	Pass
Barbara	7.999417	7.17968347	Pass	Pass

4.7. Sensitivity Analysis

A good encryption algorithm should be sensitive to the plain image and the secret keys [30].

4.7.1. Plain image sensitivity

For calculation of NPCR and UACI, let us assume two cipher images C_1 and C_2 whose corresponding plain images have only one-pixel difference. NPCR and UACI are defined through the following formula as:

$$NPCR = \frac{\sum_i \sum_j D(i,j)}{T} \times 100 \quad (11)$$

$$UACI = \left[\sum_i \sum_j \frac{|C_1(i,j) - C_2(i,j)|}{LT} \right] \times 100 \quad (12)$$

The symbols T and L denote the number of pixels in the cipher image and the largest allowed pixel intensity, respectively. $D(i,j)$ is defined as:

$$D(i,j) = \begin{cases} 0 & C_1(i,j) = C_2(i,j) \\ 1 & C_1(i,j) \neq C_2(i,j) \end{cases} \quad (13)$$

Table 7 shows the real values of NPCR and UACI of cipher images by changing a pixel of the plain image.

Table 7. Comparison of NPCR and UACI of Cipher Images by Changing a Pixel of Plain Image

Image	method	NPCR%	UACI%
Lena	Proposed	99.626668	33.567139
	Clifford system	99.622414	33.472513
	Ref.[16]	99.60	99.66
	Ref.[25]	99.61	33.48

4.7.2. Key sensitivity

A 128×128 gray image CT scan of brain is encrypted by using $a = -1.85$, $b = 1.48$, $c = -1.55$, $d = -1.87$, $e = -4.32$, $f = 0.63$, $r_1 = 4$, $r_2 = 4$, $r_3 = 3.95$ as the first set of key. The key is changed slightly to be used to encrypt the same plain image. Slightly different kinds of key are listed below.

1. $a = -1.8500001$, $b = 1.48$, $c = -1.55$, $d = -1.87$, $e = -4.32$, $f = 0.63$, $r_1 = 4$, $r_2 = 4$, $r_3 = 3.95$
2. $a = -1.85$, $b = 1.4800001$, $c = -1.55$, $d = -1.87$, $e = -4.32$, $f = 0.63$, $r_1 = 4$, $r_2 = 4$, $r_3 = 3.95$
3. $a = -1.85$, $b = 1.48$, $c = -1.5500001$, $d = -1.87$, $e = -4.32$, $f = 0.63$, $r_1 = 4$, $r_2 = 4$, $r_3 = 3.95$
4. $a = -1.85$, $b = 1.48$, $c = -1.55$, $d = -1.8700001$, $e = -4.32$, $f = 0.63$, $r_1 = 4$, $r_2 = 4$, $r_3 = 3.95$
5. $a = -1.85$, $b = 1.48$, $c = -1.55$, $d = -1.87$, $e = -4.3199999$, $f = 0.63$, $r_1 = 4$, $r_2 = 4$, $r_3 = 3.95$
6. $a = -1.85$, $b = 1.48$, $c = -1.55$, $d = -1.87$, $e = -4.32$, $f = 0.6300001$, $r_1 = 4$, $r_2 = 4$, $r_3 = 3.95$
7. $a = -1.85$, $b = 1.48$, $c = -1.55$, $d = -1.87$, $e = -4.32$, $f = 0.63$, $r_1 = 4.0000001$, $r_2 = 4$, $r_3 = 3.95$
8. $a = -1.85$, $b = 1.48$, $c = -1.55$, $d = -1.87$, $e = -4.32$, $f = 0.63$, $r_1 = 4$, $r_2 = 3.9999999$, $r_3 = 3.95$
9. $a = -1.85$, $b = 1.48$, $c = -1.55$, $d = -1.87$, $e = -4.32$, $f = 0.63$, $r_1 = 4$, $r_2 = 4$, $r_3 = 3.9500001$

Table 8 shows the NPCR and UACI between the cipher images with the correct key and slightly different keys mentioned above. The cipher images with the correct and slightly different keys are compared pixel-by-pixel. This table shows that more than 99.60 % pixels in the cipher image change their gray rate when one of the keys just changes 10^{-6} . If a slightly modified key is used to decrypt the cipher image, the

decryption fails completely.

Table 8. NPCR and UACI between Cipher Image with Key $a = -1.85$, $b=1.48$, $c = -1.55$, $d = -1.87$, $e= -4.32$, $f = 0.63$, $r_1 = 4$, $r_2 = 4$, $r_3 = 3.95$ and Other Cipher Images with Slightly Different Keys

Kind of modified key	NPCR %	UACI%
(1)	99.664306	33.157169
(2)	99.627685	33.602558
(3)	99.603271	33.536951
(4)	99.609375	33.728769
(5)	99.664306	33.591356
(6)	99.676513	33.635636
(7)	99.609375	33.191253
(8)	99.633789	33.302504
(9)	99.688720	33.790785

Fig. 10. Key sensitivity analysis for encryption process for CT scan of brain Image (a) Plain Image (b) Cipher image with correct key (c) Cipher image by changing parameter b , $b=1.4800001$ (d) Cipher image by changing parameter $r_3 = 3.9500001$.

Comparison of NPCR and UACI for five cipher images by using proposed and Clifford systems is listed in Table 9.

Table 9. Comparison of NPCR and UACI between Cipher Image with Key $a = -1.85$, $b=1.48$, $c = -1.55$, $d = -1.87$, $e= -4.32$, $f = 0.63$, $r_1 = 4$, $r_2 = 4$, $r_3 = 3.95$ and Other Cipher Images by Changing Parameter b , $b=1.4800001$

Image	Proposed method		Clifford system	
	NPCR%	UACI%	NPCR %	UACI%
Lena	99.626668	33.567139	99.599711	33.402580
Puppy	99.611780	33.523781	99.623159	33.488289
Mandrill	99.620887	33.480585	99.606999	33.518280
Elaine	99.639835	33.498231	99.613497	33.452525
Barbara	99.613511	33.521040	99.607467	33.476234

4.7.3 Randomness test for NPCR and UACI

Recently, the NPCR and UACI randomness tests [31] were derived, and thus these scores can be used for the qualitative test as well. The critical NPCR score N_{α}^* associated with one-side hypothesis test under

the α level of significance is shown in (14), where the $\Phi^{-1}(\cdot)$ is the inverse CDF of the standard Normal distribution $N(0,1)$. If an actual NPCR score is above N_{α}^* , then the null hypothesis that the difference of cipher images from the slightly different plain images are random-like. In other words, the NPCR test is passed if the actual NPCR score is greater than N_{α}^* .

$$N_{\alpha}^* = \frac{L - \Phi^{-1}(\alpha) \sqrt{L/T}}{L+1} \quad (14)$$

In addition, the critical UACI scores associated with the two-side hypothesis test under α -level of significance are shown in (15). As a result, the UACI test is passed if the actual UACI score falls in the interval of $(U_{\alpha}^{*-}, U_{\alpha}^{*+})$.

$$\begin{cases} U_{\alpha}^{*-} = \mu_U - \Phi^{-1}(\alpha/2) \sigma_U \\ U_{\alpha}^{*+} = \mu_U + \Phi^{-1}(\alpha/2) \sigma_U \end{cases}, \quad \mu_U = \frac{L+2}{3L+3}, \quad \sigma_U^2 = \frac{(L+2)(L^2+2L+3)}{18(L+1)^2 LT} \quad (15)$$

The results of NPCR and UACI scores from the proposed image encryption method are shown in Table 10.

Table 10. NPCR and UACI Tests for Cipher Images

Image name	NPCR scores%			UACI scores%		
	Actual NPCR	Theoretical NPCR		Actual UACI	Theoretical UACI	
		$N_{0.01}^*$ 99.5810	$N_{0.05}^*$ 99.5893		$U_{0.01}^{*-} / U_{0.01}^{*+}$ 33.3445/33.5826	$U_{0.05}^{*-} / U_{0.05}^{*+}$ 33.3730/33.5541
Lena 512×512	99.803924	Pass	Pass	25.048828	Fail	Fail
Barbara 512×512	99.803924	Pass	Pass	33.384517	Pass	Pass
Boat 512×512	99.587991	Pass	Fail	33.412356	Pass	Pass

4.8. MSE, MAE and PSNR

The difference between the plain image and cipher image can be measured with mean square error (MSE) and mean absolute error (MAE) criteria. MSE and MAE values are stated in (15) and (16), respectively.

$$MSE = \frac{\sum_i \sum_j (P(i,j) - C(i,j))^2}{T} \times 100 \quad (16)$$

$$MAE = \frac{\sum_i \sum_j |P(i,j) - C(i,j)|}{T} \times 100 \quad (17)$$

The symbol T denotes the number of pixels in the cipher image. The larger value of MAE is better for the image security. Peak signal-to-noise ratio (PSNR) is the ratio between the plain image and the cipher image. The PSNR is defined as below:

$$PSNR = 10 \log_{10} \left(\frac{MAX_I^2}{MSE} \right) \quad (18)$$

where MAX_I is the maximum possible pixel value of the plain image. The higher PSNR, the closer the encrypted image is to the original. In general, a higher PSNR value should correlate to a higher quality image. In good encryption, scheme PSNR should be as low as possible. MSE, MAE and PSNR values of the cipher image are reported in Table 11.

Table 11. The Results of MSE, MAE and PSNR Tests

Image	MSE	MAE	PSNR
Clock	12228.46	59.013153	7.257085
Airplane	10915.78	66.1103363	7.750255
Moon surface	6252.83	67.188827	10.170032
Woman	11973.73	89.400894	7.348506
Peppers	8313.78	75.225265	8.932815

From Table 11, the results of MSE and MAE tests are high values which can ensure the resistance of the algorithm against any differential attacks.

5. Conclusion

The reported paper aimed at developing a secure algorithm for image encryption. The first step of implementation is image scrambling that the position of the pixels in the plain image is changed by using Logistic map and Clifford attractor. After quantization step, positions and modification value are gained.

A computer simulation is used to evaluate and compare the encrypted images by the proposed algorithm with other methods. The results of computer simulation show good results in the proposed method by using the nature of Chaos mechanism to achieve the improvements of the Clifford system. These comparisons were based on chi-square test of histogram, correlation coefficients of pixels, the number of pixel change rate (NPCR), entropy test, unified average changing intensity (UACI), mean square error (MSE), mean absolute error (MAE), peak signal-to-noise ratio (PSNR), key space, and sensitivity analysis.

In ideal condition, there should not be any relation between the pixels of the encrypted image. As shown in Fig. 9, Table 2 and Table 3, the absolute value of correlation coefficient of the proposed system is less than image encryption algorithms in [28], [10], [13] and [27]. Furthermore, the block entropy test also supports that the proposed method is good for image encryption. The results of statistical tests provided sufficient evidence for the superiority of the proposed chaotic image encryption system over other algorithms.

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